System Behavior Analysis in the Urea Fertilizer Industry

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Abstract. This paper models and analyses a urea plant having subsystems of different operational nature for system parameters using Regenerative Point Graphical Technique (RPGT). A common cause failure is also considered in modeling. Problem is formulated and solved for constant failure/repair rates for each subsystem. A state diagram of the system depicting the transition rates is drawn and expressions for path probabilities mean sojourn times are derived. Analytical discussion is carried out by tables and graphs. Behavioral inferences have been drawn which may useful to concerned industrial personals.

Keywords: Fuzzy logic · RPGT · System parameters

1 Introduction

In this paper, behavior analysis of a urea fertilizer industry consisting of six sub-units Ammonia Making Section (A), Medium Pressure Section (B), Low Pressure Section (C), Pre-vacuum Section (D), Vacuum Section (E) & Periling Section (F) and three pressure units named low, medium and high pressure units (P_1, P_2 & P_3 respectively) is carried out. Fuzzy logic is used to determine good state of a unit. When low or high pressure units fail, then low and medium pressure steams are obtained from high pressure unit by scientific logic and however when high pressure unit fail then whole system fails. The failed pressure units can be repaired only when the whole system is in failed state. System works in full capacity when all units are good and fails when any one of the six sub-systems fails. A common cause failure which may cause the system to complete failure has also been considered in the study. All the units have sub units in series, whenever any one of the sub unit fails then that unit fails causing the system failure. There is single repair facility which is always available. Taking failure/repair rates constant and independent, a transition diagram of the system is drawn in Fig. 1 to find Primary circuits, Secondary circuits & Tertiary circuits and Base state. System behaviour analysis is discussed for different repair and failure rates of sub-units.

A number of researchers have analyzed availability parameters of various industrial system using different methods. In this research paper sensitivity analysis of Urea Fertilizer Industry has been discussed using regenerative point graphical technique

B. Panda et al. (Eds.): REDSET 2017, CCIS 799, pp. 3–16, 2018.
https://doi.org/10.1007/978-981-10-8527-7_1

(RPGT). Reporting related research in brief, Kumar et al. [1] have discussed the concept of preventive maintenance for a single unit system. Lilu, K. et al. [2] some interconnected studies were also found such as, Nakagawa, T. et al. [3], Goel and Sing [4], Gupta et al. [5], Kumar. S and Goel P. [6], Gupta et al. [7], Nidhi et al. [8], Sharma and Goel [9], Goyal and Goel [10], Gupta et al. [11], Sethi, Rachita and Garg D. [12], Garg and Yadav [13], Kumar and Ritikesh [14] have discussed behavior with perfect and imperfect switch over of system using various techniques.

Assumptions:

1. Repaired unit is as good as new one.
2. When the system is in failed state nothing can fail further.

Notations:

$\left(i \xrightarrow{s_k} j\right)$:	k-th simple path from ith-state to jth-state.
$\left(\xi \xrightarrow{sff} i\right)$:	Simple failure free path from ξth-state to ith-state.
$V_{m,k}$:	probability of mth-state reachable from the terminal state kth-state.
μ_i:	Sojourn mean time in state i, before visiting any other states;
μ_i:	$= \int_0^\infty R_i(t)dt$.
n_i:	$W_i^*(0)$ at zero time.
ξ:	Base state of the system.
f_j:	Fuzziness measure of the j-state.
○:	Full Capacity Working State
□:	Failed State
$\alpha_i, \beta_i (1 \le i \le 6)$:	Constant failure/repair rates of subunit A, B, C, D, E, F respectively.
$\alpha_7, \alpha_8, \alpha_9$:	Constant failure rate of pressure unit P_1, P_2 and P_3 respectively.
α_0:	Study failure rate of entire system from working state.
h:	Steady repair rate of system on failure of pressure unit P3.
c:	Steady repair rate of system due to common cause failure.
a:	subsystem A is failed, similarly for other units.
S_0:	Initial state when all units are good.
S_{21}:	Failed state due to the failure of pressure unit P_3.
S_{22}:	Failed state due to the common cause failure.
S_i, S_{7+i}, S_{14+i}, S_{23+i}:	Failed states due to the failure of subsystem A, B, C, D, E, F respectively for i = 1, 2, 3, 4, 5 and 6.

2 Research Problem Statement

A urea fertilizer industry consists of six sub-units Ammonia Making Section (A), Medium Pressure Section (B), Low Pressure Section (C), Pre-vacuum Section (D), Vacuum Section (E) & Periling Section (F) and three pressure units low pressure, medium pressure and high pressure units (P_1, P_2 & P_3), whenever low or high pressure units fail, then low and medium pressure steams can be obtained from high pressure unit and whenever high pressure unit fail then high pressure steam cannot be obtained, thus the whole system is failed. The failed pressure units are repaired only when the whole system is in failed state. System works in full capacity when all units are good and fails when any one of the six sub-systems fails. Common cause failures may also lead the system to complete failure. All the units have sub units in series, so any one of the sub unit fails then the unit fails and the system fails when the number of failed units is more than two. In this research the *Fuzzy concept* is used to determine failure/working state of a unit. Taking failure rates/repair rates constant & independent and taking into consideration various probabilities, a transition diagram of the system is drawn in Fig. 1 to find Primary circuits, Secondary circuits & Tertiary circuits and Base state. Problem is solved using RPGT to determine system parameters. System behavior is discussed with the help of graphs and tables. Particular cases are also taken for different repair and failure rates of the system.

The state transition Fig. 1, display that there are maximum number of primary circuits and minimum number of secondary circuits; hence vertex '0' is the base state (Table 1).

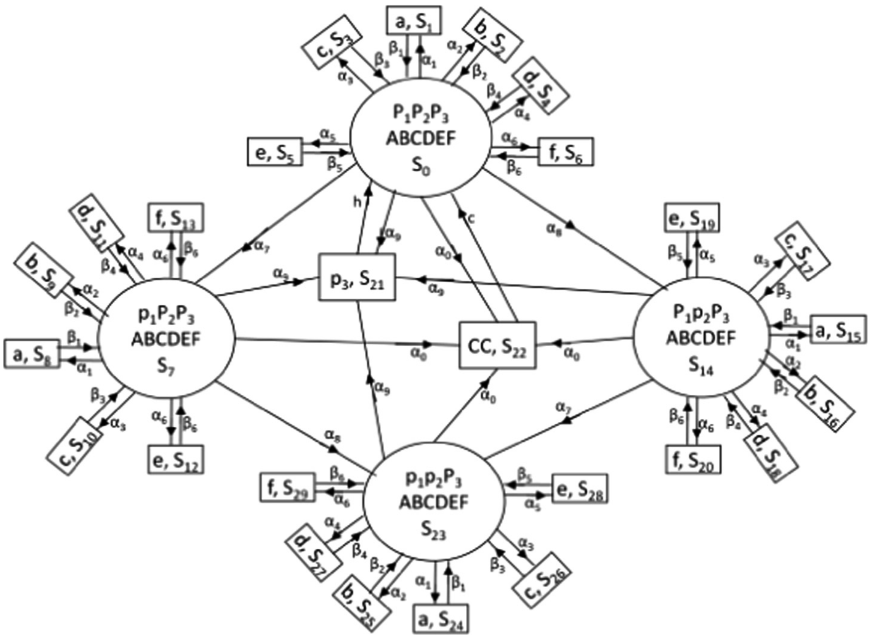

Fig. 1. Transition diagram of the system design

Table 1. Primary, secondary, tertiary circuits w. r. t. the simple paths (Base-State '0')

Vertex j	$\left(\mathbf{0} \xrightarrow{S_r} \boldsymbol{j}\right)$: ($P_0$)	(P_1)
0	$\left(0 \xrightarrow{S_0} 0\right)$: (0,1,0) $\left(0 \xrightarrow{S_1} 0\right)$: (0,2,0) $\left(0 \xrightarrow{S_2} 0\right)$: (0,3,0) $\left(0 \xrightarrow{S_3} 0\right)$: (0,4,0) $\left(0 \xrightarrow{S_4} 0\right)$: (0,5,0) $\left(0 \xrightarrow{S_5} 0\right)$: (0,6,0) $\left(0 \xrightarrow{S_6} 0\right)$: (0,22,0) $\left(0 \xrightarrow{S_7} 0\right)$: (0,21,0) $\left(0 \xrightarrow{S_8} 0\right)$: (0,14,22,0) $\left(0 \xrightarrow{S_9} 0\right)$: (0,23,22,0) $\left(0 \xrightarrow{S_{10}} 0\right)$: (0,7,21,0) $\left(0 \xrightarrow{S_{11}} 0\right)$: (0,7,22,21,0) $\left(0 \xrightarrow{S_{12}} 0\right)$: (0,7,23,22,0) $\left(0 \xrightarrow{S_{13}} 0\right)$: (0,7,22,0) $\left(0 \xrightarrow{S_{14}} 0\right)$: (0,14,21,0)	- - - - - - - - - - (14,19,14), (14,16,14), (14,17,14), (14,20,14) (14,15,14), (14,18,14) (14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23), (23,28,23), (23,29,23) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14)
1	$\left(0 \xrightarrow{S_1} 1\right)$: (0,1)	-
2	$\left(0 \xrightarrow{S_1} 2\right)$: (0,2)	-
3	$\left(0 \xrightarrow{S_1} 3\right)$: (0,3)	-
4	$\left(0 \xrightarrow{S_1} 4\right)$: (0,4)	-
5	$\left(0 \xrightarrow{S_1} 5\right)$: (0,5)	-

(*continued*)

Table 1. (*continued*)

6	$\left(0 \xrightarrow{S_1} 6\right)$: (0,6)	-
7	$\left(0 \xrightarrow{S_1} 7\right)$: (0,7)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7)
8	$\left(0 \xrightarrow{S_1} 8\right)$: (0,7,8)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (8,7,8)
9	$\left(0 \xrightarrow{S_1} 9\right)$: (0,7,9)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (9,7,9)
10	$\left(0 \xrightarrow{S_1} 10\right)$: (0,7,10)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (10,7,10)
11	$\left(0 \xrightarrow{S_1} 11\right)$: (0,7,11)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (11,7,11)
12	$\left(0 \xrightarrow{S_1} 12\right)$: (0,7,12)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (12,7,12)
13	$\left(0 \xrightarrow{S_1} 13\right)$: (0,7,13)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (13,7,13)
14	$\left(0 \xrightarrow{S_1} 14\right)$: (0,14)	(14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14)
15	$\left(0 \xrightarrow{S_1} 15\right)$: (0,14,15)	(14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14), (15,14,15)
16	$\left(0 \xrightarrow{S_1} 16\right)$: (0,14,16)	(14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14), (16,14,16)
17	$\left(0 \xrightarrow{S_1} 17\right)$: (0,14,17)	(14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14), (17,14,17)
18	$\left(0 \xrightarrow{S_1} 18\right)$: (0,14,18)	(14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14), (18,14,18)
19	$\left(0 \xrightarrow{S_1} 19\right)$: (0,14,19)	(19,14,19), (14,19,14), (14,17,14), (14,15,14) (14,16,14), (14,18,14), (14,20,14)
20	$\left(0 \xrightarrow{S_1} 20\right)$: (0,14,20)	(14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14), (20,14,20)

(*continued*)

Table 1. (*continued*)

21	$\left(0 \xrightarrow{S_1} 21\right)$: (0,21) $\left(0 \xrightarrow{S_2} 21\right)$: (0,14,21) $\left(0 \xrightarrow{S_3} 21\right)$: (0,7,21) $\left(0 \xrightarrow{S_4} 21\right)$: (0,7,23,21) $\left(0 \xrightarrow{S_5} 21\right)$: (0,14,23,21)	- (14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23) (14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23), (23,28,23), (23,29,23)
22	$\left(0 \xrightarrow{S_1} 22\right)$: (0,22) $\left(0 \xrightarrow{S_2} 22\right)$: (0,14,22) $\left(0 \xrightarrow{S_3} 22\right)$: (0,7,22) $\left(0 \xrightarrow{S_4} 22\right)$: (0,7,23,22)	- (14,19,14), (14,17,14), (14,15,14), (14,16,14) (14,20,14), (14,18,14) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23)
23	$\left(0 \xrightarrow{S_1} 23\right)$: (0,7,23) $\left(0 \xrightarrow{S_2} 23\right)$: (0,14,23)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23) (14,15,14), (14,16,14), (14,17,14), (14,18,14) (14,19,14), (14,20,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23), (23,28,23), (23,29,23)
24	$\left(0 \xrightarrow{S_1} 24\right)$: (0,14,23,24) $\left(0 \xrightarrow{S_2} 24\right)$: (0,7,23,24)	(14,15,14), (14,16,14), (14,17,14), (14,18,14) (14,19,14), (14,20,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23), (23,28,23), (23,29,23) (24,23,24) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23), (24,23,24)
25	$\left(0 \xrightarrow{S_1} 25\right)$: (0,14,23,25) $\left(0 \xrightarrow{S_2} 25\right)$: (0,7,23,25)	(14,15,14), (14,16,14), (14,17,14), (14,18,14) (14,19,14), (14,20,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23), (23,28,23), (23,29,23) (25,23,25) (7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23), (25,23,25)

(*continued*)

Table 1. (*continued*)

26	$\left(0 \xrightarrow{S_1} 26\right)$: (0,14,23,26)	(14,15,14), (14,16,14), (14,17,14), (14,18,14) (14,19,14), (14,20,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23), (23,28,23), (23,29,23) (26,23,26)
	$\left(0 \xrightarrow{S_2} 26\right)$: (0,7,23,26)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23), (26,23,26)
27	$\left(0 \xrightarrow{S_1} 27\right)$: (0,14,23,27)	(14,15,14), (14,16,14), (14,17,14), (14,18,14) (14,19,14), (14,20,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23), (23,28,23), (23,29,23) (27,23,27)
	$\left(0 \xrightarrow{S_2} 27\right)$: (0,7,23,27)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23), (27,23,27)
28	$\left(0 \xrightarrow{S_1} 28\right)$: (0,14,23,28)	(14,15,14), (14,16,14), (14,17,14), (14,18,14) (14,19,14), (14,20,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23), (23,28,23), (23,29,23) (28,23,28)
	$\left(0 \xrightarrow{S_2} 28\right)$: (0,7,23,28)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23), (28,23,28)
29	$\left(0 \xrightarrow{S_1} 29\right)$: (0,14,23,29)	(14,15,14), (14,16,14), (14,17,14), (14,18,14) (14,19,14), (14,20,14), (23,24,23), (23,25,23) (23,26,23), (23,27,23),(23,28,23), (23,29,23) (29,23,29)
	$\left(0 \xrightarrow{S_2} 29\right)$: (0,7,23,29)	(7,8,7), (7,9,7), (7,10,7), (7,11,7), (7,12,7), (7,13,7) (23,24,23), (23,25,23), (23,26,23), (23,27,23) (23,28,23), (23,29,23), (29,23,29)

There are no tertiary circuits for all simple paths.

Transition Probability and the Sojourn Mean times Definitions (Table 2).

$q_{i,j}(t)$: Probability density function from a state ‘i’ to a state ‘j’.

$p_{i,j}$: Transition probability in moving from ith-state to jth-state $p_{i,j} = q^*_{i,j}(0)$;

Sojourn Mean Times (Table 3).

$R_i(t)$: Reliability at time t at state i.

μ_i: Sojourn mean time spent in state i.

Table 2. Transition probabilities

$q_{i,j}(t)$	$P_{ij} = q^*_{i,j}(0)$
$q_{0,i}(t) = \alpha_i e^{-kt}$ $q_{0,14}(t) = \alpha_8 e^{-kt}$ $q_{0,21}(t) = \alpha_9 e^{-kt}$ $q_{0,22}(t) = \alpha_0 e^{-kt}$ Where i = 1 to 7	$p_{0,i} = \alpha_i/k$ $p_{0,14} = \alpha_8/k$ $p_{0,21} = \alpha_9/k$ $p_{0,22} = \alpha_0/k$ $k = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_9 + \alpha_8 + \alpha_0$
$q_{i,0}(t) = \beta_i e^{-\beta_i t}, q_{7+i}(t) = \beta_{7+i} e^{-\beta_i t}$ $q_{14+i}(t) = \beta_{14+i} e^{-\beta_i t}, q_{21+i}(t) = \beta_{21+i} e^{-\beta_i t}$	$p_{7+i} = 1, p_{14+i} = 1, p_{21+i} = 1$ $p_{i,0} = 1$, where $1 \le i \le 6$
$q_{7,7+i}(t) = \alpha_i e^{-(k-\alpha_7)t}$ $q_{7,23}(t) = \alpha_8 e^{-(k-\alpha_7)t}$ $q_{7,21}(t) = \alpha_9 e^{-(k-\alpha_7)t}$ $q_{7,22}(t) = \alpha_0 e^{-(k-\alpha_7)t}$	$p_{7,7+i} = \alpha_i/(k-\alpha_7)$ $p_{7,23} = \alpha_8/(k-\alpha_7)$ $p_{7,21} = \alpha_9/(k-\alpha_7)$ $p_{7,22} = \alpha_0/(k-\alpha_7)$
$q_{7+i,7}(t) = \beta_1 e^{-\beta_1 t}$	$p_{7+i,7} = 1$, where $1 \le i \le 6$
$q_{14,14+i}(t) = \alpha_i e^{-(k-\alpha_8)t}$ $q_{14,22}(t) = \alpha_0 e^{-(k-\alpha_8)t}$ $q_{14,23}(t) = \alpha_7 e^{-(k-\alpha_8)t}$ $q_{14,21}(t) = \alpha_9 e^{-(k-\alpha_8)t}$	$p_{14,14+i} = \alpha_i/(k-\alpha_8)$ $p_{14,22} = \alpha_0/(k-\alpha_8)$ $p_{14,23} = \alpha_7/(k-\alpha_8)$ $p_{14,21} = \alpha_9/(k-\alpha_8)$
$q_{14+i,14}(t) = \beta_1 e^{-\beta_1 t}, q_{23+i,23}(t) = \beta_i e^{-\beta_i t}$	$p_{14+i,14} = 1, p_{23+i,23} = 1$, where $1 \le i \le 6$
$q_{21,0}(t) = he^{-ht}, q_{22,0}(t) = ce^{-ct}$	$p_{21,0} = 1, p_{22,0} = 1$
$q_{23,22}(t) = \alpha_0 e^{-(k-\alpha_7-\alpha_8)t}$ $q_{23,23+i}(t) = \alpha_i e^{-(k-\alpha_7-\alpha_8)t}$ $q_{23,21}(t) = \alpha_9 e^{-(k-\alpha_7-\alpha_8)t}$	$p_{23,22} = \alpha_0/(k-\alpha_7-\alpha_8)$ $p_{23,23+i} = \alpha_i/(k-\alpha_7-\alpha_8)$ $p_{23,21} = \alpha_9/(k-\alpha_7-\alpha_8)$

Table 3. Sojourn mean times

$R_i(t)$	$\mu_i = R_i^*(0)$
$R_0(t) = e^{-kt}$	$\mu_0 = 1/k$
$R_{n+i}(t) = e^{-\beta_i t}$	$\mu_i = 1/\beta_i$, where $1 \le i \le 6$, n = 0, 7, 14, 23
$R_j(t) = e^{-(k-\alpha_7)t}$, where j = 7, 14, 23	$\mu_j = 1/(k-\alpha_7)$
$R_{21}(t) = e^{-ht}, R_{22}(t) = e^{-ct}$	$\mu_{21} = 1/h$, $\mu_{22} = 1/c$

The following paragraphs outline definitions of the parameters evaluation, MTSF (T_0), Availability of the system (A_0), Busy period of the server, and Expected Fractional Number of Inspection by the repair man. With some modifications, the notations are adapted from Kumar and Ritikesh [11].

Parameters Evaluation: The transition probability of states from the base state 'ξ' = '0' are:

Probabilities from state '0' to different vertices are given as

$$V_{0,0} = 1, V_{0,1} = (0,1) = p_{0,1}, V_{0,2} = (0,2) = p_{0,2}, V_{0,3} = (0,3) = p_{0,3}, V_{0,4} = (0,4) = p_{0,4}$$
$$V_{0,5} = (0,5) = p_{0,5}, V_{0,6} = (0,6) = p_{0,6}, V_{0,7} = (0,7)/L$$
$$V_{0,8} = (0,7,8)/L(1-L_7)$$

Where L = $(1-L_1)$ $(1-L_2)$ $(1-L_3)$ $(1-L_4)$ $(1-L_5)$ $(1-L_6)$, here L_i are cycles. Similarly other path probabilities are evaluated.

MTSF (T_0): The system is in working states (from initial state) before failure states are 0, 7, 14, 23, hence MTSF is given by

$$\text{MTSF } (T_0) = \left[\sum\nolimits_{i,sr} \left\{ \frac{\left\{ pr\left(\xi \xrightarrow{sr(sff)} i\right)\right\} \mu i}{\Pi_{m_1 \neq \xi}\left\{1 - V_{\overline{m_1 m_1}}\right\}} \right\}\right] \div \left[1 - \sum\nolimits_{sr} \left\{ \frac{\left\{ pr\left(\xi \xrightarrow{sr(sff)} \xi\right)\right\}}{\Pi_{m_2 \neq \xi}\left\{1 - V_{\overline{m_2 m_2}}\right\}} \right\}\right]$$

$$\begin{aligned}
T_0 = &\left(V_{0,0}\mu_0 + V_{0,7}\mu_7 + V_{0,14}\mu_{14} + V_{0,23}\mu_{23}\right)/[\{1 - (0,7,21,0) - (0,7,23,21,0) \\
&-(0,7,23,22,0) - (0,14,21,0) - (0,14,22,0) - (0,14,23,21,0) - (0,14,23,22,0)\}] \\
= &\left(V_{0,0}\mu_0 + V_{0,7}\mu_7 + V_{0,14}\mu_{14} + V_{0,23}\mu_{23}\right)/(1 - p_{0,7}p_{7,21}p_{21,0} - p_{0,7}p_{7,23}p_{23,21}p_{21,0} \\
&- p_{0,7}p_{7,23}p_{23,22}p_{22,0} - p_{0,14}p_{14,21}p_{21,0} - p_{0,14}p_{14,22}p_{22,0} - p_{0,14}p_{14,23}p_{23,21}p_{21,0} \\
&- p_{0,14}p_{14,23}p_{23,22}p_{22,0})
\end{aligned}$$

Availability of the System (A_0): The states at which the system is available are 'j' = 0, 7, 14, 23 taking 'ξ' = '0' the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum\nolimits_{j,sr} \left\{ \frac{\{pr(\xi^{sr\rightarrow} j)\} fj, \mu j}{\Pi_{m_1 \neq \xi}\left\{1 - V_{\overline{m_1 m_1}}\right\}} \right\}\right] \div \left[\sum\nolimits_{i,s_r} \left\{ \frac{\{pr(\xi^{sr\rightarrow} i)\} \mu_i^1}{\Pi_{m_2 \neq \xi}\left\{1 - V_{\overline{m_2 m_2}}\right\}} \right\}\right]$$

$$\begin{aligned}
A_0 &= \left[\sum\nolimits_j V_{\xi,j}, f_j, \mu_j\right] \div \left[\sum\nolimits_i V_{\xi,i}, f_j, \mu_i^1\right] \\
&= \left(V_{0,0}\mu_0 + V_{0,7}\mu_7 + V_{0,14}\mu_{14} + V_{0,23}\mu_{23}\right)/D
\end{aligned}$$

Where

$$\begin{aligned}
D = (&V_{0,0}\mu_0 + V_{0,1}\mu_1 + V_{0,2}\mu_2 + V_{0,3}\mu_3 + V_{0,4}\mu_4 + V_{0,5}\mu_5 + V_{0,6}\mu_6 + V_{0,7}\mu_7 + V_{0,8}\mu_8 \\
&+ V_{0,9}\mu_9 + V_{0,10}\mu_{10} + V_{0,11}\mu_{11} + V_{0,12}\mu_{12} + V_{0,13}\mu_{13} + V_{0,14}\mu_{14} + V_{0,15}\mu_{15} + V_{0,16}\mu_{16} \\
&+ V_{0,17}\mu_{17} + V_{0,18}\mu_{18} + V_{0,19}\mu_{19} + V_{0,20}\mu_{20} + V_{0,21}\mu_{21} + V_{0,22}\mu_{22} + V_{0,23}\mu_{23} + V_{0,24}\mu_{24} \\
&+ V_{0,25}\mu_{25} + V_{0,26}\mu_{26} + V_{0,27}\mu_{27} + V_{0,28}\mu_{28} + V_{0,29}\mu_{29})
\end{aligned}$$

Busy Period of the Server: The states where the server is busy are S_i, S_{7+i}, S_{14+i}, S_{23+i}, where $1 \le i \le 6$, S_{21}, S_{22} taking ξ = '0', the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum\nolimits_{j,sr} \left\{ \frac{\{pr(\xi^{sr\to}j)\}, nj}{\Pi_{m_1 \neq \xi}\{1 - V_{\overline{m_1 m_1}}\}} \right\}\right] \div \left[\sum\nolimits_{i,s_r} \left\{ \frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{m_2 \neq \xi}\{1 - V_{\overline{m_2 m_2}}\}} \right\}\right]$$

$$B_0 = \left[\sum\nolimits_j V_{\xi,j}, n_j\right] \div \left[\sum\nolimits_i V_{\xi,i}, \mu_i^1\right]$$

$$\begin{aligned} B_0 = (&V_{0,1}\mu_1 + V_{0,2}\mu_2 + V_{0,3}\mu_3 + V_{0,4}\mu_4 + V_{0,5}\mu_5 + V_{0,6}\mu_6 + V_{0,8}\mu_8 + V_{0,9}\mu_9 \\ &+ V_{0,10}\mu_{10} + V_{0,11}\mu_{11} + V_{0,12}\mu_{12} + V_{0,13}\mu_{13} + V_{0,15}\mu_{15} + V_{0,16}\mu_{16} + V_{0,17}\mu_{17} \\ &+ V_{0,18}\mu_{18} + V_{0,19}\mu_{19} + V_{0,20}\mu_{20} + V_{0,21}\mu_{21} + V_{0,22}\mu_{22} + V_{0,24}\mu_{24} + V_{0,25}\mu_{25} \\ &+ V_{0,26}\mu_{26} + V_{0,27}\mu_{27} + V_{0,28}\mu_{28} + V_{0,29}\mu_{29})/D \end{aligned}$$

Expected Fractional Number of Inspections by the Repair Man: The states where the repairman do visit's a fresh are S_i, S_{7+i}, S_{14+i}, S_{23+i}, where $1 \leq i \leq 6$, S_{21}, S_{23}taking 'ξ' = '0', the number of visit by the repair man is given by

$$V_0 = \left[\sum\nolimits_{j,sr} \left\{ \frac{\{pr(\xi^{sr\to}j)\}}{\Pi_{k_1 \neq \xi}\left\{1 - V_{\overline{k_1 k_1}}\right\}} \right\}\right] \div \left[\sum\nolimits_{i,s_r} \left\{ \frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{k_2 \neq \xi}\left\{1 - V_{\overline{k_2 k_2}}\right\}} \right\}\right]$$

$$V_0 = \left[\sum\nolimits_j V_{\xi,j}\right] \div \left[\sum\nolimits_i V_{\xi,i}, \mu_i^1\right]$$

Analytical Example with Particular Cases: Data Analysis and Regenerative Point Graphical Results

$$\alpha_i = \alpha,\ (0 \leq i \leq 9), \quad \beta_i = \beta, (1 \leq i \leq 6), \quad h = 1, \quad c = 1$$

Mean Time to System Failure (T_0) are (Table 4):

Table 4. Mean time to system failure (T_0)

T_0	β = 0.5	β = 0.6	β = 0.7	β = 0.8
α = 0.1	40.5644	40.5644	40.5644	40.5644
α = 0.2	20.2821	20.2821	20.2821	20.2821
α = 0.3	13.5212	13.5212	13.5212	13.5212
α = 0.4	10.1410	10.1410	10.1410	10.1410

Mean Time to System Failure Graph is (Fig. 2):

From the above table and graph we analyze that MTSF is constant with the increase in repair rates (Horizontal) i.e. MTSF is independent of repair rates and decreases with increases in failure rates (Vertical) which is the practical trend. Given below in figure

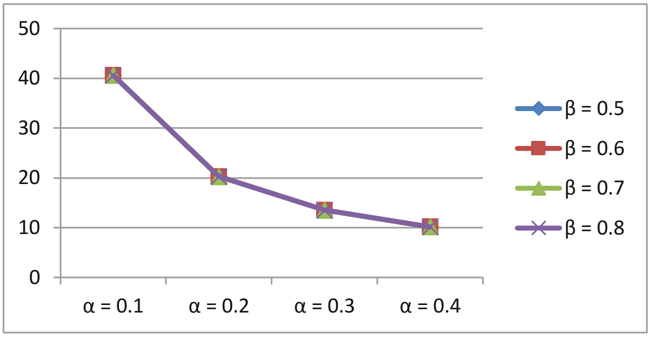

Fig. 2. Mean time to system failure graph

Availability of the System (A_0) (Table 5):

Table 5. Availability of the system (A_0) table

A_0	β = 0.5	β = 0.6	β = 0.7	β = 0.8
α = 0.1	0.40790	0.44597	0.47782	0.50487
α = 0.2	0.25620	0.28697	0.31391	0.33767
α = 0.3	0.18674	0.2155	0.23372	0.25366
α = 0.4	0.14692	0.16752	0.18617	0.20313

The Availability of the System (A_0) Graph shown below (Fig. 3):

From the above table and graph while looking horizontally it is seen that with the increase of repair rate availability and with the increase in failure rate, while observing vertically availability decreases which is the practical trend, so for optimum value of availability the repair rates of units should be kept maximums far as possible and failure rates of units should be minimum.

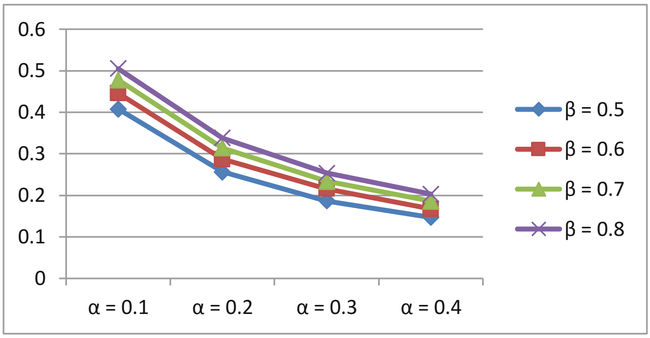

Fig. 3. Availability of the system (A_0) graph

The Busy Period of the Server (B_0) tabulated below (Table 6):

Table 6. Busy period of the server (B_0) table

B_0	β = 0.5	β = 0.6	β = 0.7	β = 0.8
α = 0.1	0.74116	0.71701	0.69679	0.67963
α = 0.2	0.83742	0.81789	0.80080	0.78572
α = 0.3	0.88149	0.86575	0.85168	0.83903
α = 0.4	0.90677	0.89369	0.88186	0.87110

The Busy Period of the Server Graph shown below (Fig. 4):

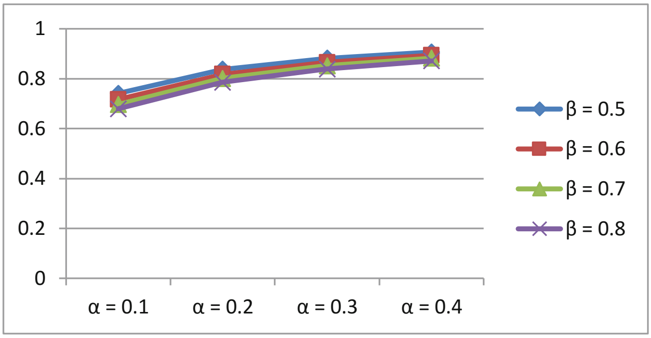

Fig. 4. Busy period of the server graph

From the above table (Horizontally) we see that busy period of the server decreases with the increase in the repair rate, but having a look vertically it increases with the increase in failure rate which match the practical situations.

The Expected Fractional Number of Inspections by the Repairman (V_0) are:

The Expected Fractional Number of Inspections by the Repairman Graph are:

From the Table 7 and Fig. 5 we see that expected fractional value of server increases significantly with the increase in failure rates in comparison to increase in repair rates.

Table 7. Expected fractional number of inspection by the repairman (V_0) table

V_0	β = 0.5	β = 0.6	β = 0.7	β = 0.8
α = 0.1	0.13091	0.14313	0.15336	0.16204
α = 0.2	0.16445	0.18421	0.20150	0.21675
α = 0.3	0.17981	0.22500	0.22505	0.24425
α = 0.4	0.21005	0.22607	0.23901	0.24847

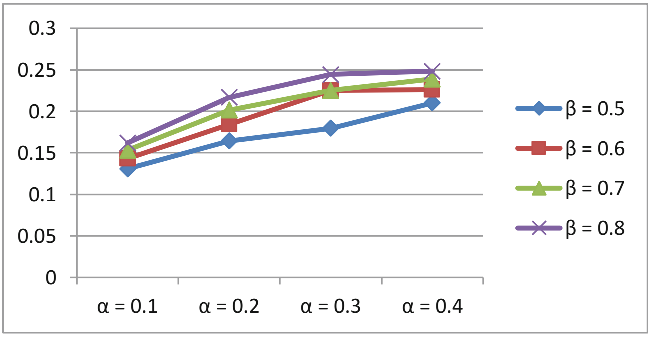

Fig. 5. Expected fractional number of inspection by the repairman graph

3 Resulting Conclusion

In order for the urea fertilizer factory system, to have optimum value of system parameters management can control the failure and repair rates of units depending upon the availability of finances and market circumstances. In particular, the optimum value of availability repair rates of units should be kept maximum as far as possible and failure rates of units should be minimum. Also, the expected fractional value of server increases significantly with the increase in failure rates in comparison to increase in repair rates.

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