An Approach to Multi-criteria Decision Making Problems Using Dice Similarity Measure for Picture Fuzzy Sets

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Abstract. This paper presents an approach for multi criteria decision making problems based on the Dice similarity measure and weighted Dice similarity measure for picture fuzzy sets (PFSs). To illustrate the application of the proposed method, a practical problem has been considered and the results are compared and verified with an existing method. The results are well matched and the calculations are compact and much easier to analyze.

Keywords: Picture fuzzy sets · Dice similarity measure
Weighted Dice similarity measure · Multi-criteria decision making

1 Introduction

The theory of intuitionistic fuzzy sets (IFSs) proposed by *Atanassov* [1] has been successfully applied in various fields like decision making, logic programming, pattern recognition, medical diagnosis and more [2,3]. Although, IFS theory has been successfully applied in different areas, but there are situations in real life which cannot be represented by IFS [4]. Voting could be a good example of such situation as the human voters may be divided into four groups of those who; vote for, vote against, abstain and refusal of voting. Nevertheless, the IFS theory care to those who vote for or vote against, and consider those who abstain and refusal are equivalent. This concept is particularly effective in approaching the practical problems in relation to the synthesis of ideas; make decision such as voting, financial forecasting and risks in business [5].

Similarity measures are common tools used widely in measuring the deviation and closeness degree of different arguments. *Dengfeng et al.* [3] introduced the degree of similarity between IFS to propose several new similarity measures and applied those new measures into pattern recognition. A new measure of similarity for IFSs, considering the distance to its complement to analyze the extension of agreement in group of experts was proposed by *Szmidt et al.* [6]. *Ye* [7] extended the concept of the cosine similarity measure for fuzzy sets and therefore

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proposed the cosine similarity measure for IFSs. He also proposed the concept of the reduct intuitionistic fuzzy set of interval valued intuitionistic fuzzy set with respect to adjustable weight vectors and the dice similarity measure based on the reduct IFS to explore the effects of optimism, neutralism and pessimism in decision making [8]. Some cosine similarity measures and weighted cosine similarity measures between picture fuzzy sets were discussed by *Wei* [9] and are applied to strategic decision making problem for selecting optimal production strategy. Cosine similarity measure is undefined when one vector is zero, since it is defined as the inner product of their lengths. In this case, the dice similarity measure for PFSs can be used which is an extension of Dice similarity measure for IFSs. Therefore, the purpose of this study is to propose the Dice similarity measure for picture fuzzy sets (PFSs) and utilize it in decision making problems.

This paper is organized as follows: In Sects. 2, 3 and 4, we review definitions of picture fuzzy set (PFS), cosine similarity measure and Dice similarity measure. In Sect. 5, we propose a Dice similarity measure for picture fuzzy sets (PFSs). The proposed Dice similarity measure based MCDM method in Sect. 6 is also implemented on a practical problem of known criteria weights in Sect. 7. Final results are compared with the result of other existing methods in Sect. 8 and finally the conclusions are presented at the end of the paper.

2 Picture Fuzzy Sets

Cuong *et al.* [2] proposed picture fuzzy sets, which is defined as follows: A picture fuzzy set A on a universe X is defined as an object of the following form;

$$A = \{<x, \mu_A(x), \eta_A(x), \nu_A(x)> : x \in X\} \tag{1}$$

where the functions $\mu_A(x)$, $\eta_A(x)$, $\nu_A(x)$ are respectively called the degree of positive membership, the degree of neutral membership, the degree of negative membership of x in A, and following conditions are satisfied;

$$0 \leq \mu_A(x), \eta_A(x), \nu_A(x) \leq 1$$

$$\mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1 \forall x \epsilon X$$

Then, $\forall x \in X : 1 - (\mu_A(x) + \eta_A(x) + \nu_A(x))$ is called the degree of refusal membership of x in A.

3 Cosine Similarity Measure for PFSs

Wei [9] introduced similarity measures for PFSs based on the concept of cosine function. Suppose there are two PFSs given as follows:

$$A = \{<x, \mu_A(x), \eta_A(x), \nu_A(x)> : x \in X\}$$

$$B = \{<x, \mu_B(x), \eta_B(x), \nu_B(x)> : x \in X\}$$

in the universe of discourse $X = \{x_1 x_2 x_3 \ldots\ldots x_n\}$. Then cosine similarity measure between PFSs is as follows:

$$PFC\,(A,B) = \frac{1}{n}\sum_{j=1}^{n} \frac{\mu_A(x_j)\,\mu_B\,(x_j) + \eta_A\,(x_j)\,\eta_B\,(x_j) + \nu_A(x_j)\,\nu_B\,(x_j)}{\sqrt{\mu_A^2\,(x_j) + \eta_A^2\,(x_j) + \nu_A^2\,(x_j)}\sqrt{\mu_B^2\,(x_j) + \eta_B^2\,(x_j) + \nu_B^2\,(x_j)}} \tag{2}$$

4 Dice Similarity Measure

Let $X = \{x_1, x_2, x_3, \ldots\ldots\ldots x_n\}$ and $Y = \{y_1, y_2, y_3, \ldots\ldots\ldots y_n\}$ be two vectors of length n where all the coordinates are positive. Then the Dice similarity measure [4] is defined as follows:

$$D = \frac{2X.Y}{\|X\|_2^2 + \|Y\|_2^2} = \frac{2\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2}$$

where $X.Y = \sum_{i=1}^{n} x_i y_i$ is the inner product of the vectors X and Y and $\|X\|_2 = \sqrt{\sum_{i=1}^{n} x^2}$ and $\|Y\|_2 = \sqrt{\sum_{i=1}^{n} y^2}$, are the Euclidean norms of X and Y. The Dice similarity measure takes value in the interval [0, 1]. However, it is undefined if $x_i = y_i = 0$ $(i = 1, 2, \ldots n)$.

5 Dice Similarity Measure for PFS

In this section, the dice similarity measure for PFS is proposed as a generalization of the Dice similarity measure [4] in vector space. Let A and B be two PFSs in the universe of discourse $X = \{x_1, x_2, \ldots\ldots x_n\}$. Based on the extension of the Dice similarity measure [4], the Dice similarity measure between picture fuzzy sets A and B is proposed in vector space as follows;

$$D_{PFS}\,(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{2(\mu_{Ai}\mu_{Bi} + \eta_{Ai}\eta_{Bi} + \nu_{Ai}\nu_{Bi})}{\mu_{Ai}^2 + \eta_{Ai}^2 + \nu_{Ai}^2 + \mu_{Bi}^2 + \eta_{Bi}^2 + \upsilon_{Bi}^2} \tag{3}$$

The Dice similarity measure between two PFSs A and B satisfies the following properties:

1. $0 \leq D_{PFS}\quad(A,B) \leq 1$
2. $D_{PFS}\quad(A,B) = D_{PFS}\quad(B,A)$
3. $D_{PFS}\quad(A,B) = 1$ if and only if $A = B$, i.e., $\mu_{Ai} = \mu_{Bi}, \upsilon_{Ai} = \nu_{Bi}$ and $\eta_{Ai} = \eta_{Bi}$

5.1 Proof

1. Let us consider the i^{th} item of the summation in Eq. (3)

$$D_i\,(A_i,B_i) = \frac{2(\mu_{Ai}\mu_{Bi} + \eta_{Ai}\eta_{Bi} + \nu_{Ai}\nu_{Bi})}{\mu_{Ai}^2 + \eta_{Ai}^2 + \nu_{Ai}^2 + \mu_{Bi}^2 + \eta_{Bi}^2 + \upsilon_{Bi}^2} \tag{4}$$

   It is obvious that $D_i\,(A_i,B_i) \geq 0$ and $\mu_{Ai}^2 + \eta_{Ai}^2 + \nu_{Ai}^2 + \mu_{Bi}^2 + \eta_{Bi}^2 + \upsilon_{Bi}^2 \geq 2(\mu_{Ai}\mu_{Bi} + \eta_{Ai}\eta_{Bi} + \nu_{Ai}\nu_{Bi})$ according to the inequality $\left(a^2 + b^2 \geq 2ab\right)$. Thus $0 \leq D_i\,(A_i,B_i) \leq 1$.
2. It is obvious that the property is true.

3. When $A = B$, there are $\mu_{Ai} = \mu_{Bi}, \upsilon_{Ai} = \nu_{Bi}$ and $\eta_{Ai} = \eta_{Bi}$, for $i = 1, 2, \ldots\ n$. So there is $D_{PFS}\ (A, B) = 1$. When $D_{PFS}\ (A, B) = 1$, there are $\mu_{Ai} = \mu_{Bi}, \upsilon_{Ai} = \nu_{Bi}$ and $\eta_{Ai} = \eta_{Bi}$, for $i = 1, 2, \ldots\ n$. So there is $A = B$.

If we consider the weight of x_i, the weighted Dice similarity measure between PFSs A and B is proposed as follows:

$$W_{PFS}\,(A, B) = \sum_{i=1}^{n} w_i \frac{2(\mu_{Ai}\mu_{Bi} + \eta_{Ai}\eta_{Bi} + \nu_{Ai}\nu_{Bi})}{\mu_{Ai}^2 + \eta_{Ai}^2 + \nu_{Ai}^2 + \mu_{Bi}^2 + \eta_{Bi}^2 + \upsilon_{Bi}^2} \tag{5}$$

where $w_i \in [0, 1]$, $i = 1, 2, \ldots\ldots\ldots\ n$, and $\sum_{i=1}^{n} w_i\ = 1$.

6 MCDM Based on Proposed Similarity Measure

In this section, a decision making method by using above defined Dice similarity measure for PFSs has been presented followed by an illustrative example for demonstrating the approach.

Let a set of m alternatives denoted by $A = \{A_1, A_2 \ldots\ldots\ldots\ldots A_m\}$ which has been evaluated by the decision maker under the set of the different criteria $C = \{C_1, C_2 \ldots\ldots\ldots C_n\}$ whose weight vectors are $w = \{w_1, w_2 \ldots\ldots\ldots w_m\}$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Assume that the decision maker gave his preference in the form of PFNs $\alpha_{ij} = \langle \mu_{ij}, \upsilon_{ij}, \eta_{ij} \rangle$. Then, in the following, we develop an approach based on the proposed similarity measure for MCDM problem, which involve the following steps.

Step 1. Construct a picture fuzzy decision matrix $D = (\alpha_{ij})_{m \times n}$ by the preference given by decision maker towards the alternative A_i.

Step 2. If there are different types of criteria, namely cost (C) and benefit (B), then we normalize it by using the following equation;

$$r_{ij} = \begin{cases} \alpha_{ij}, j \in B \\ \alpha_{ij,} j \in C \end{cases} \tag{6}$$

Step 3. Define an ideal picture fuzzy set for each criterion in the ideal alternative A^* as $C_j^* = (1, 0, 0)$ for "excellence". Then applying Eq. (5), we can obtain the weighted dice similarity measure between the ideal alternative A^* and an alternative A_i $(i = 1, 2, \ldots\ m)$. The bigger the value of $w_i(A^*, A_i)$, the better the alternative A_i, as the alternative A_i is closer to the ideal alternative A^*.

7 Practical Applications

A practical MCDM problem has been taken from Sect. 5.1 of *Garg* [5]. Suppose a multinational company in India is planning its financial strategy for the next year, according to the group strategy objective. For this, the four alternatives are obtained after their preliminary screening and are defined as $A_{1:}$ to invest in the "Southern Asian markets"; A_2: to invest in the "Eastern Asian markets";

A_3: to invest in the "Northern Asian markets"; A_4: to invest in the "Local markets". This evaluation proceeds from four aspects, namely as C_1: the growth analysis; C_2: the risk analysis; C_3: the social-political impact analysis; C_4: the environmental impact analysis whose weight vector is $w = (0.2,\ 0.3,\ 0.1,\ 0.4)$. The following steps have been performed to compute the best one;

Step 1: Picture fuzzy decision matrix given by following:

	C_1	C_2	C_3	C_4
A_1	$\langle 0.2, 0.1, 0.6\rangle$	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.4, 0.3, 0.2\rangle$
A_2	$\langle 0.1, 0.4, 0.4\rangle$	$\langle 0.6, 0.3, 0.1\rangle$	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.2, 0.1, 0.7\rangle$
A_3	$\langle 0.3, 0.2, 0.2\rangle$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.3, 0.3, 0.4\rangle$
A_4	$\langle 0.3, 0.1, 0.6\rangle$	$\langle 0.1, 0.2, 0.6\rangle$	$\langle 0.1, 0.3, 0.5\rangle$	$\langle 0.2, 0.3, 0.2\rangle$

Step 2: Since the criteria C2 and C3 are the cost criteria while C1 and C4 are benefit criteria, so we get normalized decision matrix using Eq. (6) as follows:

	C_1	C_2	C_3	C_4
A_1	$\langle 0.6, 0.1, 0.2\rangle$	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.2, 0.3, 0.4\rangle$
A_2	$\langle 0.4, 0.4, 0.1\rangle$	$\langle 0.6, 0.3, 0.1\rangle$	$\langle 0.5, 0.2, 0.2\rangle$	$\langle 0.7, 0.1, 0.2\rangle$
A_3	$\langle 0.2, 0.2, 0.3\rangle$	$\langle 0.6, 0.2, 0.1\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.4, 0.3, 0.3\rangle$
A_4	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.1, 0.2, 0.6\rangle$	$\langle 0.1, 0.3, 0.5\rangle$	$\langle 0.2, 0.3, 0.2\rangle$

Step 3: Computing the values of weighted Dice similarity measure by applying Eq. (5).

$$w\left(A^*, A_1\right) = 0.59053$$

$$w\left(A^*, A_2\right) = 0.8057$$

$$w\left(A^*, A_3\right) = 0.6259$$

$$w\left(A^*, A_4\right) = 0.3585$$

Ranking of all alternatives is obtained in accordance with the descending values of weighted Dice similarity measure, as $A_2 > A_3 > A_1 > A_4$.

Hence the best financial strategy is A_2 *i.e.* to invest in Asian market. We compare our result with method given in Sect. 5.1 of *Garg* [5], and our result is same as obtained using the PFWA operator defined by *Garg* [5] i.e. $A_2 > A_3 > A_1 > A_4$.

8 Comparison with Other Methods

1. Comparing our results with the method using cosine similarity measure given by Wie [9], we get following values of weighted cosine similarity measure

$$w^c\left(A^*, A_1\right) = 0.7835$$

$$w^c(A^*, A_2) = 0.8510$$
$$w^c(A^*, A_3) = 0.7557$$
$$w^c(A^*, A_4) = 0.5806$$

Ranking all the alternatives in accordance with the descending values of weighted cosine similarity measure, as $A_2 > A_1 > A_3 > A_4$. Hence the best alternative is A_2, which is same as our result.

2. We compare our result with method given in Sect. 5.1 of *Garg* [5], and our result is same as obtained using the PFWA operator defined by *Garg* [5] i.e. $A_2 > A_3 > A_1 > A_4$.

9 Conclusion

In this paper, we have presented a new method for handling multi criteria decision making problems, where the characteristics of the alternative are represented by picture fuzzy sets. Furthermore, a decision making method was established by the use of proposed Dice similarity measure and weighted Dice similarity measure for picture fuzzy sets. Illustrative example demonstrated the feasibility of the proposed method in practical applications. The proposed method differs from previous approaches for multi criteria decision making not only due to fact that the proposed method uses PFS theory rather than other fuzzy theories, but also due to the Dice similarity measure and weighted Dice similarity measure based on PFS, the calculations are compact and much easier to analyze.

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