An Exact Synthesis of Pick-and-Place Mechanisms Using a Planar Four-Bar Linkage

Aravind Baskar and Sandipan Bandyopadhyay

Abstract For repetitive material-handling operations in various industries, fixed automation using single-degree-of-freedom mechanisms can often serve as a low-cost alternative to multi-degrees-of-freedom robots. Therefore, developing design procedures for inexpensive fixed automation solutions may be highly relevant in the context of developing as well as underdeveloped economies. A design methodology to analytically synthesise a planar pick-and-place system for displacement and velocity requirements using a planar four-bar mechanism is carried out in this work. A methodology to establish the availability of kinematic defect-free solutions in terms of two free design parameters is also proposed and illustrated with a numerical example.

Keywords Kinematic synthesis · Pick-and-place mechanism · Circuit defect
Branch defect

1 Introduction

The problem of kinematic path synthesis using planar four-bar linkage for positional requirements is well studied in the literature both analytically (see, e.g., [1]) and numerically using continuation techniques (for example, see [2, 3]). Holte et al. [4] presented a closed-form solution for a two precision-points problem with velocity requirements at one of the points and used the available free parameters for velocity approximations at several other positions, for assisting a technician in a laboratory environment. Robson and McCarthy [5] solved a synthesis problem matching three positions and two velocities by adjusting the velocity specifications iteratively to find

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defect-free solution. In this work, a closed-form solution to a pick-and-place application in an industrial setting is presented, for exact position and velocity requirements at two precision-points, in addition to a requirement of finite angular displacement of the end-effector by a desired angle. This problem falls under PP-PP category with two sets of two infinitesimally separated positions, as termed by Tesar [6]. A pair of infinitesimally separated positions represents a positional requirement and a velocity requirement at the same position.

Figure 1 depicts the problem schematically. The objective of this work is to design a pick-and-place system to transfer a component from a moving conveyor 1 to another moving conveyor 2, which is a common and highly repetitive task in many industries. While performing the pick-and-place operation, the following requirements are to be met:

1. The mechanism should include the pick and drop locations in the path of a designated coupler point, where a gripper would be attached.
2. The said coupler point (i.e., the gripper) should match the velocities of the corresponding conveyors as it picks and drops the object to reduce mechanical impact during operation.
3. The object to be handled should be rotated by a finite angle before the drop at the destination, as depicted with two different reference frames $\boldsymbol{p}_1 - \boldsymbol{X}_1\boldsymbol{Y}_1$ and $\boldsymbol{p}_2 - \boldsymbol{X}_2\boldsymbol{Y}_2$ at the pick and drop locations, respectively.
4. Additionally, the mechanism must be free of any kinematic defects such as the branch defect, circuit defect and order defect, and it should be of Grashof type.

Although kinematic synthesis of pick-and-place mechanisms have been discussed in the literature, only subsets of the requirements listed above have been addressed. For example, the requirements 1 and 2 are partially addressed in [4, 5] with little emphasis on kinematic defects. To the best of the authors' knowledge, all the four requirements are not addressed in an exact manner in the existing literature. The

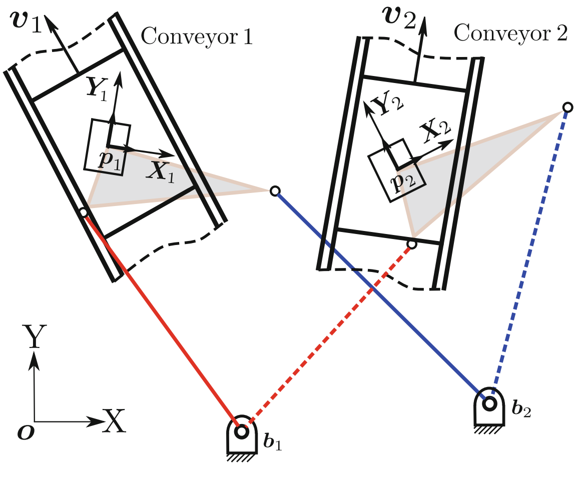

Fig. 1 Schematic of a two-conveyor system with a pick-and-place mechanism

proposed comprehensive methodology to find a feasible design free of the defects can be potentially extended to other class of problems.

The rest of the paper is organised as follows: a closed-form solution to the aforementioned problem is proposed in Sect. 2. Feasibility analysis for obtaining defect-free solutions using the available free design parameters is carried out in Sect. 3. The solution methodology is illustrated with a numerical example of a pick-and-place problem in Sect. 4. Section 5 concludes the paper.

2 Problem Formulation

A planar four-bar mechanism is modeled as a combination of two vector-dyads, represented by the complex variables (z_1, z_2) and (z_3, z_4), with z_5 as the ground link, as shown in Fig. 2. The complex numbers r_1 and r_2 represent the position vectors of the pick and drop locations $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, respectively, in the global frame of reference $\boldsymbol{o} - \boldsymbol{XY}$. Let the conveyors be moving at constant velocities defined by v_1 and v_2, respectively[1] (refer to Fig. 1). Further requirement to rotate the object by a fixed angle λ, between the two positions, imposes an additional constraint. The angular velocity of the crank is considered to be a constant ω, which is the frequency of the operation cycle. This leaves the problem with two free real parameters, namely, the crank displacement, ϕ and the follower displacement, μ (see Fig. 2), of which the details are explained in the following. These parameters are utilised at a later stage to eliminate branch defects and to identify Grashof four-bars that are devoid of circuit defects, as classified by Chase and Mirth [7].

Formulation through complex variables enables easier computation of the solution in the closed-form. The procedure adopted is a hybrid one in that the complex variables are eliminated from the vector equations first and then the real variables are solved for by splitting the real and imaginary components of the residual equations. Loop-closure equations using complex numbers offer a concise way to pose the problem. First, the displacement constraints are posed mathematically. With z_6 as the reference vector of the base point $\boldsymbol{b}_1$, the following equations can be written for the first precision-point in terms of its position vector r_1:

$$z_6 + z_1 + z_2 = r_1, \tag{1}$$

$$z_6 + z_5 + z_3 + z_4 = r_1. \tag{2}$$

Analogous equations can be written for the second position, r_2:

$$z_6 + z_1 e^{i\phi} + z_2 e^{i\lambda} = r_2, \tag{3}$$

$$z_6 + z_5 + z_3 e^{i\mu} + z_4 e^{i\lambda} = r_2, \tag{4}$$

[1] Vectors expressed as a pair of real numbers are represented in bold fonts, while those expressed as complex numbers are not.

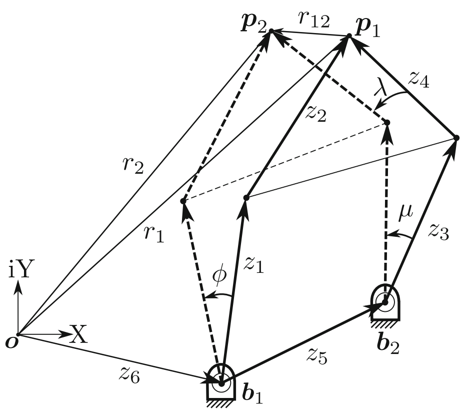

Fig. 2 Vector loop for two finitely separated positions $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, which represent the pick and drop locations, respectively

where i is the imaginary unit. As z_2 and z_4 are rigidly embedded in the coupler, they are displaced through the same angle λ between the two positions, thus completing the four-bar linkage architecture. Eliminating the variables z_5 and z_6 by subtracting Eqs. (1) and (2) from Eqs. (3) and (4), respectively, *the loop-closure conditions of the path* are obtained, with the *increment vector* [2] defined by $r_{12} = r_2 - r_1$:

$$z_1(e^{i\phi} - 1) + z_2(e^{i\lambda} - 1) = r_{12}, \tag{5}$$

$$z_3(e^{i\mu} - 1) + z_4(e^{i\lambda} - 1) = r_{12}. \tag{6}$$

The time derivative of the Eqs. (1)–(4) yields the velocity constraints, Eqs. (7)–(10), in terms of v_k, the conveyor velocity vector at position k, $k = 1,\ 2$:

$$z_1\omega + z_2\omega_{21} = -iv_1, \tag{7}$$

$$z_3\omega_{31} + z_4\omega_{21} = -iv_1, \tag{8}$$

$$z_1e^{i\phi}\omega + z_2e^{i\lambda}\omega_{22} = -iv_2, \tag{9}$$

$$z_3e^{i\mu}\omega_{32} + z_4e^{i\lambda}\omega_{22} = -iv_2, \tag{10}$$

where ω_{jk} refers to the angular velocity of the vector z_j, $j = 2,\ 3$ at the precision-point k, $k = 1,\ 2$.

The system of equations to be solved can be summarised as follows:

- Number of complex equations: 6 (Eqs. 5–10)
- Number of complex variables: 4 ($z_1,\ z_2,\ z_3,\ z_4$)
- Number of real variables: 4 ($\omega_{21}, \omega_{22}, \omega_{31}, \omega_{32}$).

Thus, the problem is formulated as $6 \times 2 = 12$ scalar equations and it can be solved for $4 \times 2 + 4 = 12$ scalar variables as listed. This leaves the designer with two free real design parameters, namely, ϕ and μ. From Eqs. (7)–(10), the complex variables z_1, z_2, z_3 and z_4 can be solved for linearly, in terms of the angular displacement variables ϕ and μ, and angular velocity variables $\omega_{21}, \omega_{22}, \omega_{31}$ and ω_{32}:

$$z_1 = -i(v_2\omega_{21} - e^{i\lambda}v_1\omega_{22})/(\omega(e^{i\phi}\omega_{21} - e^{i\lambda}\omega_{22})), \tag{11}$$

$$z_2 = -i(e^{i\phi}v_1 - v_2)/(e^{i\phi}\omega_{21} - e^{i\lambda}\omega_{22}), \tag{12}$$

$$z_3 = -i(v_2\omega_{21} - e^{i\lambda}v_1\omega_{22})/(e^{i\mu}\omega_{21}\omega_{32} - e^{i\lambda}\omega_{22}\omega_{31}), \tag{13}$$

$$z_4 = -i(v_2\omega_{31} - e^{i\mu}v_1\omega_{32})/(e^{i\lambda}\omega_{22}\omega_{31} - e^{i\mu}\omega_{21}\omega_{32}). \tag{14}$$

On substituting the solutions of Eqs. (11)–(14) in Eqs. (5) and (6), four scalar equations in terms of the four angular velocity variables are obtained by separating the real and imaginary parts of the equations. The variables ω_{21}, ω_{22}, ω_{31} and ω_{32} appear linearly, pairwise, in the resulting equations, which are not presented for the sake of brevity. They can be solved through Cramer's rule, in terms of the components of the increment vector $r_{12} = \delta_a + i\delta_b$ and the conveyor velocity vector $v_j = v_{ja} + iv_{jb}$, $j = 1,\ 2$, where δ_a, δ_b, v_{ja} and v_{jb} are real. The solutions to the angular velocity variables along with the dyadic solutions given by Eqs. (11)–(14) constitute a closed-form solution to the PP-PP problem parametrised in terms of the design parameters ϕ and μ. Since the problem has a unique solution for each point on the parameter space, the problem has ∞^2 solutions.

3 Parametric Analysis to Find Feasible Solutions

Although a closed-form solution is obtained for the problem, it is not guaranteed that the solution obtained is feasible. In other words, the mechanism may suffer from kinematic defects that are inherent to any synthesis problem. It has been well established in literature that, three types of defects can occur in kinematic synthesis, namely, branch defect, circuit defect (including Grashof defect) and order defect (see, for example, [8]). In problems of path synthesis, order defects occur only in problems with four precision-points or higher. In three precision-point problems, order defect can be circumvented by changing the direction of the input motion and in two-point problems, order defects do not occur at all. Thus, in the following sections, only branch and circuit defects are addressed for the pick-and-place problem using the two design parameters available to find a feasible solution.

3.1 Branch Defect

Branch errors cannot be avoided during synthesis, due to the inherent quadratic nature of trigonometric functions in the loop-closure constraints. A methodology is proposed here to identify branch transition linkages in the design parameter space, which mark the transition of a precision-point between the two branches of the four-bar mechanism. A two precision-point problem with points A and B can have the following branch behaviour regions:

1. A_1B_1: Both precision-points lying on the first branch.
2. A_1B_2: Point A lying in the first branch and point B in the second.
3. A_2B_1: Point B lying in the first branch and point A in the second.
4. A_2B_2: Both precision-points lying on the second branch.

Among the four cases listed, the first and the last cases represent the branch defect-free scenarios. For the problem at hand, the design parameters ϕ and μ represent the angular displacements of the crank and the follower, respectively, that vary from 0 to 2π radians. Since it is a PP-PP problem with only two finitely separated points and the design parameters can take any set of values in the continuous domain, the branch transition points occurring at the two design positions should split the two-dimensional parameter space into four distinct regions of branch behaviour as enumerated. Consequently, to find the corresponding branch transition points in the parameter space, the condition for collinearity of the vectors that represent the coupler and the follower at the design points are studied. For the first design point $\boldsymbol{p}_1$, using the following identity for parallel vectors:

$$(z_2 - z_4)\bar{z}_3 = (\bar{z}_2 - \bar{z}_4)z_3, \tag{15}$$

and solving it in conjunction with the Eqs. (11)–(14) derived in the previous section, the condition for branch transition corresponding to the first precision-point $\boldsymbol{p}_1$ is obtained, as shown below:

$$\omega_{21}\omega_{32}\sin(\phi - \mu) - \omega_{22}\omega_{31}\sin(\phi - \lambda) + \omega_{22}\omega_{32}\sin(\mu - \lambda) = 0. \tag{16}$$

Identical analysis at the second design point $\boldsymbol{p}_2$ yields the transition condition for the second precision-point, given by Eq. (17):

$$\omega_{22}\omega_{31}\sin(\phi - \mu) - \omega_{21}\omega_{32}\sin(\phi - \lambda) + \omega_{21}\omega_{31}\sin(\mu - \lambda) = 0. \tag{17}$$

These two conditions divide the parameter space into four regions and facilitate identification of the feasible regions that are free of branch defect. There may be degenerate points in the parameter space where one or more link lengths become zero or tend to infinity. Hence, care must be taken to stay clear of these degenerate points while choosing a solution.

3.2 Circuit Defect

Elimination of branch error does not ensure mechanical feasibility, as the resulting mechanism can still encounter circuit defect. Chase and Mirth [7] termed a circuit as "all possible orientations of the links which can be realised without disconnecting any of the joints". Even if the precision-points lie on the same branch or phase of the mechanism, they may or may not lie on the same circuit. Murray et al. [9] showed that critical points in the parameter space, where the circuit behaviour changes, occur in a planar four-bar linkage when all the links are collinear. For identifying those critical points of the design parameter space that change the circuit behaviour, a formulation similar to the one presented in [10] is followed.

4 Numerical Results and Discussions

Consider the following specifications of a desired pick-and-place scenario,

$$\delta_a = -1\ \text{m},\ \delta_b = -1\ \text{m},\ v_{1a} = -1/\sqrt{2}\ \text{ms}^{-1},\ v_{1b} = -1/\sqrt{2}\ \text{ms}^{-1},$$
$$v_{2a} = -\sqrt{3}/2\ \text{ms}^{-1},\ v_{2b} = -1/2\ \text{ms}^{-1},\ \lambda = 3\pi/4\ \text{rad},\ \omega = 2\ \text{rad s}^{-1}.$$

For the above set of values, the parametric analysis to find feasible solutions is demonstrated for a test value of crank displacement[2] $\phi = 5\pi/3$ and the feasible regions are identified in the parametric space of μ. Without any loss of generality, the first precision-point $\boldsymbol{p}_1$ is assumed to coincide with the origin $\boldsymbol{o}$. Conditions for branch transition derived in the Sect. 3.1 yield the following points:

$$\mu_{\boldsymbol{p}_1} = 1.648,\ \mu_{\boldsymbol{p}_2} = 2.946,$$

where the subscripts denote the precision point with which each transition point is associated. Although the bounds are established, the directionality of the branch defect-free region can only be found by testing a sample value of μ. For this problem, $(1.648,\ 2.946)$ forms the range of parameter values that avoids the branching problem. Some common degeneracies, where one of the link-lengths becomes zero, occur when μ takes the values 0, λ and ϕ, and also at 2.297 and 5.439 for this example.

The condition for circuit transition linkage (derived following [10]) yields eight non-degenerate roots for the parameter μ, with six real roots for the numerical example addressed. The six real roots of μ that alter the circuit behaviour are listed as follows:

$$\{1.185,\ 2.201,\ 2.269,\ 2.317,\ 5.397,\ 5.482\}.$$

Following the methodology explained in [9], it can be established that Grashof-type four-bar mechanisms occur for the values of μ represented by $(1.185,\ 2.201)$. Thus, the intersection of the two domains, given by $\mu \in (1.648,\ 2.201)$, defines the range of values μ can take for defect-free Grashof four-bar solution. A sample plot for a feasible parameter value is shown in Fig. 3. The value of the parameter μ can be chosen so as to address secondary considerations such as foot-print and transmission capability.

[2] All the angles, namely, ϕ, λ and μ, are represented in radians and the units are omitted.

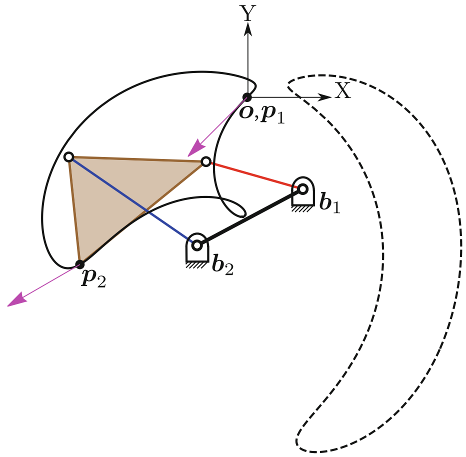

Fig. 3 Coupler-plot of defect-free four-bar mechanism for $\phi = 5\pi/3$ and $\mu = 2$. The numerical solution is given by $z_1 = -0.432 - 0.419i$, $z_2 = 0.098 + 0.968i$, $z_3 = 0.800 + 0.478i$, $z_4 = -0.502 + 0.408i$, $\boldsymbol{b}_1 = [0.334, -0.549]^\top$ and $\boldsymbol{b}_2 = [-0.299, -0.886]^\top$. The arrows indicate the direction of the conveyor velocity at the precision-points, scaled-down for representation

The methodology presented here does not take into account the angular velocity of the coupler at the pick and drop locations. However, the freedom in choosing the value of the parameter μ can be utilised to minimise the angular velocities at the pick and drop locations.

5 Contributions and Future Scope

This work presents a formulation for the exact synthesis of two sets of two infinitesimally separated points using a planar four-bar mechanism. The problem is formulated using complex variables to enable the derivation of the solution in the closed-form. Feasibility analysis using the available free design parameters, in the form of crank and follower angular displacements between two positions is carried out. Branch defect-free regions and degeneracies in the domain of the free parameter space are obtained. Circuit defect-free regions are identified through a characterisation scheme presented in [9]. Analytical formulation allows the user to design pick-and-place systems accurately using a single-degree-of-freedom mechanism. Once the feasible domains are identified, secondary objectives such as zero angular velocities at the pick-and-drop locations and foot-print considerations may be addressed. For double-dwell problems, the prescribed formulation breaks down and the classical solution involving double-cusp four-bar mechanisms could be considered (for example, see [11]). Using six-bar mechanisms for pick-and-place systems may offer additional variables to include acceleration constraints.

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