Energy-Efficient Optimum Design for Massive MIMO

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1 Introduction

Over the past few decades, wireless access has been increased due to advancement of applications of cellular applications and internet of things. It leads to exponential growth in network traffic. Environmental and economic concerns demand to improve energy efficiency therefore research has been diverted towards optimizing energy efficiency. Future wireless communication system, e.g., 5G network which will be deployed up to year 2020 expected to achieve energy efficiency through various techniques, e.g., use of more number of base station in the smaller area, use of unused mm wave spectrum and massive MIMO [3, 4, 6, 13, 14].

In channel estimation and precoding, the circuit power consumption was very high [4]. Looking toward energy-efficienct operations in calculating channel estimation and precoding has been changed in the proposed work. A single cell of downlink massive MIMO channel is considered. A new refine model highlights the total number of complex operation performed to estimate channel information and ZF precoding. Results of the proposed work shows that by using ZF precoding and MRT technique at base station in massive MIMO regime gives area throughput that changes with increasing number of antenna at base station and also optimum value of energy efficiency with respect to optimum number of antennas at the base station and optimum number of user terminals.

The paper is organized as follows: The system model is discussed in Sect. 2. Power consumption model is described in Sect. 3. These are then used to compute optimal number of base station antennas, the optimal number of user terminals and transmit power under the assumption of perfect channel state information by using zero forcing precoding. In Sect. 4, numerical results are used to confirm the theoretical analysis. Finally, conclusion and future scopes are described in Sect. 5.

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2 System Model

Consider the downlink of a massive MIMO system of single cell, where a BS having M antennas transfer information to K user terminals (UTs) having single antenna. The channel is Rayleigh fading channel with zero mean and λ_k variance (Fig. 1).

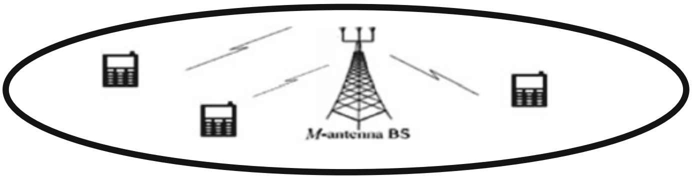

Fig. 1. A massive MIMO scenario: a circular cell with M-antenna BS and K single-antenna UTs

Let ‘x’ is Mx1 precoded vector of the complex information symbol transmitted from antenna of base station. The signal received by the user antenna $y \in C^{Kx1}$ is then given as [13]

$$y = \sqrt{\rho} H^H x + n \tag{1}$$

where H represents the M × K channel matrix between the M antenna at BS and the K user terminals. n $\in C^{KX1}$ is the additive white gaussian noise (AWGN) having zero mean and variance $\sigma^2 = N_0 B$. Here channel bandwidth and power spectral density of the AWGN denotes by B (Hz) and N_0 (W/Hz) respectively.

Channels are static within time–frequency coherence blocks of $T = B_c T_c$ symbols (channel uses) where B_c is coherence bandwidth and T_c is coherence time. The assumption is taken that BS and UTs are perfectly synchronized and using the time-division duplex (TDD) protocol to acquire the channel state information (CSI). The pilot signaling occupies τK symbols in uplink. Pilot signals length $\tau K < T$ for each transmission where $\tau \geq 1$ to enable orthogonal pilot sequences among the UT [2, 8, 13]. The case undertaken when the BS has perfect CSI, i.e., the BS has full knowledge of the instantaneous channel realization and possibly of the interferences statistics at the UT.

3 Generic Modal of Energy Efficiency (EE)

EE (bits/joule) is the ratio of the total number of bits transferred and total power consumed. EE might not increase only by reducing transmit power because additional power consumed from digital signal processing and analog filters used for RF and baseband processing. EE is given as

$$EE = \frac{R}{\frac{\rho KBA_{\lambda}}{\eta} + P_{CKT}} \tag{2}$$

where information rate (R) for ZF precoding for all K user is given below as [3]

$$\mathrm{R} = BK\left(1 - \frac{\tau \mathrm{K}}{T}\right)\log_2(1 + \rho(M - K)) \tag{3}$$

where $\left(1 - \frac{\tau \mathrm{K}}{T}\right)$ is pre-log factor accounts for pilot overhead. This term is included due to the fact that in training period τK time slots are used to estimate CSI during one coherence block of T. T − τK shows the number of data transmission slots and obtained information rate is to be averaged over T channel uses. Information rate can be increased by using precoding technique. For zero forcing precoding $\mathrm{V} = H(H^{H}\mathrm{H}\ )^{-1}$ and for MRT V = H.

Power amplifier (PA) power is $\frac{\rho KBA_{\lambda}}{\eta}$ [3] where ρ is radiated power and $0 < \eta \leq 1$ is PA efficiency. The user distribution and propagation environment is indicated by A_{λ} as remark 1 from [3]. P_{CKT} is power consumed by other circuit systems which have described in power consumption model.

3.1 Circuit Power Consumption Model

A massive MIMO transceiver is consumed power M $\mathrm{P_{tx}}$ + K $\mathrm{P_{rx}}$ + $\mathrm{P_{syn}}$ watt [4]. $\mathrm{P_{tx}}$ is power consumed by component of each antenna attached to BS (as converters, mixers, and filters). $\mathrm{P_{rx}}$ power is need by circuit component of each single-antenna UT (as amplifiers, oscillator, mixer, and filters). $\mathrm{P_{syn}}$ is power consumed by single local oscillator which is used for all BS antenna. power is required for channel coding and decoding RK($\mathrm{P_{cod}}$ + $\mathrm{P_{decd}}$) watt [10], where $\mathrm{P_{cod}}$ and $\mathrm{P_{decd}}$ are the power needed for coding and decoding respectively. Fixed power $\mathrm{P_{fix}}$ is required for control signaling, site-cooling and by backhaul infrastructure that is independent of load power and baseband processors [4]. $\mathrm{P_{bh}}$ is power required for load dependent backhaul infrastructure [3]. The complete complex operations performed in the channel estimation and ZF multiuser detection, i.e., ($\mathrm{2MKT + 4MK^2 + (8\ K^3/3)}$) [11] that are less computed operation than assumed in [3, 4].

(1) Total number of operations calculated $\mathrm{2MK^2}\tau$ by multiplying M × τK matrix with τK × K matrix for computing the channel matrix ($\widehat{H}$) [11].

(2) To calculate the pseudo-inverse of $\widehat{H}$ i.e. $V = \widehat{H}\left(\widehat{H}^{H}\widehat{H}\right)^{-1}$, {$\mathrm{4MK^2 + (8K^3/3)}$} total number of operations needed that is explained below:

- $\mathrm{2MK^2}$ number of operations need to calculate $\left(\widehat{H}^{H}\widehat{H}\right)$ by multiplying K × M matrix with M × K matrix.
- $\mathrm{8K^3/3}$ number of operations need to calculate $\left(\widehat{H}^{H}\widehat{H}\right)^{-1}$ by taking inversion of K × K matrix [5].

- 2MK^2 number of operations need to calculate $\widehat{H}\left(\widehat{H}^H\widehat{H}\right)^{-1}$ by multiplying M × K with K × K matrix.

(3) 2MK (T − Kτ) number of operations need for data phase(T − Kτ channel uses) ZF multiuser detection by multiplying K × M matrix with M × 1 vector [11].

Let a single complex operation is need L_0 J energy. Total number of operations described above are computed in T_c s. Hence, the average power need to compute these many operations is described below as [11].

$$P_c = 2\mathrm{MK}L_0B_c + 4\mathrm{M}K^2\frac{L_0}{T_c} + \frac{8K^3}{3}\frac{L_0}{T_c} \tag{4}$$

Total power needs to estimate channel information and to detect multiuser by MRT technique is $2\mathrm{MK}L_0B_c$ [3, 12].

3.2 Energy Efficiency Optimization with ZF Processing

This section is designed to elaborate the EE model in more compact form for better understanding and comprehensibility using ZF precoding. Analytic processing to get optimize value of M, K, and ρ is proposed.

$$EE = \frac{BK\left(1-\frac{\tau K}{T}\right)\log_2(1+\rho(M-K))}{\frac{\rho KBA_\lambda}{\eta} + \sum_{i=0}^{3} C_iK^i + M\sum_{i=0}^{2} D_iK^i + EKB\left(1-\frac{\tau K}{T}\right)\log_2(1+\rho(M-K))} \tag{5}$$

Björnson et al. [4] proposed EE is given in (5) and proposed circuit power consumption coefficients C_i, D_i and E for ZF are $C_0 = P_{syn} + P_{fix}$, $C_1 = P_{rx}$, $C_2 = 0$, $C_3 = \frac{8L_0}{3T_C}$, $E = P_{cod} + P_{dec} + P_{bt}$, $D_0 = P_{tr}$, $D_1 = 2L_0B_c$, $D_2 = 4\frac{L_0}{T_c}$ and for MRT are $C_0 = P_{syn} + P_{fix}$, $C_1 = P_{rx}$, $C_2 = 0$, $C_3 = 0$, $E = P_{cod} + P_{dec} + P_{bt}$, $D_0 = P_{tr}$, $D_1 = 2L_0B_c$, $D_2 = 0$.

Lemma 3 from [4] gives the solution of the EE optimization problem explained in (5) and the behavior of z^{opt} determines by lemma 4 from [4] need to study the behavior of optimized solution of M, K, and ρ.

Optimal Transmit Power

Transmit power ρ is directly proportional to SINR and also to power amplifier (PA) transmit power under ZF processing.

Proposition 1 Optimized value of ρ is given by (6) for maximizing EE explained in (5).

$$\rho^{opt} = \frac{e^{W\left(\frac{\eta}{BA_\lambda}\frac{(M-K)(C\prime+MD\prime)}{e}-\frac{1}{e}\right)+1}-1}{M-K} \tag{6}$$

$$\mathrm{C}' = \frac{\sum_{i=0}^{3} C_i K^i}{K} \quad \text{and} \quad \mathrm{D}' = \frac{\sum_{i=0}^{2} D_i K^i}{K} \tag{7}$$

$\mathrm{C}' > 0$ and $\mathrm{D}' > 0$ are defined above. The proof of Proposition 1 is very similar to Theorem 3 from [4]. Only circuit power coefficients have changed. W(x) is a lambert W function of x [7].

Optimal Number of Base Station Antennas

Proposition 2 We find optimum number of base station antenna for given value of $\rho \, and \, K$ as $M^{opt} = \lfloor M \rceil$. M is given by Eq. (8)

$$M = \frac{e^{\mathcal{W}\left(\frac{\rho\left(\frac{\rho B A_{\lambda}}{\eta} + C\prime\right)}{D\prime e} + \frac{\rho K - 1}{e}\right) + 1} + \rho K - 1}{\rho} \tag{8}$$

where $\mathrm{C}' > 0$ and $\mathrm{D}' > 0$ are defined in (7). The proof of proposition 2 is very similar to theorem 2 from [4]. Only circuit power coefficients have changed. Optimum value of M is noninteger value got in a solution of objective function, but quasiconcavity tells that the M^{opt} is attained at one of the two closest integers.

Optimal Numbers of User Terminals

Proposition 3 We look forward to find optimal number of user terminals when M and ρ are given. We assume $\rho K = \bar{\rho} \, and \, \frac{M}{K} = \bar{\beta}$ as constant with $\bar{\rho} > 0$ and $\bar{\beta} > 1$. We found

$$K^{opt} = \max_i \lfloor K_i \rceil \tag{9}$$

K_i is real positive roots of the quartic equation given below

$$K^4 - \frac{2T}{\tau} K^3 - \mu_1 K^2 - 2\mu_0 K + \frac{T\mu_0}{\tau} = 0 \tag{10}$$

where

$$\mu_1 = \frac{\frac{T}{\tau}\left(C_2 + \bar{\beta} D_1\right) + C_1 + \bar{\beta} D_0}{C_3 + \bar{\beta} D_2} \quad \text{and} \quad \mu_0 = \frac{C_0 + \frac{A_{\lambda} B \bar{\rho}}{\eta}}{C_3 + \bar{\beta} D_2} \tag{11}$$

The notation $\lfloor . \rceil$ in (9) tells that optimal value K^{opt} is either the closest smaller or closest larger integer to K_i, that can be find out with the comparison of corresponding EE. The proof of proposition 3 is very similar to theorem 1 from [4]. Only circuit power coefficients changed.

Table 1 Simulation Parameters

Cell radius (single cell): d_{max}	250 m	P_{fix}	18 W
Minimum distance: d_{min}	35 m	P_{syn}	2 W
Path loss at distance d: λ	$\frac{10^{-3.53}}{d^{3.76}}$	P_{rx}	1 W
B	20 MHz	P_{tx}	1 W
B_c	180 kHz	P_{cod}	0.1 W/(Gbit/s)
T_c	10 ms	P_{dec}	0.8 W/(Gbit/s)
L_0 (Joule/operation)	10^{-9}	P_{bt}	0.25 W/(Gbit/s)
Pilot length τ	2	M	280
		K	180

4 Numerical Results

Simulation parameter values are taken from [1, 9] and summarized in Table 1. We made an assumption that user is uniformly distributed in a 250 m radius circular cell. The propagation parameter $\boldsymbol{A}_{\lambda}$ is calculated as in remark 1 from [3].

Figure 2 depicts that the obtained EE for various value of M and K under ZF precoding (note that $M \geq K$ due to ZF). Each value of M and K used to find the value of ρ from (6) where EE is maximized. The figure illustrates that there is a global optimal point at M = 231 and K = 180 with ρ = 1.1466 and EE = 76.25. Standard alternating optimization algorithm is taken from [4] to search the combined global optimum. Figure 2 is concave and smooth. EE is improved two times the results of Bj$\ddot{o}$rnson et al. [4]. The algorithm initiation point is M = 3, K = 1, ρ = 1 taken. The algorithm converged after 8 iterations to a suboptimal solution.

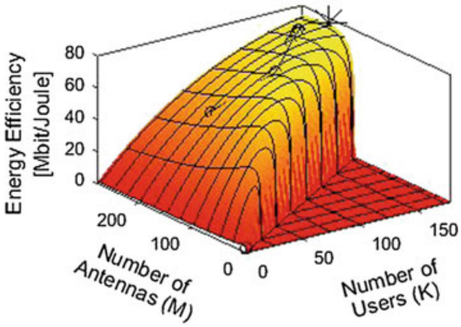

Fig. 2. Under the consideration of single-cell scenario, the variation of energy efficiency (in Mbit/J) for different combinations of M and K with ZF precoding. The global optimum is indicated by star while the circles shows the obtained convergence point of alternating optimization algorithm

For comparisons Fig. 3 shows energy efficiency for MRT precoding. Its result was generated by Monte Carlo simulations while Fig. 2 was computed using our analytic results. EE optimal value is achieved at M = 123, K = 99. EE in MRT case is improved 6.4% than the result of Bj*ö*rnson et al. [4].

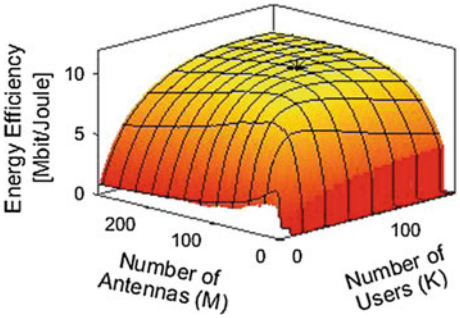

Fig. 3. Under the consideration of single-cell scenario, the variation of energy efficiency (in Mbit/J) for different combinations of M and K with MRT. The global optimum is marked with a star

By comparing Fig. 2, 3 it is deduce that EE is better in ZF than MRT under perfect CSI in single-cell scenario. Figure 4 shows the area throughput at maximizing point of the EE for different M and at that point area throughput is increased than result of Bj*ö*rnson et al. [4]. Area throughput is very high for ZF than MRT processing. Area throughput at optimizing point of EE is 181.2 Gbits/s. for ZF case and 9.99 Gbits/s. for MRT case which shows three and two times improvement than the result of Bj*ö*rnson et al. [4] respectively.

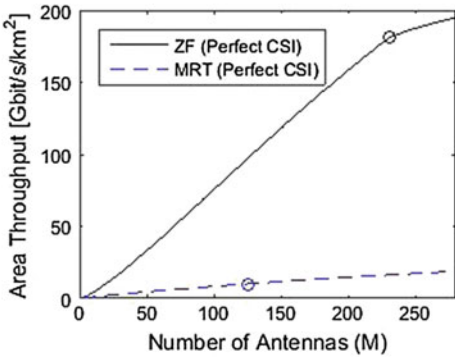

Fig. 4. Under the consideration of single–cell scenario, the variation of Area Throughput at the EE-maximizing solution for different number of BS antenna M

A brief summary of work done and proposed methodology in massive MIMO for ZF and MRT described in Table 2. From the analysis we deduce that optimal point of EE is improved with increasing M and K up to a particular limit.

Table 2. Comparison on various parameter employed in different literature

Properties	Björnson et al. [3]		Björnson et al. [4]		Proposed work	
	ZF	MRC	ZF	MRC	ZF	MRC
Energy efficiency	7	1.5	30.7	9.86	76.25	10.49
M^{opt}	165	4	165	81	231	123
K^{opt}	85	1	104	77	180	99

5 Conclusion

In this paper, circular single cell equipped with M number of antenna at base station and K user terminals with single antenna. Information rate is used under ZF precoding. A new refine model for power consumption is acquired hence the EE is increased with the increment of optimum number of antenna at base station and user terminals. Linear precoding scheme only useful to mitigate intracell interference. In multi-cell scenario, pilot contamination diminishes the linear precoder advantage. To mitigate inter-user interference complex nonlinear precoding technique like dirty paper coding and maximal likelihood detection technique should be used. Multi-cell scenario considered as future work.

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