



Static Fatigue or Maturing of Contacts in Silica Sand

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Abstract. Silica sands are known to exhibit a time-dependent response to applied loads, particularly after they were disturbed, for example, due to compaction. This behavior was documented by a time-dependent increase in shear wave velocity in sand subjected to sustained loads. The change of material properties with increasing time is often referred to as sand *aging*. While several hypotheses have been proposed to explain the aging process, none has been generally accepted by the research community. The hypothesis advocated in this paper is that static fatigue at contacts between the grains may be a key factor in time-dependent behavior of silica sand. An apparatus was constructed to load individual sand grains, and the time-dependent deflection under sustained load was monitored. The rate of deflection was found dependent chiefly on the surface texture of the grains (roughness), with rougher surfaces at contacts being more susceptible to larger deflection. The process of static fatigue occurring at the contacts is also referred to in this presentation as *contact maturing*. The results of grain scale testing in the custom-constructed apparatus are consistent with the hypothesis, which implies that contact maturing is a plausible contributor to aging of silica sand.

Keywords: Static fatigue · Sand aging · Grain-scale testing · Surface texture

1 Introduction

Disturbed silica sands are known to exhibit time-dependent response when subjected to sustained loads. Afifi and Woods (1971) found the speed of shear wave in sand to increase in time under constant loads, indicating increasing stiffness. A delayed increase in cone penetration resistance in sand after compaction through vibrations or blasting was documented by Mitchell and Solymar (1984). Another example of time-dependent response of disturbed sand is in the *setup* of displacement piles. The capacity of the shaft resistance may double in the first three months after the pile is installed (Chow et al. 1997). While several hypotheses have been proposed in the past to elucidate these phenomena (Schmertmann 1991, Mesri et al. 1990, Lade and

Karimpour 2010), new test results indicate that static fatigue at contacts between grains (or *contact maturing*) may play a key role in the evolution of the stiffness of sand.

Static fatigue will be briefly discussed and experimental results will be shown consistent with the hypothesis indicating static fatigue as the key cause of sand *aging*. Some effort toward simulations of this phenomenon will also be presented.

2 Static Fatigue at Contacts

Examples of texture of silica grain surfaces are presented in Fig. 1. When two grains with a rich surface texture come into contact and become loaded, the response is fracturing and crushing the textural features on the surfaces in contact.

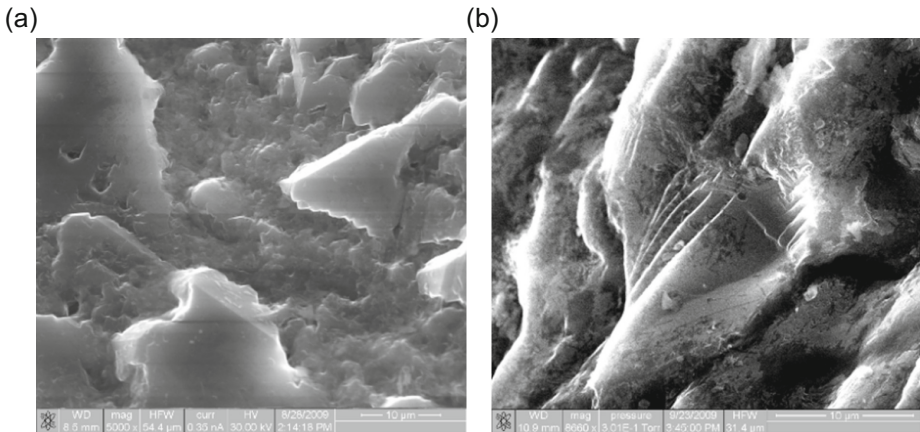


Fig. 1. Silica sand surface texture at micron scale; see 10 µm bar at lower-right of the images: (a) Lake Michigan Empire Beach Sand, and (b) Ottawa 20–30 Sand

A hypothesis was suggested by Michalowski and Nadukuru (2012) that time-dependent fracturing of the microscopic features at contacts (asperities, crystalline fragments) is a key contributor to time-dependent changes of properties in sand subjected to sustained loads, often referred to as *aging*.

The underpinning of this concept is in the rate process theory applied to fracture kinetics (Bažant and Pang 2007), and it has its origins in the Svante Arrhenius equation for the rate constant of chemical reactions.

It has been documented that the process of sand aging leads to an increase in the macroscopic stiffness of sand (Afifi and Woods 1971), which is beneficial when considering engineering properties. In order to avoid the pejorative connotation associated with the term *fatigue*, we will refer to the process more often as *contact maturing*, rather than static fatigue.

Experimental investigation of the response of contacts between silica sand grains was carried out using a custom-designed apparatus.

3 Testing for Contact Maturing

3.1 Apparatus

Reports on grain-scale testing and modeling are available in the literature, but they are focused predominantly on the immediate response of grains to loads, fracture, and crushability of grains (e.g., Cole and Peters 2008, Cavarretta et al. 2011, Senetakis and Coop 2013, Einav 2007). The apparatus used in this research was custom-designed to monitor the long-term (weeks) behavior of grains under low to moderate (several Newtons) sustained loads.

Several generations of the device for testing time-dependent response of contacts between silica grains were constructed (Wang and Michalowski 2015) before the final designed used in this research was completed. A photographic view of the apparatus is presented in Fig. 2(a).

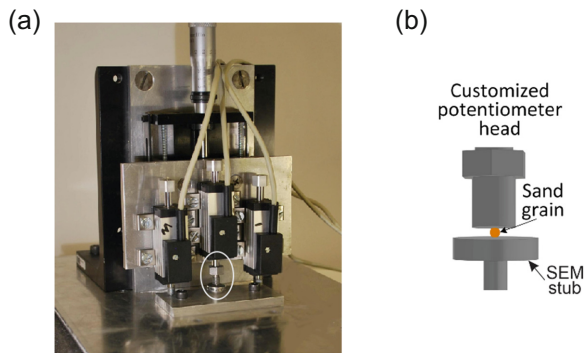


Fig. 2. (a) Photographic image of the device constructed for testing sand contacts, and (b) loading detail with customized head of the central potentiometer

In order to avoid uncertainties associated with creep of glue used for mounting grains on opposite platens, this apparatus measures the simultaneous response of two contacts on one grain with two relatively smooth loading platens, thus avoiding using glue to mount the grains. The fundamental component of the apparatus is the central potentiometer, which applies the required force to the grain and measures the time-dependent *convergence* of the two platens (or deflection of the grain).

3.2 Sand Grain Texture

Contacts between grains are not Hertzian-type interfaces; rather, they are characterized by irregular asperities and debris. An example of a contact between two grains is shown in Fig. 3(a), and an Atomic Force Microscopy scan of a grain surface is illustrated in Fig. 3(b).

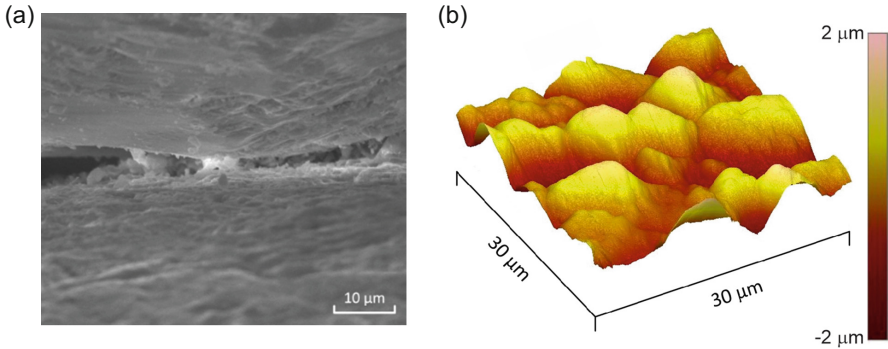


Fig. 3. (a) Contact between two grains, and (b) Atomic Force Microscopy scan of Ottawa 20–30 sand grain surface

The richness of the texture is expected to play a significant role in the process of static fatigue or contact maturing. The texture can be characterized with the root mean square of the surface elevation, defined as

$$RMS = \sqrt{\frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (z_{ij} - \mu)^2} \quad (1)$$

where μ is the average elevation, z_{ij} is an elevation identified by the i^{th} and j^{th} point in the x and y direction of the scan, and m and n are the number of scanned points in the x and y directions, respectively. The scan in Fig. 3(b) represents the roughness expressed by $RMS = 577$ nm.

3.3 Results

A series of tests was carried out on Ottawa 20–30 grains of various roughness, under various loads. The outcome was found to be affected by relative humidity and temperature; therefore, all tests were performed in an environmental chamber with controlled relative humidity ($30 \pm 5\%$) and temperature ($20 \pm 1^\circ\text{C}$). We present selected results from a larger program of testing, discussed in Michalowski et al. (2017). The noise in the signal presented in Fig. 4, was filtered using a moving average with a five-point span, which reduced the noise band from over 100 nm to about 50 nm.

It is apparent that the initial roughness (expressed as the root mean square of the surface elevation) has a key influence on the time-dependent process. The richer the texture the more intense the contact maturing in the early stages of the process (larger rate of convergence). The deflection of the grain that was the smoothest ($RMS = 28.6$ nm) appears to increase linearly with time. This is probably caused by the response of the core material of the grain (creep) with lesser contribution of the contacts. It is interesting to notice that, after about 18 days, the rate of convergence dropped below 1 nm/h for all tests, regardless of the initial roughness and the load.

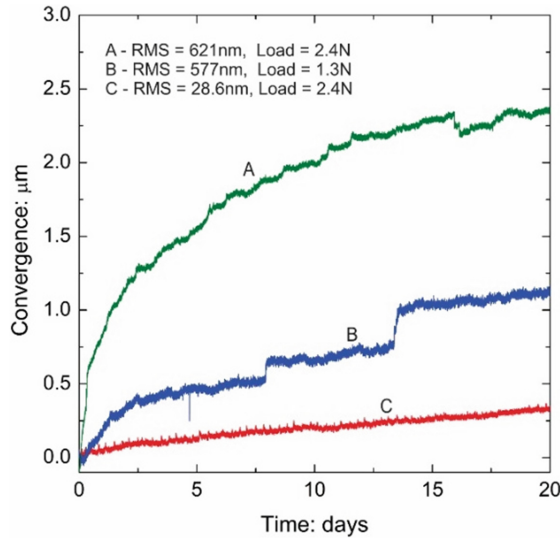


Fig. 4. Deflection of a single grain (convergence of loading platens) from three tests on grains with representative initial roughness expressed by the root mean square of the average elevation of 621 nm, 577 nm, and 28.6 nm (data from Michalowski et al. 2017)

This small rate of convergence is likely owed predominantly to creep of the core material of the grains and less to the contact maturing process.

All tests were performed for air-dry contacts. Chemistry of the environment has influence on the process of contact maturing, thus also on the response of contact to loads (Krauskopf 1959, Hu and Hueckel 2007), but these effects have not been investigated.

4 Simulation of Time-dependent Contact Behavior

Simulations of time-dependent behavior of contacts have been attempted using the distinct element method. An individual grain is constructed of sub-particles bonded together by parallel bonds subjected to stress corrosion cracking (static fatigue). The bonds can transmit forces and moments, including torsion. Should tension be induced in the bond, the stress corrosion process will cause gradual reduction of the size of the bond \bar{D} (Potyondy 2007), possibly leading to the loss of the bond, interpreted as a crack initiation. The rate of bond size reduction (bond width in 2D or bond diameter in 3D simulations) is defined by constants β_1 and β_2 in the model

$$\frac{d\bar{D}}{dt} = \begin{cases} 0 & \text{if } \bar{\sigma} < \bar{\sigma}_a \\ -\beta_1 \exp\left\{\beta_2 \frac{\bar{\sigma}}{\bar{\sigma}_c}\right\} & \text{if } \bar{\sigma}_a < \bar{\sigma} < \bar{\sigma}_c \\ -\infty & \text{if } \bar{\sigma} > \bar{\sigma}_c \end{cases} \quad (2)$$

where $\bar{\sigma}$ is the maximum tensile stress acting on the parallel-bond, $\bar{\sigma}_a$ is the threshold stress below which the stress-corrosion ceases, and $\bar{\sigma}_C$ is the parallel-bond tensile strength.

For the purpose of the demonstration, a 2D simulation of two half-grains in contact is presented in Fig. 5. The number of sub-particles used in the simulation was 23,190, and the simulated grain width was 0.6 mm. The two grains were loaded with a force of 12 N. The threshold tensile stress $\bar{\sigma}_a$ was taken as 70 MPa, and the damage-rate constants β_1 and β_2 , Eq. (2), were selected as $5 \cdot 10^{-17}$ m/s, and 30 (dimensionless), respectively (the remaining parameters, characteristic in DEM modeling can be found in Wang 2016).

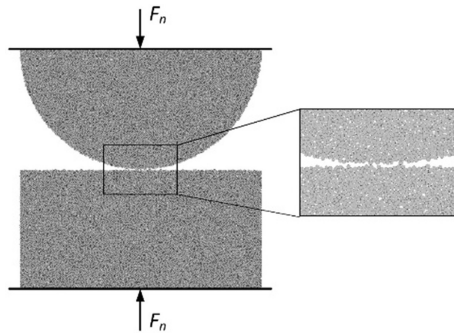


Fig. 5. Simulated interaction of two grains

The outcome of the simulation is illustrated in Fig. 6. Once the load of 12 N was applied to the half-grains (at time $t = 0$), the interactive forces were transmitted through three micro-force chains, while 27 micro-cracks (collapse of inter-particle bonds) were recorded, Fig. 6(a). Within five hours, a distinct macro-crack formed as a result of coalescence of micro-cracks. At that time the number of micro-cracks in the neighborhood of the contact increased to 260, and the number of force chains intersecting the contact doubled (Fig. 6(b)). The two half-grains fractured after 2.81 days, with 683 micro-cracks and 12 micro-force chains across the contact.

It is plausible that an increase in the small-strain stiffness observed in sand during aging is a result of evolution of the contacts between the grains, demonstrated in the simulation as an increase in the number of force chains intersecting the contact (increase in the number of “contact points” within a single nominal contact). We have referred to those as micro-force chains, to distinguish those from the typical use of the term (one force chain per one nominal contact).

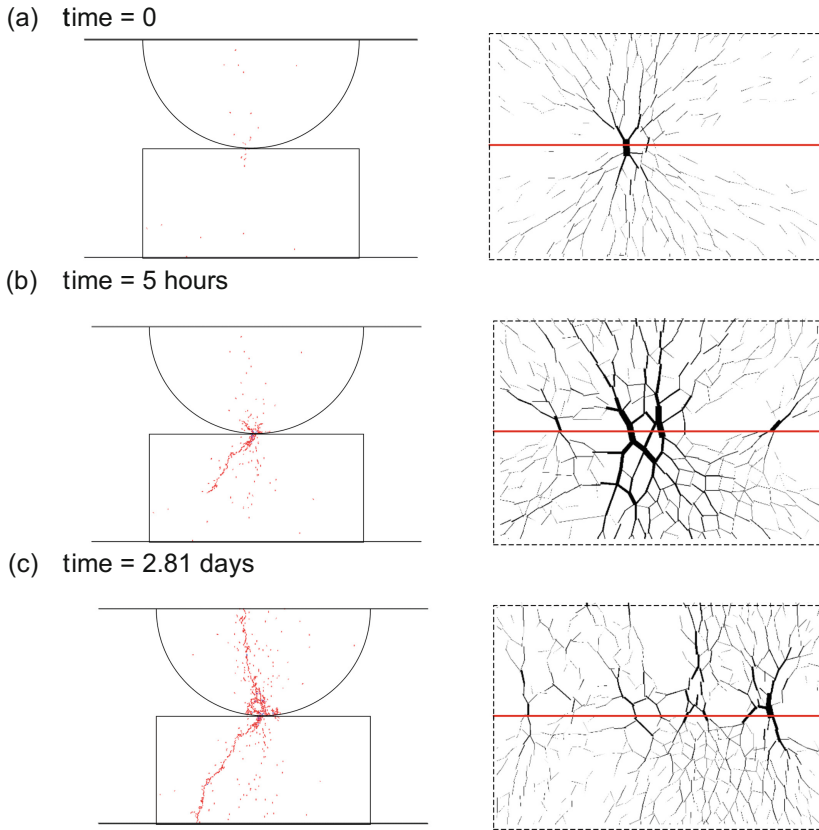


Fig. 6. Evolution of micro-cracks and micro-force chains within one nominal contact: (a) $t = 0$, 27 micro-cracks, 3 sub-contacts, (b) $t = 5$ h, 260 micro-cracks, 6 sub-contacts, and (c) $t = 2.81$ days, 683 micro-cracks, 12 sub-contacts

5 Conclusions

Testing of contacts under constant loads of about 2 N yields a time-dependent response where, at first, the deflection at contacts increases at a relatively high rate to drop to about 1 nm/h or less, after about 18 days. This process has been suggested as the cause of aging of silica sands. While more experimental evidence is needed, grain-scale testing carried out so far indicates a rather realistic plausibility of the contact maturing hypothesis. The process was captured in preliminary simulations using distinct element method.

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