Probabilistic Analysis of Responses of Tunnel Under the Surcharge Considering Soil Vertical Spatial Variability

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Abstract. Surface surcharge is regarded to be one of the main factors that will cause large deformation of tunnel. On the other hand, it is widely accepted that the soil exhibits significant spatial variability that consequently causes the large variation of structural performance of embedded tunnels. This paper aims to investigate the influence of soil vertical spatial variability on tunnel subjected to the surface surcharge. Herein, the random finite difference method (RFDM) is used. Random fields of elastic modulus E_s of soils are generated and mapped into finite difference analysis to reveal the impact of spatial variability. Given the modeling specifics mentioned above, some results of the numerical simulations are found: (1) the spatial variability may be underestimated if the discretized points in simulating the vertical random field are too coarse; (2) there exists two critical scale of fluctuation: 15 m and 10 m for COV of E_s at 0.15 and 0.35 when evaluating the crown settlement. The estimated value will be high if neglecting spatial variability; (3) the run number for Monte Carlo simulation (MCS) also plays an important role; a converged run number means that the COV of generated data is not sensitive to the run number. In this study, it is about 300 in this sense; (4) the different combination of scale of fluctuation and limiting value will lead to wide range difference when evaluating the probability of exceedance and reliability index. Therefore, it is necessary to consider the spatial variability on analyzing the effect of surface surcharge on tunnel.

Keywords: Spatial variability · Random field · Surcharge · Crown settlement

1 Introduction

With rapid urbanization, more tunnels are being constructed in highly congested areas. Tunnels under such conditions are bound to be affected by other engineering activities. The excavation-induced ground deformation in homogenous soil has been analyzed during the past decades [4, 6]. This topic was also investigated considering surcharge loading [5], but they are all purely deterministic.

Homogeneous soil will ignore the spatial variability of soil parameters, so that the results are only the state of the mean. The average level of results may miss the true failure mechanisms and ignore the weakest part of soils in the sense of randomness of soil properties. Therefore, it is necessary to consider the soil spatial variability on probabilistic analysis. The spatial variability is often modeled by random field theory.

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Vanmarcke has been discussed the effect of soil spatial variability on geotechnical systems using random field method [1]. Huang et al. showed that tunnel differential settlement is significantly affected by the variation and scale of fluctuation of soil in longitudinal direction [2]. Nevertheless, few previous researches have been devoted to the effect of soil spatial variability on tunnel considering surface surcharge.

This paper is organized as follows. First, the FDM for modeling shield tunnel is presented. Second, the RFDM is introduced to simulate the spatial variation of soil properties of the ground under the tunnel. Third, several cases are implemented to demonstrate how the tunnel responses, such as crown settlement, probability of exceedance and reliability index are affected by the spatial variation of soil properties.

2 Finite Difference Method for Modeling Shield Tunnel

In this study, finite difference analyses are performed using the FLAC3D software. The numerical model considered in the present work is shown in Fig. 1. The plane-strain condition is assumed for this finite difference method analysis. A continuous loading is applied to the surface of the domain, with smooth interface conditions, in order to determine the surcharge load *F*.

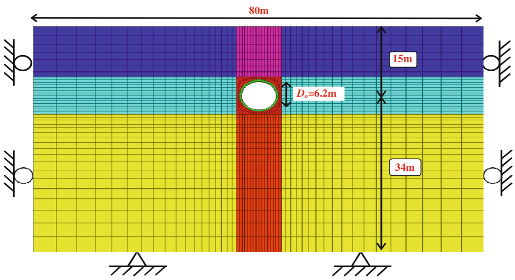


Fig. 1. Geometry of the finite difference model of shield tunnel.

The boundary conditions are shown in Fig. 1 in all case scenarios. The bottom boundary is fixed while the vertical boundaries are fixed in horizontal direction. The soil is treated as elastic-plastic materials with a Mohr–Coulomb failure criterion.

The tunnel is modeled as elastic homogeneous ring. There is no joint because of the use of the shell element, so we use the reduction coefficient in this paper, the reduction coefficient is 0.7. Details of input parameters are listed in Table 1. There are 2164 soil zones and 4452 grid points. The sand layer is defined into 1 layer for the ease of assigning input soil parameters in the RFDM. Figure 2 shows the profiles of simulated using finite difference analysis.

Table 1. Soil and tunnel parameters adopted in finite difference modeling.

Parameter	Notation	Value	Unit
Elastic modulus (soil/tunnel)	E_s/E_t	30/34500	MPa
Cohesion	c	4	kPa
Friction	φ	30.5	°
Density (soil/tunnel)	ρ_s/ρ_t	1800/2450	kg/m^3
Poisson's ratio (soil/tunnel)	v_s/v_t	0.31/0.2	—
Surcharge load	F	30	kN
Outside (internal) diameter	D_o (D_i)	6.2(5.5)	m
Depth of tunnel	H	12	m
Thickness of tunnel	t	0.35	m

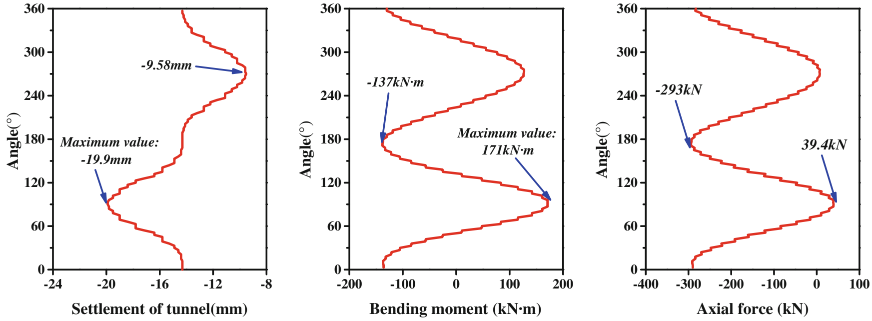


Fig. 2. Profiles of simulated using finite difference analysis

3 Spatial Variability of Soil Property

There is a lot of uncertainty in the construction of tunnel engineering. Traditional design method usually adopts single safety factor to consider many uncertainty factors. It is fails to consider the effect of spatial variability on engineering safety risk.

3.1 Modeling of Spatial Variability

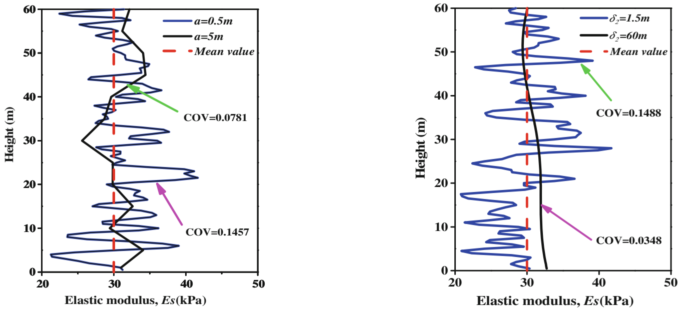
Scale of fluctuation is an important concept of geotechnical parameters in the random field modeling. It can well reflect the spatial variability of the soil. In this study, the correlation matrix is built with the Gaussian autocorrelation function:

$$\rho(\tau_1, \tau_2) = exp\left[-\pi\left(\frac{\tau_1^2}{\delta_1^2} + \frac{\tau_2^2}{\delta_2^2}\right)\right] \tag{1}$$

Where τ_1 and τ_2 are horizontal and vertical distances between two points, respectively, δ_1 and δ_2 are correlation distances in horizontal and vertical direction, and

$\rho(\tau_1,\tau_2)$ is the correlation coefficient between two points. The Karhunen-Loeve expansion technique is used to discretize the random field.

As in Fig. 3(a), the COV of the generated date of a = 5 m (a = 0.5 m) is 0.0781 (0.1488), which means the spatial variability of the soil may be greatly underestimated if the discretized points in simulating the vertical random field is too coarse. As in Fig. 3(b), a smaller scale of fluctuation leads to relatively more drastically variation of E_s in the random field, indicating a high level of spatial variability.



(a) δ_1=10m and δ_2=1.5m (b) δ_1=10m and a=0.5m

Fig. 3. Example of simulated spatial variability of E_s of random field modeling

3.2 Evaluation of Exceedance Probability and Reliability Index

In this study, we think the tunnel is no serviceability when the result of index (S) (such as crown settlement) exceeds the limiting value (S_{lim}). In order to examine the spatial effect statistically, MCS needs to be adopted. As will be seen later in this study, the crown settlement may be approximated by a lognormal distribution. The probability of exceedance and reliability index are as follows:

$$P_e = P(S > S_{lim}) = P(lnS > lnS_{lim}) = 1 - \Phi\left(\frac{lnS_{lim} - \mu_{lnS}}{\sigma_{lnS}}\right) = \Phi\left(\frac{\mu_{lnS} - lnS_{lim}}{\sigma_{lnS}}\right) \tag{2}$$

$$\beta = \frac{lnS_{lim} - \mu_{lnS}}{\sigma_{lnS}} \tag{3}$$

Where μ_{lnS} and $\sigma_{lnS_{lim}}$ are mean and standard deviation of ln_S.

4 Effect of Spatial Variability on Shield Tunnel

4.1 Random Field Modeling of Elastic Modulus

In this study, only the elastic modulus E_s is considered to be a spatially random property. Random fields of soil E_s are generated and mapped into finite difference analysis. The COV of E_s and δ_2 have many different combinations. The COV has two choices: 0.15 and 0.35. The δ_2 has six choices: 1.5, 5, 10, 15, 20 and 60 m.

Figure 4 shows an example of the simulated distribution of crown settlement for E_s with COV = 0.15 and δ_2 = 1.5 m. It shows that the empirical cumulated distribution function (CDF) and the CDF of the lognormal random variable are very close, indicating the crown settlement in such a case may be approximated by a lognormal distribution.

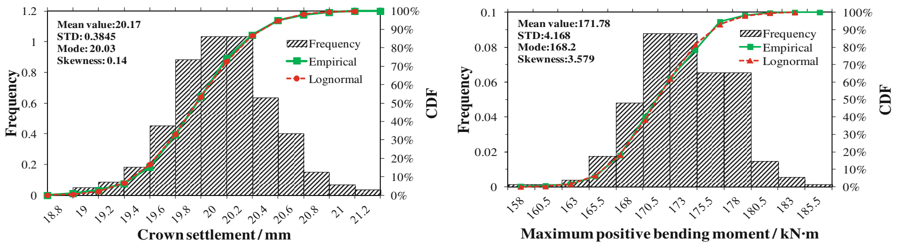


Fig. 4. Histogram of crown settlement and maximum positive bending moment

4.2 Effect of Number of Monte Carlo Simulation Runs

Generally speaking, the number of MCS has a great effect on estimated P_e. A small probability event may not happen if the number is too small. On the contrary, the computational efficiency would be lower if number is too large. Hence, we should find a suitable number to meet our demands and improve the efficiency of calculation (Table 2).

Table 2. Effect of number of runs on predicted value of crown settlement for COV of *Es* = 0.15.

Scale of fluctuation (δ_2/m)	Number of MCS	Predicted value of crown settlement (mm)	
		Mean	COV
1.5	60	11.9172	0.01771
	150	11.9679	0.01634
	300	11.9022	0.01523
	600	11.9356	0.01518
	900	11.9414	0.01487
	1200	11.9361	0.01485

The effect of number of MCS is shown in Fig. 5. It is observed that the COV of crown settlement decreases with the increase of the number of MCS. Further, there is an obvious turn point when number at 300. When number is smaller, the COV is higher relatively. The contribution of decreasing variation is tiny by increasing the number of MCS when it is larger than 300. Hence, we can adopt the 300 MCS in this study.

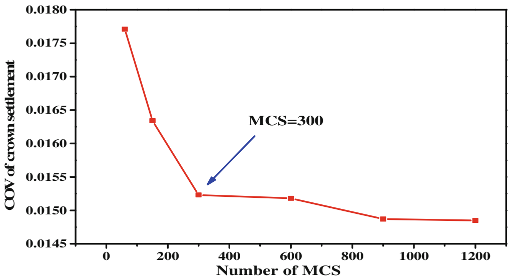


Fig. 5. Effect of number of MCS on the estimated COV of crown settlement

4.3 Effect of Spatial Variability on the Crown Settlement

In this study, the crown settlement is examined, since it acts as the key design parameters for the safety of tunnel. Through FDM analysis of the generated 300 realizations, simulations for various combinations of COV and δ_2 are implemented and the predicted crown settlement is summarized and analyzed statistically.

Generally, it can be noticed from Fig. 6 that the mean value and COV of crown settlement become larger as the δ_2 increase. In Fig. 6(a), when the COV is 0.15, the mean value slightly increases with δ_2 increases, and this trend levels off after δ_2 exceeds 15 m. When the COV is 0.35, the amplitude of increase is larger than the COV is 0.15; meanwhile, the turn point is 10 m. The result is slightly greater than the deterministic analysis result (19.9 mm) when considering the spatial variability. In Fig. 6(b), when the COV of E_s is 0.35, the COV value is significantly greater than 0.15. It is also clearly observed that the turn point of δ_2 is basically same when the COV of E_s is same, and meanwhile, the larger COV of soil E_s, the smaller value of the turn point.

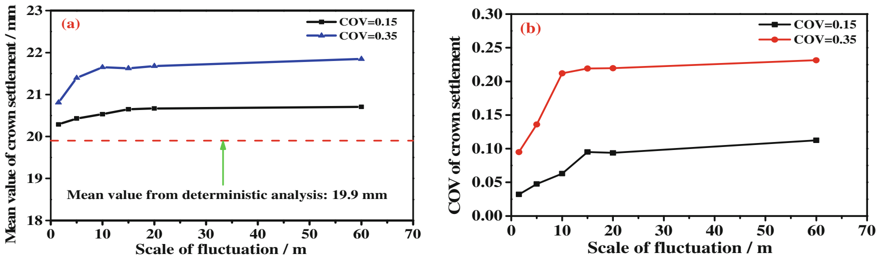


Fig. 6. Effect of spatial variability on crown settlement: (a) Mean value; (b) COV

In summary, the predicted mean value will be overestimated if the spatial variability is not considered. This conclusion is consistent with the findings by the previous investigators in their research on random field modeling [3, 4]. In reference to Figs. 6, there exists two critical δ_2: 15 m and 10 m for COV of E_s at 0.15 and 0.35, respectively.

4.4 Probability of Exceedance and Reliability Index of Crown Settlement

In order to calculate the P_e, the key point is the limiting value (S_{lim}). Using the data generated by MCS, the P_e and β can be estimated with Eqs. (2) and (3), respectively. The calculated results for various levels of S_{lim} are shown in Figs. 6 and 7.

As shown in Fig. 7, it is observed that P_e decreases with the increase of the specified S_{lim}. Comparing Fig. 7, we can find that there is an intersection. When COV = 0.15, the critical value of S_{lim} is about 20 mm; the value is about 21 mm while COV at 0.35.

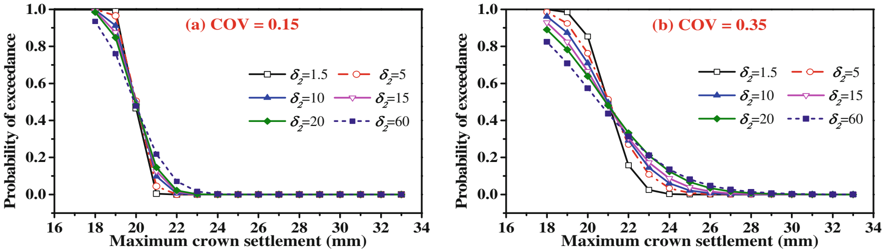


Fig. 7. The probability of exceedance with respect to various maximum crown settlement

It should also be observed in Fig. 7(a) that, for S_{lim}< 20 mm, the P_e of δ_2 = 1.5 is large. When S_{lim}> 20 mm, the P_e of δ_2 = 1.5 is small. For higher value of δ_2, the relationship between S_{lim} and P_e is quite insensitive. On the other hand, we can also see that the P_e is about 0 when S_{lim}> 24 mm in Fig. 7(a). The critical value is 30 mm in Fig. 7(b).

Figure 8 shows the effect of spatial variability on β. When the S_{lim}< the critical value, the β of smaller δ_2 is small; when the S_{lim}> the critical value, the β of smaller δ_2

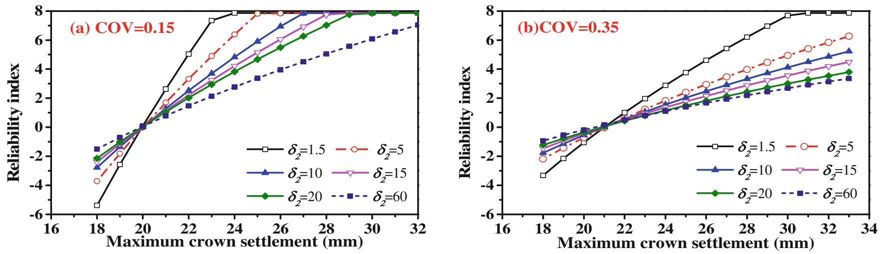


Fig. 8. The reliability index with respect to various maximum crown settlement

is large. Meanwhile, the effect is also related to the selection of limiting value. Therefore, considering the spatial variability will make the results more reasonable when evaluating the P_e and β of tunnel.

5 Concluding Remarks

Based on the research results presented, the following conclusions can be drawn:

(1) The spatial variability may be underestimated if discretized points in simulating vertical random field are too coarse. Meanwhile, a small scale of fluctuation leads to dramatically variation of E_s in random field, indicating large spatial variability.
(2) The number of MCS also plays an important role in this study, a suitable number of MCS means that the COV of the generated data changes little when you increase the number of MCS. In this study, this run number is about 300 in this sense.
(3) Neglecting spatial variability can result in larger uncertainty in the estimated statistics; the predicted value will be overestimated, especially when the variation of soil parameter is relatively high. There exists two critical scale of fluctuation: 15 m and 10 m for COV of E_s at 0.15 and 0.35 when evaluating the crown settlement.
(4) The spatial soil variability has an important impact on P_e and β. Meanwhile, the effect is also related to the selection of limiting value. The different combination of scale of fluctuation and limiting value will lead to wide range difference when evaluating the P_e and β. Therefore, considering the spatial variability will make the results more reasonable when evaluating the P_e and β of the tunnel.

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