A Constitutive Model for Unsaturated Soils Using Degree of Capillary Saturation and Effective Interparticle Stress as Constitutive Variables

Annan Zhou(✉), Shengshen Wu, and Jie Li

Royal Melbourne Institute of Technology (RMIT), Melbourne 3000, Australia
annan.zhou@rmit.edu.au

Abstract. This paper discusses the role of the degree of capillary saturation in modelling the coupled hydro-mechanical behaviour of unsaturated soils and proposes a new constitutive model for unsaturated soils by using the degree of capillary saturation and the effective inter-particle stress. In the model, the shear strength, yield stress and deformation behaviour of unsaturated soils are governed directly by the above two constitutive variables. The model is then validated against a variety of experimental data in the literature, and the results show that a reasonable agreement can be obtained using this new constitutive model.

Keywords: Unsaturated soil · Degree of capillary saturation
Constitutive model

1 Introduction

The objective of this study is to establish a constitutive model for unsaturated soils by acknowledging that (i) pore water consists of capillary water and adsorbed water, and (ii) they contribute very differently to the constitutive behaviour of unsaturated soils. Because the capillary water exists among soil particles and the pressure of capillary water affects the contact stress among soil particles. Therefore, the stress carried by the capillary water is classified as an inter-particle stress. Compared with the capillary water, the contribution from the adsorbed water to the shear strength and deformation of a soil is very limited [1–3]. In principle, this is because adsorbed water wraps the surface of each soil particle. Thus, the stress carried by the adsorbed water is reasonably treated as an intra-particle stress.

One of the most widely-used constitutive variables for unsaturated soil modelling is the Bishop effective stress [4]. The formulation of σ'_{ij} can be written as follows

$$\sigma'_{ij} = \sigma_{ij} + \chi s \delta_{ij} = \bar{\sigma}_{ij} - u_{\mathrm{a}} + \chi(u_{\mathrm{a}} - u_{\mathrm{w}})\delta_{ij} \quad (1)$$

where σ_{ij} is the net stress with $\sigma_{ij} = \bar{\sigma}_{ij} - u_{\mathrm{a}}$, $\bar{\sigma}_{ij}$ is the total stress, u_{a} is the pore air pressure, χ is the effective stress parameter, s is the suction ($s = u_{\mathrm{a}} - u_{\mathrm{w}}$), u_{w} is the pore water pressure and δ_{ij} is the Kronecker delta. In this study, the degree of capillary

L. Hu et al. (Eds.): GSIC 2018, *Proceedings of GeoShanghai 2018 International Conference: Multi-physics Processes in Soil Mechanics and Advances in Geotechnical Testing*, pp. 79–86, 2018.
https://doi.org/10.1007/978-981-13-0095-0_9

saturation (S') is selected as the basic constitutive variable to highlight that only the capillary water affects the strength and deformation of unsaturated soils. Specifically, on one aspect, the degree of capillary saturation is used for the effective stress parameter, i.e., $\chi = S'$, and the effective stress when $\chi = S'$ is referred to as the effective inter-particle stress (σ'_{ij}) to emphasize that the intra-particle stress associated with the adsorbed water pressure has been ruled out. On the other aspect, the slope of the NCLs is also a function of the degree of capillary saturation, i.e., $\lambda = \lambda(S')$ to underline that the mechanical state of an unsaturated soil is related to the capillary water only. It is important to note that, although the constitutive relationship is initially established in the space of $\left\{\sigma'_{ij}, S'\right\}$, it can be generalised in the space of primary variables $\{\sigma_{ij}, s, S\}$ that is in accordance with variables adopted for finite element methods [5–7].

2 Constitutive Equations

2.1 Hydraulic Equations

Sharing the theoretical concept delivered by Or and Tuller [8] and practical method by Khlosi et al. [9], a simple equation of water retention curve was proposed to consider capillarity and adsorption separately [1], which can be written as follows:

$$S = S' + S'' \tag{2}$$

where S is the degree of saturation; S' and S'' are the capillary component and adsorptive component of the degree of saturation respectively. Following Khlosi et al. [9], the two-parameter equation proposed by Kosugi [10] to quantify the capillary component (i.e., capillary water retention curve, CWRC) was employed here:

$$S' = (1 - S'')C(s), \text{ and } C(s) = \frac{1}{2}\operatorname{erfc}\left((\sqrt{2}\zeta)^{-1}\ln(s/s_{\mathrm{m}})\right) \tag{3}$$

where erfc() the complementary error function, s_{m} the suction that corresponds to the median pore radius (r_{m}), and ζ^2 the variance of the log-transformed pore radius. s_{m} varies from s_{mR} corresponding to the main drying branch to s_{mA} corresponding to the main wetting branch. Zhou [11] stated that the contact angle (θ) for the main drying branch and the main wetting branch are equal to the receding contact angle (θ_{R}) and advancing contact angle (θ_{A}), respectively. Also, only on the main drying/wetting branches, the contact angle (θ) is independent of the change of the suction. For the scanning processes, the contact angle is approaching to θ_{R} for drying and to θ_{A} for wetting. Based on the concept introduced by Zhou [11], the term $C(s)$ can be revised to consider hysteretic behaviour due to the variation of contact angle caused by the suction change as follows

$$C(s) = \frac{1}{2}\operatorname{erfc}\left((\sqrt{2}\zeta)^{-1}\ln[(s\cos\theta_{\mathrm{R}})/(s_{\mathrm{mR}}\cos\theta)]\right) \tag{4}$$

where θ_R and θ_A are the receding and advancing contact angles respectively. For simplicity, θ_R is usually assumed to be 0 and θ_A can be calibrated by the main wetting branch. s_{mR} stands for the suction that corresponds to the median pore radius (r_m) in the main drying process, which can be calibrated by the main drying branch. The variation of the contact angle due to suction has been provided by Zhou [11].

$$d\theta = -\frac{\beta ds}{s\tan\theta} \text{ and } \beta = \begin{cases} [(\cos\theta - \cos\theta_R)/(\cos\theta_A - \cos\theta_R)]^b & ds \geq 0 \\ [(\cos\theta_A - \cos\theta)/(\cos\theta_A - \cos\theta_R)]^b & ds < 0 \end{cases} \quad (5)$$

where b is a parameter to adjust the rate of contact angle change due to suction change, which can be calibrated by scanning wetting or drying tests.

In addition to the contact angle hysteresis, the mechanical loading changes soil's pore distribution and further affects its water retention behaviour [12–17]. The mechanical compression due to net stress increase decreases the median pore radius (r_m) as well as the variance (ζ^2) [18]. This phenomenon is termed as the mechanical shift of the water retention curve. The following equation is proposed

$$r_m = r_{m0}(1 - \varepsilon_{v\sigma})^a \text{ and } \zeta^2 = \zeta_0^2(1 - \varepsilon_{v\sigma})^a \quad (6)$$

where r_{m0} and ζ_0^2 is the median pore radius and the variance at the reference state (i.e., $\varepsilon_{v\sigma} = 0$), $\varepsilon_{v\sigma}$ is the volumetric strain due to the mechanical loading, a is a parameter to consider the mechanical effects on pore size distribution. Per the Young-Laplace equation, for a given contact angle, suction is in inverse proportion to pore radius. Therefore, term $C(s)$ can be further upgraded to the following expression to consider the mechanical shift.

$$C(s) = \frac{1}{2}\mathrm{erfc}\left(\left[\sqrt{2}\zeta_0(1 - \varepsilon_{v\sigma})^{a/2}\right]^{-1}\ln[(s(1 - \varepsilon_{v\sigma})^a)/(s_{mR0}\cos\theta)]\right) \quad (7)$$

where s_{mR0} is the suction that corresponds to the median pore radius at the reference state ($\varepsilon_{v\sigma} = 0$) and the receding contact angle ($\theta = \theta_R$, ζ_0^2) the variance of the log-transformed pore radius at the reference state ($\varepsilon_{v\sigma} = 0$). Both can be easily calibrated by test results obtained from the drying branch of conventional water retention experiments with a constant net stress. The volumetric strain due to hydraulic loading (ε_{vs}) has been considered when we calibrate s_{mR0} and ζ_0^2. This is the reason why only the volumetric strain due to mechanical loading ($\varepsilon_{v\sigma}$) is involved in Eq. (7).

The adsorptive component S'' (i.e., adsorbed water retention curve, AWRC) can be described by the following equation [1]:

$$S'' = (1 - P_{cc})\Theta_a/\Theta_s \quad (8)$$

where Θ_s is the volumetric water content at the fully saturated state, Θ_a is the maximum volumetric water content due to adsorption ignoring capillary condensation due to the mutual influence of adjacent adsorptive water films, and P_{cc} stands for the possibility of capillary condensation ($0 \leq P_{cc} \leq 1$). The simplest equation meets the requirement of P_{cc} can be written as: $P_{cc} = S'$. Specifically, Θ_a can be described by the equation proposed by Campbell and Shiozawa [19], i.e.,

$$\Theta_a = \Theta_s A(s), \text{ and } A(s) = \alpha(1 - \ln s / \ln s_d) \tag{9}$$

where α is the parameter that is related to the maximum degree of saturation due to adsorption (without considering capillary condensation) when the suction is equal to 1 kPa. s_d is the suction at oven dryness. Experimental results have shown that oven dryness generally corresponds to a finite suction of 10^6 kPa. Therefore, the adsorbed water retention curve can be specified as

$$S'' = A(s)(1 - S') \tag{10}$$

Combining Eqs. (2), (3) and (10) yields the following closed-form equations for WRC, CWRC and AWRC:

$$\begin{cases} \text{WRC:} \quad S = S' + S'' = \frac{C(s) + A(s) - 2C(s)A(s)}{1 - C(s)A(s)} \\ \text{CWRC:} \quad S' = \frac{C(s) - C(s)A(s)}{1 - C(s)A(s)}, \text{ and AWRC: } S'' = \frac{A(s) - C(s)A(s)}{1 - C(s)A(s)} \end{cases} \tag{11}$$

2.2 Mechanical Equations

Realising the inter-particle water (or water bridges or menisci) is composed of the capillary water only while the adsorptive water forms the water film wrapping particles, the effective inter-particle stress equation for the shear strength of unsaturated soil was suggested by Zhou et al. [1], highlighting only the capillary water contributes to the shear strength under a given suction.

$$\sigma'_{ij} = \sigma_{ij} + S' s \delta_{ij} \tag{12}$$

where σ'_{ij} is the effective inter-particle stress, and S' can be determined by Eq. (11). In the space of deviator stress (q) and mean effective inter-particle stress (p'), the shear strength of unsaturated soil can be written as [20, 21]

$$q = Mp'. \tag{13}$$

The experimental validation for Eqs. (12) and (13) on predicting unsaturated soil strength can be found in [1].

Realising that the degree of capillary saturation contributes much more to unsaturated soil's mechanical behaviour than the degree of adsorptive saturation, the NCLs of unsaturated soils are proposed as below, by adopting the degree of capillary saturation (S') other than the degree of saturation (S) as the key variable.

$$v = N - \lambda \ln p', \tag{14}$$

where p' is the mean effective inter-particle stress defined in Eq. (12), v the specific volume, N the intercept of the NCL with the v-axis when $\ln p' = 0$, and λ the elastoplastic compression index representing the slope of the NCL, which is assumed as a function of the degree of capillary saturation, i.e. $\lambda = \lambda(S')$.

$$\lambda = \lambda_0 - (\lambda_0 - \lambda_d)(1 - S')^k, \tag{15}$$

where λ_0 is the elastoplastic compression index for the fully saturated soil, λ_d is the elastoplastic compression index for when the soil contains zero capillary water (i.e. oven dryness), and k is a coupling parameter which can be determined by a drying test. For the elastic response, the following equation is employed.

$$\mathrm{d}v = -\kappa \mathrm{d}p'/p', \tag{16}$$

where κ is the elastic compression index representing the slope of the unloading and reloading line (URL). For κ, an equation like λ is employed.

$$\kappa = \kappa_0 - (\kappa_0 - \kappa_d)(1 - S')^k, \tag{17}$$

where κ_0 is the elastoplastic compression index for the fully saturated soil, and κ_d is the elastoplastic compression index for when the soil contains zero capillary water (i.e., oven dried). For the clayey soil, the capillary water can only be fully removed by very high suction and the compressibility of the clayey soil at a very high suction is far less than its compressibility at the fully saturated state. Therefore, for simplicity, in this paper, we assume $\lambda_d = \kappa_d = 0$ to simplify Eqs. (15) and (17) as

$$\kappa = \kappa_0 - \kappa_0(1 - S')^k, \text{ and } \lambda = \lambda_0 - \lambda_0(1 - S')^k. \tag{18}$$

The yield surface (or the loading collapse surface) function in the isotropic stress states

$$f_{LC} = (p')^{1-(1-S')^k} - p'_c = 0. \tag{19}$$

The Modified Cam-clay model is employed here to extend the isotropic yield surface to a triaxial stress state [22–24]

$$f = p' + \frac{q^2}{M^2 p'} - (p'_c)^\delta = 0, \text{ where } \delta = \frac{1}{1 - (1 - S')^k}. \tag{20}$$

3 Validations

Li et al. [25] presented a series of suction controlled triaxial tests on the compacted Zaoyang clay. The model parameters for predictions are listed in Table 1.

Table 1. Model parameters for the compacted unsaturated Zaoyang clay

Mechanical parameters (4)				Hydraulic parameters (6)					
λ_0	κ_0	M	k	s_{mR0}	ζ_0	α	θ_A	a	b
0.11	0.02	0.95	5.0	400 kPa	3.0	0.45	70°[a]	13	0.5[a]

[a]Note: θ_A (for wetting) and b (for scanning) are not necessary because only constant suction tests are involved in this section.

Figures 1 and 2 present a series of unsaturated triaxial compression tests on the compacted Zaoyang clay. The suction is set to be 100 kPa and 200 kPa in Figs. 1 and 2, respectively. For each figure (i.e., for each suction level), the stress-strain relationship (q vs ε_1), volume change (v vs ε_1) and the change of the degree of saturation (S vs ε_1) in constant-suction triaxial compression are presented in subfigure a, b and c, respectively. In addition, for each figure (i.e., for each suction level), three different

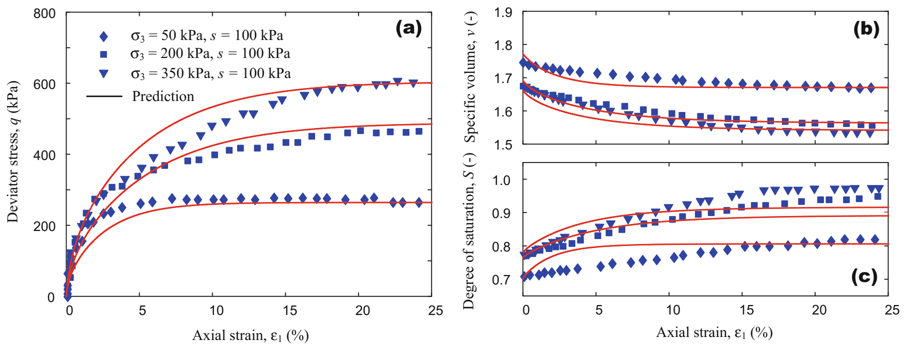

Fig. 1. Stress-strain relationship, volume change and saturation change for the unsaturated Zaoyang clay (s = 100 kPa) in triaxial compressions and model simulations.

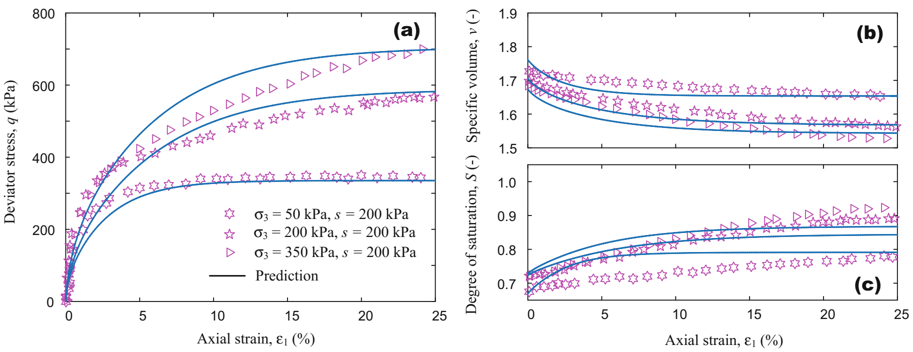

Fig. 2. Stress-strain relationship, volume change and saturation change for the unsaturated Zaoyang clay (s = 200 kPa) in triaxial compressions and model simulations.

confining stresses (net stresses = 50, 200 and 350 kPa) are involved in the constant suction triaxial compressions. The different symbols are used to present the observed test results and solid curves are employed to stand for model predictions. As shown in Figs. 1 and 2, the model predictions quantitatively capture the test data reasonably well for the compacted samples with various suctions (100 and 200 kPa) and various confining stresses (50, 200 and 350 kPa).

4 Conclusions

A new constitutive model for unsaturated soils using the degree of capillary saturation and effective inter-particle stress is proposed in this paper, where the shear strength, yield stress and deformation behaviour of unsaturated soils are governed by two constitutive variables. The proposed constitutive model can capture the observed mechanical and hydraulic behaviours with a limited number of parameters. The capacity of the model has been validated against a variety of experimental data in the literature.

References

1. Zhou, A., Huang, R.-Q., Sheng, D.: Capillary water retention curve and shear strength of unsaturated soils. Can. Geotech. J. **53**(6), 974–987 (2016)
2. Lu, N., Godt, J.W., Wu, D.T.: A closed-form equation for effective stress in unsaturated soil. Water Resour. Res. **46**(5), W05515 (2010)
3. Konrad, J.-M., Lebeau, M.: Capillary-based effective stress formulation for predicting shear strength of unsaturated soils. Can. Geotech. J. **52**(12), 2067–2076 (2015)
4. Bishop, A.W.: The principle of effective stress. Teknisk Ukeblad **106**(39), 859–863 (1959)
5. Sheng, D., Sloan, S.W., Gens, A., Smith, D.W.: Finite element formulation and algorithms for unsaturated soils. Part I: theory. Int. J. Numer. Anal. Meth. Geomech. **27**(9), 745–765 (2003)
6. Zhou, A.N., Zhang, Y.: Explicit integration scheme for non-isothermal elastoplastic model with convex and nonconvex subloading surfaces. Comput. Mech. **55**, 943–961 (2015)
7. Zhang, Y., Zhou, A.: Explicit integration of a porosity-dependent hydro-mechanical model for unsaturated soils. Int. J. Numer. Anal. Meth. Geomech. **40**(17), 2353–2382 (2016)
8. Or, D., Tuller, M.: Liquid retention and interfacial area in variably saturated porous media: Upscaling from single-pore to sample-scale model. Water Resour. Res. **35**(12), 3591–3605 (1999)
9. Khlosi, M., Cornelis, W.M., Douaik, A., van Genuchten, M.T., Gabriels, D.: Performance evaluation of models that describe the soil water retention curve between saturation and oven dryness. Vadose Zone J. **7**(1), 87–96 (2008)
10. Kosugi, K.: Lognormal distribution model for unsaturated soil hydraulic properties. Water Resour. Res. **32**(9), 2697–2703 (1996)
11. Zhou, A.N.: A contact angle-dependent hysteresis model for soil–water retention behaviour. Comput. Geotech. **49**, 36–42 (2013)
12. Sheng, D., Zhou, A.N.: Coupling hydraulic with mechanical models for unsaturated soils. Can. Geotech. J. **48**(5), 826–840 (2011)

13. Zhou, A.N., Sheng, D.: An advanced hydro-mechanical constitutive model for unsaturated soils with different initial densities. Comput. Geotech. **63**, 46–66 (2015)
14. Zhou, A.N., Sheng, D., Sloan, S.W., Gens, A.: Interpretation of unsaturated soil behaviour in the stress-saturation space, I: volume change and water retention behaviours. Comput. Geotech. **43**, 178–187 (2012)
15. Zhou, A.N., Sheng, D., Sloan, S.W., Gens, A.: Interpretation of unsaturated soil behaviour in the stress-saturation space, II: constitutive relationships and validations. Comput. Geotech. **43**, 111–123 (2012)
16. Zhou, A.N., Sheng, D., Carter, J.P.: Modelling the effect of initial density on soil-water characteristic curves. Geotechnique **62**(8), 669–680 (2012)
17. Zhou, A.N., Sheng, D., Li, J.: Modelling water retention and volume change behaviours of unsaturated soils in non-isothermal conditions. Comput. Geotech. **55**, 1–13 (2014)
18. Oualmakran, M., Mercatoris, B.C.N., François, B.: Pore-size distribution of a compacted silty soil after compaction, saturation, and loading. Can. Geotech. J. **53**(12), 1902–1909 (2016)
19. Campbell, G.S., Shiozawa, S.: Prediction of hydraulic properties of soils using particle-size distribution and bulk density data. In: van Genuchten, M.T., Leij, F.J., Lund, L.J. (eds.) Indirect Methods for Estimating the Hydraulic Properties of Unsaturated Soils, pp. 317–328. Univ. of California, Riverside (1992)
20. Yao, Y., Hu, J., Zhou, A., Luo, T., Wang, N.: Unified strength criterion for soils, gravels, rocks, and concretes. Acta Geotech. **10**(6), 1–11 (2015)
21. Sheng, D., Zhou, A.N., Fredlund, D.G.: Shear strength criteria for unsaturated soils. Geotech. Geol. Eng. **29**(2), 145–159 (2011)
22. Yao, Y.P., Hou, W., Zhou, A.N.: UH model: three-dimensional unified hardening model for overconsolidated clays. Geotechnique **59**(5), 451–469 (2009)
23. Yao, Y.P., Kong, L.M., Zhou, A.N., Yin, J.H.: Time-dependent unified hardening model: three-dimensional elastoviscoplastic constitutive model for clays. J. Eng. Mech. **141**(6), 04014162 (2015)
24. Yao, Y.P., Sun, D.A., Matsuoka, H.: A unified constitutive model for both clay and sand with hardening parameter independent on stress path. Comput. Geotech. **35**(2), 210–222 (2008)
25. Li, J., Yin, Z.-Y., Cui, Y., Hicher, P.-Y.: Work input analysis for soils with double porosity and application to the hydromechanical modeling of unsaturated expansive clays. Can. Geotech. J. **54**(2), 173–187 (2016)