Repeated Burst Error Correcting Linear Codes Over GF(q); q = 3

Vinod Tyagi[1] and Subodh Kumar[2](✉)

[1] Shyam Lal College (Eve.), University of Delhi, Delhi, India
vinodtyagi@hotmail.com
[2] Shyam Lal College, University of Delhi, Delhi, India
subodh05031981@gmail.com

Abstract. In this paper, we develop a simple matrix method of constructing a parity check matrix for non binary (5k, k; b, q, m) linear codes capable of correcting m repeated burst errors of length b or less.

Keywords: Repeated burst · Burst error · Open loop and closed loop bursts
Parity check digits · Error patterns and syndrome

1 Introduction

At a very early stage in the history of coding theory, codes were meant for detecting and correcting single errors only. But later on, it was noticed that in almost all communication channels errors occur more in adjacent positions and quite less in random manner. Adjacent error correcting codes were introduced by Abramson [1]. The generalization of this idea was put in the category of errors that is now known as burst error in the literature of coding theory. But the nature of burst errors differ from channel to channel depending upon the behavior of the channel and therefore different type of codes were developed to deal with different type of burst errors. Among many versions of burst errors are CT burst [3], closed loop burst, open loop burst, low density burst and high density burst etc. While studying different communication channels and type of burst errors, it was observed that among all the categories of errors, burst due to Fire [6] is the most common error that occurs during transmission. By a burst of length *b* or less, we mean a vector whose all the nonzero positions are confined to some *b* consecutive components the first and the last of which is non zero. In view of this burst error correcting codes have been developed. Some of such burst error correcting codes have found great applications in numerous areas of practical importance also and therefore have acquired important position in the literature in comparison to other variants of burst, and a good deal of research has gone into the development of bursts and multiple bursts error connecting codes. For references, see [2, 7, 9–12] and many others. Corresponding to various variants of the definition of burst, codes have been developed for correction of random burst or open loop burst errors, low and high density bursts, closed loop bursts and multiple bursts.

D. Ghosh et al. (Eds.): ICMC 2018, CCIS 834, pp. 36–41, 2018.
https://doi.org/10.1007/978-981-13-0023-3_4

In very busy and fast communication channels which is the need of present time, it has been observed that errors repeat themselves more frequently during transmission. This phenomenon has shown that the normal burst error correcting codes cannot yield any positive result for repeated burst error detection and correction. As there is not any uniform terminology for multiple bursts, repeated bursts are also put in this category.

In view of this, it was desirable to develop codes detecting/correcting errors that were in the form of repeated bursts. Dass and Verma [5] took the initiative in this direction and developed codes for repeated burst error detection and correction in binary case.

Following the old technique of parity check matrix construction given by Varshamov-Gilbert-Sacks bound [8], Dass and Verma [5] obtained upper and lower bounds on the number of parity check digits required for correcting *m* repeated burst errors over GF(*q*) (see [4, Theorem 4]). Although the method was cumbersome, bounds were derived on the number of parity check digits. This complicated synthesis procedure involving unwieldy computations particularly in case of repeated bursts and to study repeated burst error correcting codes in detail for binary and non binary cases, it was desirable to simplify the parity check matrix construction procedure.

We will call such codes as $(5k, k; b, q, m)$ linear codes throughout this paper where $n = 5k$. For *m* repeated burst errors of any length with a specific value of *m* and length of the burst *b*, the matrix in binary case comes out to be

$$\mathrm{H} = \begin{pmatrix} & I_k \\ & I_k \\ I_{4k} & \cdot \\ & \cdot \\ & I_k \end{pmatrix} \tag{1}$$

Such a matrix considered as parity check matrix shall give rise to a code that corrects *m*-repeated burst of length *b* or less. Here I_{4k} is an Identity matrix of order $(4k)$. I_k's are identity matrix of order *k*.

As an example, for $k = 3$, $m = 2$, $b = 3$, the parity check matrix for such (15, 3) binary code correcting 2 repeated bursts of length 3 or less may be written as

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

In other words this the parity check matrix for (15, 3) repeated burst correcting code in binary case. This code will always correct 2 repeated bursts of length 3. For detailed verification see [5].

We in this paper study non-binary repeated bursts error correcting codes over GF(3).

2 Non Binary Repeated Burst Error Correcting Linear Codes

Our purpose in this communication is to develop a generalized matrix method for all binary and non binary repeated burst error correcting codes as matrix formulation in (1) does not work in non binary cases. In view of this, we will have to first try to develop a different matrix over GF(3). Using hit and trial method, we came across a matrix that can be described as follows:

Let us define a diagonal matrix J whose diagonal elements are in a sequence 1, 2, 1, 2, 1, 2… etc. in such a way that a 2×2 J matrix may be written as

$$J_2 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

and $J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is a 3×3 J matrix.

Using the result given in (1), we can now write a matrix H_J for (5k, k; b, q, m) non binary linear code over GF(3) as follows:

$$H = H_J = \begin{pmatrix} & J_k \\ & \cdot \\ J_{4k} & \cdot \\ & \cdot \\ & j_k \end{pmatrix} \tag{2}$$

which is the parity check matrix of order $4k \times 5k$.

This formulation of general parity check matrix H in (2) shows that it is now easy to construct parity check matrices for (5k, k; b, q, m) non binary linear code also. Such a code will correct m repeated bursts of length b or less. Comparing this matrix with the usual procedure of constructing a parity check matrix H given by Varshamov-Gilbert-Sacks bound, it can be seen that a column h_j can be added to the matrix H_J provided that it is not a linear combination of immediately preceding $b - 1$ columns together with any $(2m - 1)b$ or less consecutive columns from the remaining $j - b$ columns. So we start with (1000…0), second column (0200…0) and keep on adding the columns in H to get a $2\,mb \times (2m + 1)b$ matrix as defined above in (3). It can be easily verified that the condition given by Dass and Verma [5] in the proof of theorem 4 is satisfied. Thus the (5k, k; b, q, m) non binary code which is the null space the matrix H as constructed above will correct all m repeated bursts of length b or less.

Also it is clear that for any given feasible integer value of the parameters k, m and b, a matrix of the type H as given in (2) can always be constructed and can be used to correct m repeated bursts of length b or less.

3 Illustration of Burst Error Correcting Linear Code for k = 3, b = 2, q = 3, m = 2

Consider a parity check matrix for (15, 3) code as shown below. It can be verified from the Error Pattern - Syndrome table that the code so constructed corrects all repeated bursts of length 2 or less.

$$H = \begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\ 0&2&0&0&0&0&0&0&0&0&0&0&0&2&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&2&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&1&0&0&0&0&0&0&0&0&2&0\\ 0&0&0&0&0&2&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&2&0&0&0&0&0&2&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&2&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&0&2&0\\ 0&0&0&0&0&0&0&0&0&0&0&2&0&0&1 \end{pmatrix}$$

The Syndromes for the parity check matrix given above can be obtained easily with the help of MS-Excel. When we check these syndromes then we will see that these all syndromes are distinct. This verifies that the (15, 3) code can correct all repeated bursts of length 2 and less.

4 Generalization of the Parity Check Matrix for (5k, k; b, q, m) Linear Codes for All Values of q

Let us define a diagonal matrix A such that A_2 is a 2×2 matrix denoted as

$$A_2 = \begin{pmatrix} 1 & 0 \\ 0 & q-1 \end{pmatrix}$$

Similarly a 3×3 matrix is given as

$$A_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & q-1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

The diagonal elements of this matrix are in the sequence 1, q – 1, 1, q – 1, 1, q – 1, 1… etc. In general the parity check matrix for (5k, k; b, q, m) repeated burst error correcting codes for all feasible values of q may be written as

$$\mathrm{H} = \mathrm{H_A} = \begin{pmatrix} & A_k \\ & . \\ A_k & . \\ & . \\ & A_k \end{pmatrix} \tag{4}$$

where A's are matrices of type (3).

5 Discussion

(a) Alternatively, matrix (4), comes out to be

$$\mathrm{H_A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1 & 0 & 0 \\ 0 & q-1 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & q-1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 1 \\ . & . & . & . & . & . & . & \ldots & . & . & . & . \\ . & . & . & . & . & . & . & \ldots & . & . & . & . \\ . & . & . & . & . & . & . & \ldots & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & q-1 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

(b) Now substituting q = 2 in matrix (5) we get

$$\mathrm{H_A} = \mathrm{H} = \begin{pmatrix} & I_k \\ & I_k \\ I_{4k} & . \\ & . \\ & I_k \end{pmatrix}$$

Which is a parity check matrix given by Dass and Verma [4] for m repeated burst error correcting linear codes in binary case.

(c) Substituting q = 3 in (5), the resultant matrix comes out to be

$$\mathrm{H_A} = \mathrm{H_J} = \begin{pmatrix} & J_k \\ & J_k \\ J_{4k} & . \\ & . \\ & J_k \end{pmatrix}$$

Where $\mathrm{H_A}$ is the matrix given above resembles with parity check matrix given in (2) for non binary linear codes for q = 3.

6 Conclusion and Open Problem

We have shown in this paper that a non binary repeated burst error correcting code exists for q = 3, m = 2 and b = 2. We have also given in (4) a parity check matrix for all possible suitable integer values of k, m and burst length b. Although we have discussed in detail the formulation of a parity check matrix for repeated burst error correcting codes over GF(q), we could only verify the matrix for q = 3. It needs further verification for larger values of q.

Acknowledgement. The authors are thankful to Bharat Garg and Preeti for their technical assistance.

References

1. Abramson, N.M.: A class of systemic codes for non independent errors. IRE Trans. Inf. Theor. **IT-5**(4), 150–157 (1959)
2. Bridwell, J.D., Wolf, J.K.: Burst distance and multiple burst correction. Bell Syst. Tech. J. **99**, 889–909 (1970)
3. Chien, R.T., Tang, D.T.: Definition of a burst. IBM J. Res. Dev. **9**(4), 292–293 (1965)
4. Dass, B.K., Verma, R.: Construction of m-repeated bursts error correcting binary linear code. Discret. Math. Algorithms Appl. **4**(3), 1250043 (2012). (7 p.)
5. Dass, B.K., Verma, R.: Repeated burst error correcting linear codes. Asian Eur. J. Math. **1**(3), 303–335 (2008)
6. Fire, P.: A class of multiple error correcting binary Codes for non independent errors, Sylvania report RSL- E-2. Sylvania Reconnaissance Systems Laboratory, Mountain View (1959)
7. Hamming, R.W.: Error detecting and error correcting codes. Bell Syst. Tech. J. **29**, 147–160 (1950)
8. Peterson, W.W., Weldon Jr., E.J.: Error Correcting Codes, 2nd edn. The MIT Press, Mass (1972)
9. Posner, E.C.: Simultaneous error correction and burst error detecting using binary linear cyclic codes. J. Soc. Ind. Appl. Math. **13**(4), 1087–1095 (1965)
10. Srinivas, K.V., Jain, R., Saurav, S., Sikdar, S.K.: Small-world network topology of hippocampal neuronal network is lost in an in vitro glutamate injury model of epilosy. Eur. Neurosci. **25**, 3276–3280 (2007)
11. Wolf, J.K.: On codes derivable from the tensor product of matrices. IEEE Trans. Inf. Theor. **IT-11**(2), 281–284 (1965)
12. Wyner, A.D.: Low-density-burst-correcting codes. IEEE Trans. Inf. Theor. **9**, 124 (1963)