Bayesian Estimation of Irregular Stochastic Volatility Model for Developed and Emerging Stock Market

Kirti Arekar, Rinku Jain and Surender Kumar

Abstract This research is to study the irregular stochastic volatility (ASV) for developed and emerging stock markets that are predictable with Markov Chain Monte Carlo (MCMC) model. Data cover daily closing prices for five years from 2010 to 2016. The study is done on secondary data. We considered six developed countries stock returns and six emerging countries stock returns. The arrangement of developed and emerging markets is created on the economic status such as GDP, GNP and per capita income. We used the reference of Human Development Index (HDI) statistics to identify the developed and emerging markets for the study where they rank the countries on the basis of their development. In this article, the developed and emerging countries considered for the study show the mature markets as compared to the countries which we excluded from the study. The data considered were taken from the value-weighted equity market indices of three developed and six emerging countries. According to the MSCI reference, there are six indices, i.e. India (S&P CNX NIFTY and BSE SENSEX), USA (DOWJONES, NASDAQ-100 and S&P-500) and UK(FTSE-100) which are classified as developed markets and six emerging countries stock returns, i.e. France(CAC-40), Spain (IBEX 35), Malaysia (KLSE), Japan (NIKKI-225), Singapore (STRAIT TIMES) and Taiwan (TAIWAN WEIGHTED) which are classified as emerging countries. The findings provided that in the developed stock markets, high volatility persistence was present in USA NASDAQ-100 and India BSE SENSEX and in the emerging stock markets high volatility was present in Malaysia and Singapore. The strong and significant leverage effect present in India S&P CNX NIFTY, USA DOWJONES, USA S&P-500 and UK FTSE-100 from the developed

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countries stock markets and the leverage effect present in France (CAC-40), Spain (IBEX 35) and Japan (NIKKE-225) from the emerging countries stock markets.

Keywords Bayesian estimation · Volatility · Monte Carlo simulation model and irregular stochastic volatility

1 Introduction

The stochastic volatility (SV) models are suppressed procedure and are commonly used in stock market returns. In stochastic model (SV), the irregular property is dependent on the undeviating correlation among the expansions in both the returns and the volatility exists in both the emerging and developed stock markets. In this specific model, some unobserved component follows some latent stochastic process.

Volatility plays a significant role in pricing, portfolio and assets allocation. Volatility of a daily stock market for emerging and developed market is investigated by using irregular stochastic model which is assessed by using Markov Chain Monte Carlo (MCMC) method. Irregular stochastic model considered unobserved components following some latent stochastic process. In a Bayesian approach, the parameters are considered as random—variables with a given priors distribution where we assume the set of unknown parameters.

The general irregular SV model (*ASV*) is as follows:

$$S(t) = \sigma(t)A_1(t)$$
$$\sigma^2(t) = \alpha + \beta\sigma^2(t) + \sigma_v A_2(t)$$

where $A_1\ (t)$ and A_2(t) are two correlated Brownian motion.

The model is expressed as follows:

$$r_t = \sigma_t u_t$$
$$\ln\ \sigma^2_{t+1} = \alpha + \phi\ \ln\ \sigma^2_t + \sigma_v v_{t+1}$$

where $r_t = \ln(S_{t+1)}/S_t)$ is the rate of return of stocks in the market and $\varphi = 1 - \beta$, σ_v is the volatility.

Yu (2004) proposed the following Gaussian nonlinear state form which indicates the correlation coefficient ρ:

$$r_t = \sigma_t u_t$$
$$\ln\ \sigma^2_{t+1} = \alpha + \phi\ \ln\ \sigma^2_t + \sigma_v \sigma_t^{-1} \rho\, r_t + \sigma_v (1 - \rho^2)^{1/2} w_{t+1}$$

The main aim of the research is to study, the irregular stochastic volatility (ASV) for developed and emerging stock markets that is assessed with Markov Chain Monte Carlo (MCMC) model. Irregular stochastic volatility (ASV) is based on the size and magnitude of the stock returns. Irregular impact has identified the relationship between the stock returns and volatility dynamics. The data cover the daily closing prices for five years from 2010 to 2016. The study is based on secondary data. We considered six developed countries stock returns, i.e. India, USA and the UK and six emerging countries stock returns, i.e. France, Spain, Malaysia, Japan, Singapore and Taiwan.

2 Literature Review

Bekaert and Wu (2000) and Wu (2001) findings say that there is a negative association which exists among the stock market index and advance volatility. Harrison and Moore (2012) measured ten stock market index of Central and Eastern European stock markets, and he also concluded that there is the presence of irregular volatility in the Central and Eastern European stock markets.

Aycan Hepsag (2016) also considered irregular stochastic models for Central and Eastern European (CEE) markets. The research identified that the high variability of volatility and high volatility persistence in stock markets of Poland and Lufthansa.

Alizadeh et al. (2002) estimated SV-LS model to identify whether there is the presence of leverage effect with respect to the association of South East Asian National Countries (ASEAN5). Hsu and Chiao (2010) used the Markov chain model by considering two levels of stochastic persistence. They also analysed the time pattern of the stock markets.

Early studies by Black (1976) and Christie (1982) concluded that increase in debt–equity ratio is due to fall in the stock prices, and the risk associated with the respective firm will increase. French et al. (1987) and Campbell and the Hents Cheal (1992) in there study shows the positive relationship among volatility and expected returns whereas Nelson (1991) and Gloston et al. (1993) shows negative relationship associated among predictable stock returns and restricted volatility.

Selcul (2004a, b) shows that there is a negative relationship between persistence in volatility and variability of volatility and also there is a negative correlation among leverage effect and persistence of volatility. Taylor (1994) proposed a stochastic model where the deterministic function of lagged squared return is used as the stochastic function of unobserved latent variables. The advantages of stochastic model are explained by the Carnero et al. (2004), and Das et al. (2009) have the capability to interpret on step ahead forecasting with respect to the stock market index.

3 Research Methodology and Findings

Volatility model for the stationary series of returns r_t is given as follows:

$$r_t = \alpha_t + Z_t$$
$$Z_t = \sigma_t \varepsilon_t$$

where ε_t is the identical independent discrete random variable with mean zero and unit variance and σ_t is the deterministic–stochastic random process which depends on the past values of the returns. α_t is constant and Z_t denotes the stochastic process.

The stochastic volatility model is represented in the following form:

$$Z_t = \exp\left(k_t/2\right)\varepsilon_t$$
$$K_t = \gamma + \theta K_{t-1} + \eta_t$$

where K_t is the latent stochastic volatility which equals to $\ln \sigma_t^2$, η_t is the i.i.d. (identical independent) a random variable with mean zero and variance σ_n^2 measures the uncertainty about future volatility. θ is the measure of the persistence of shocks to the volatility.

Asai and McAleer (2005) proposed the following specification:

$$Z_t = \exp\left(K_t/2\right)\varepsilon_t$$
$$K_{t+1} = \mu + \phi K_t + \eta_t; \qquad \varepsilon_t \sim N(0,1),\ \eta_t \sim N(0,\sigma_n^2)$$
$$E\left(\varepsilon_t \eta_t\right) = \rho\sigma_n$$

when $\rho < 0$, the type of irregularity exists is dynamic leverage stochastic volatility, whereas when $\rho = 0$, there exists no dynamic leverage among the innovations to returns and volatility (Asai and McAleer 2005).

The classification of developed and emerging markets is based on the economic status such as GDP, GNP and per capita income. We used the reference of Human Development Index (HDI) statistics to identify the developed and emerging markets for the study where they rank the countries on the basis of their development.

In this article, the developed and emerging countries considered for the study show the mature markets as compared to the countries. The data employed in the study are drawn from the value-weighted equity market indices of three developed and six emerging countries. According to the MSCI reference, there are six indices, i.e. India (S&P CNX NIFTY and BSE SENSEX), USA (DOWJONES, NASDAQ-100 and S&P-500) and UK (FTSE-100) which are classified as developed markets and six emerging countries stock returns, i.e. France (CAC-40), Spain (IBEX 35), Malaysia(KLSE), Japan (NIKKI-225), Singapore (STRAIT TIMES) and Taiwan (TAIWAN WEIGHTED) which are classifies as emerging countries.

The datasets involved daily closing price indices of developed and emerging markets. In developed countries, there are six indices, i.e. India (S&P CNX NIFTY and BSE SENSEX), USA (DOWJONES, NASDAQ-100 and S&P-500) and UK (FTSE-100) and six emerging countries stock returns, i.e. France (CAC-40), Spain (IBEX 35), Malaysia (KLSE), Japan (NIKKI-225), Singapore (STRAIT TIMES) and Taiwan (TAIWAN WEIGHTED) for the period 01/01/2010–31/12/2016, a total 1769 observations. The source of the data is www.econstats.com and www.moneymarket.com.

We considered stock returns from the stock market indices of the selected countries using $\ln\left(P_t/P_{t-1}\right) * 100$, where P_t denotes the value of the stock prices indices of each country at time t.

The descriptive statistics for the stock returns of each stock are shown in Table 1 and 2.

The descriptive statistics for the stock returns of each stock market indices for emerging and developed countries are shown in Tables 1 and 2. From the descriptive statistics of developed countries presented in Table 1, we observed that India (S&P CNX NIFTY and BSE SENSEX), USA (NASDAQ-100) and UK (FTSE) show the negative average returns, whereas USA (DOWJONES and S&P-500) shows the positive average returns. All the developed countries stock returns the standard deviation is greater than the mean of the stock returns, indicating that the all the developed countries follow a random walk process. The sample skewness is negative for the USA (DOWJONES and S&P-500). These indicate that irregular tail extends more towards negative values as compared to the positive values. And for rest of developed countries have the positive skewness. Positive skewness ranges from 0.139 India (S&P CNX NIFTY) to 0.340 USA (NASDAQ-100). The sample kurtosis approximation (the lowest, 1.477 India BSE SENSEX and highest, 3.557 USA DOWJONES) indicates that the return distributions for all the developed countries are fat-tailed.

From the descriptive statistics of emerging countries presented in Table 2, we observed that France (CAC 40), Malaysia (KLSE), Japan (Nikkei-225) and Taiwan (Taiwan weighted) show the negative average returns, whereas Singapore shows the positive average returns. All the emerging countries stock returns the standard deviation is greater than the mean of the stock returns, indicating that the all the

Table 1 Summary statistics of the developed stock markets

Countries	n	Mean	Standard deviation	Kurtosis	Skewness	Maximum	Minimum
India S&P CNX NIFTY	1712	−0.0001	0.0045	1.5129	0.139	0.026	−0.016
India BSE SENSEX	1730	−0.0002	0.0102	1.477	0.166	0.061	−0.037
USA DOWJONES	1761	0.00015	0.0039	3.557	−0.385	0.0180	−0.024
USA NASDAQ-100	1727	−0.0002	0.0047	2.815	0.340	0.027	−0.021
USA S&P-500	1790	0.00014	0.0040	4.935	−0.469	0.0299	0.0201
UK FTSE-100	1766	0.00006	0.0044	2.7388	0.253	0.026	−0.021

Table 2 Summary statistics of the emerging stock markets

Countries	n	Mean	Standard deviation	Kurtosis	Skewness	Maximum	Minimum
France (CAC-40)	1756	−0.000	0.0058	0.155	3.4971	0.036	−0.040
Spain (IBEX 35)	1792	0.000	0.0067	6.905	0.140	0.057	−0.058
Malaysia (KLSE)	1645	−0.0001	0.003	0.340	2.425	0.019	−0.014
Japan (NIKKI-225)	1696	−0.0001	0.006	0.448	6.923	0.048	−0.046
Singapore (STRAIT TIMES)	1460	0.0001	0.0035	0.340	2.425	0.019	−0.014
Taiwan (TAIWAN WEIGHTED)	1731	−0.0004	0.004	0.478	2.818	0.024	−0.019

developed countries follows a random walk process. The sample skewness for all the countries from emerging market is positive; these indicate that irregular tail extends more towards positive values as compared to the negative values. The sample kurtosis approximation (the highest, 6.293 Japan NIKKI-225 and lowest, 2.425 Malaysia and Singapore) indicates that the return distributions for all the developed countries are fat-tailed.

From Fig. 1, it is observed that for the developed countries, i.e. USA (DOWJONES, NASDAQ-100 and S&P-500) and UK (FTSE-100), volatility is increasing continuously as compared to India (BSE, NIFTY). The volatility for the Indian market is also increasing, but at the end of the Year 2016, it is decreasing as compared to other developed markets. From Fig. 2, we identified that for the emerging countries, i.e. Japan (NIKKI-225) and France (CAC-40), the volatility is continuously higher than the other emerging countries, i.e. Spain (IBEX-35), Malaysia (KLSE), Singapore (Strait Times) and Taiwan (Taiwan Weighted). Spain (IBEX-35) and Singapore (Strait Times) have a lower level of volatility. Overall, the stock market index for all developed countries seems to be consistently higher than the emerging countries. This relationship helps us to understand the volatility pattern for developed markets and emerging markets.

In this study, we used MCMC approach for estimating leverage model and we used the code provided by Yasuhiro Omori which will be used in the WinBUGS software. For the prior values we considered as $\mu \sim$ Inverse-Normal (−10,1), $\rho \sim$ Inverse-Uniform (−1,1), $\sigma_n^2 \sim$ Inverse-Gamma(2.5,0.025) and $\phi \sim$ Inverse-Beta (20,1.5) and in MCMC model we initialized the values $\mu = -9$, $\sigma_n^2 = 100$, $\phi = 0.095$ and $\rho \sim -0.4$ following also Yasuhiro Omori. We used 95% interval for the posterior means of parameter approximation.

From Table 3, it is observed that the approximations of the volatility determination coefficient ϕ are in among 0.861 USA NASDAQ-100 and 0.956 India BSE

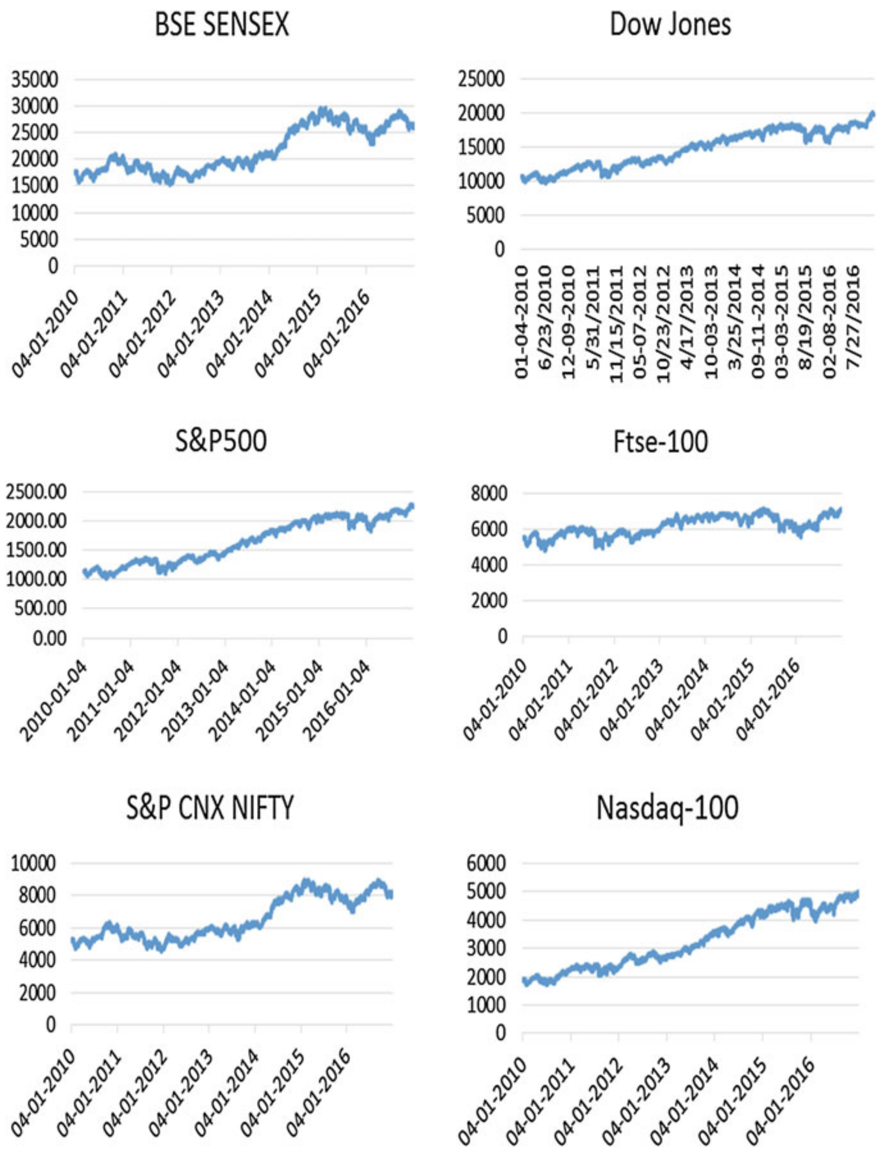

Fig. 1 Volatility pattern for developed markets

SENSEX. The results indicate that there is high volatility persistence in these stocks markets. The upper interval values of 95% credibility intervals are less than 0.99 except India BSE SENSEX and USA NASDAQ-100. These results are consistence with the results suggested by Jacquier et al. (1994, 2004).

The posterior means of the coefficient $\hat{\rho}$ indicate the correlation among shocks to return at time t, and shocks to volatility at time t + 1 are negative for all the

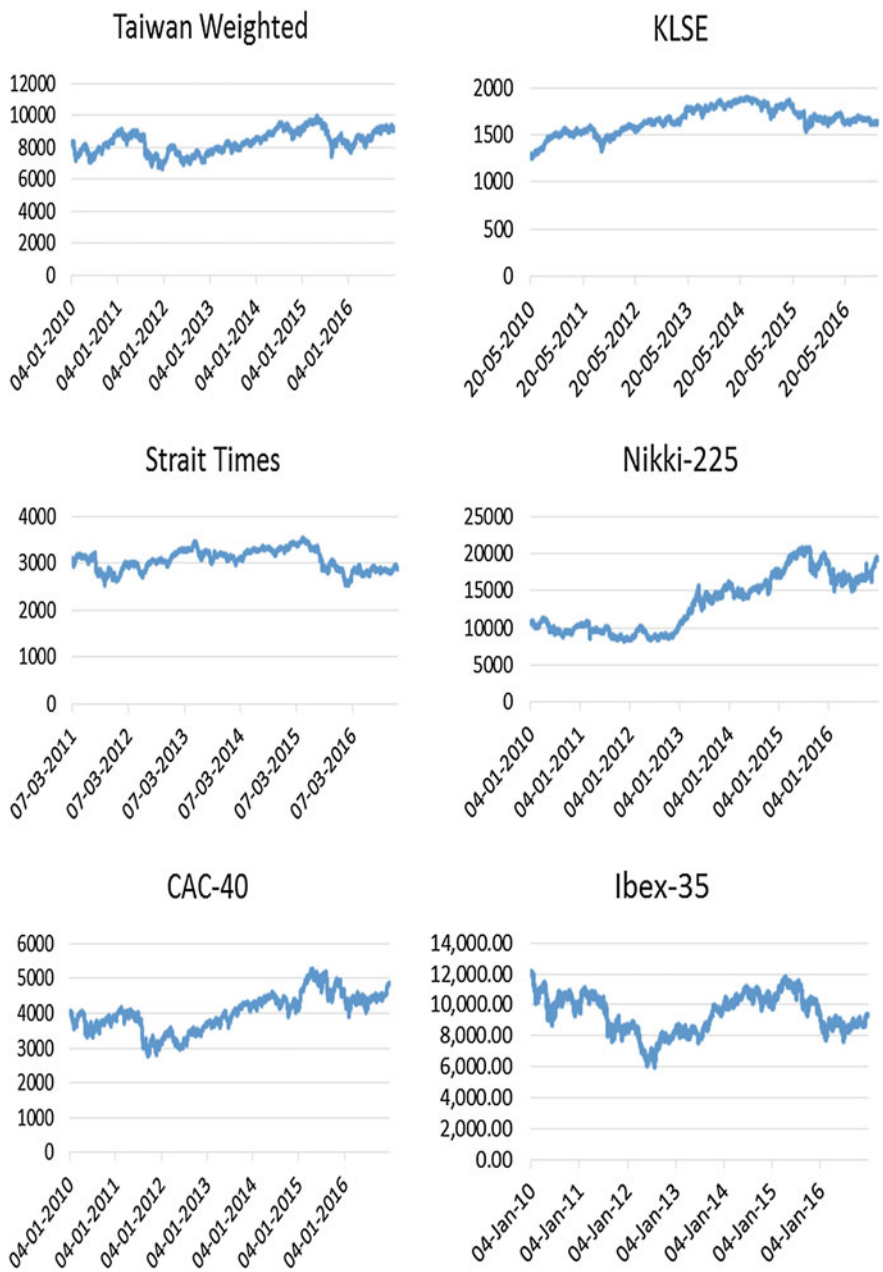

Fig. 2 Volatility pattern for emerging markets

Table 3 Estimation results of the posterior means of parameters for developed countries

Countries	$\hat{\phi}$	$\hat{\rho}$	$\hat{\sigma}$
India S&P CNX NIFTY	0.921, [0.89, 0.95]	−0.321*, [−0.45, – 0.24]	0.14*, [0.01, 0.46]
India BSE SENSEX	0.956, [0.96, 0.99]	−0.009, [−0.31, −0.06]	0.12*, [0.10, 0.34]
USA DOWJONES	0.93, [0.90, 0.96]	−0.121*, [−0.37, −0.21]	0.10*, [0.02, 0.45]
USA NASDAQ-100	0.861, [0.98, 0.99]	−0.323, [−0.31, −0.03]	0.11*,[0.05, 0.39]
USA S&P-500	0.946, [0.93, 0.95]	−0.112*, [−0.17, −0.22]	0.22*, [0.15, 0.42]
UK FTSE-100	0.921, [0.91, 0.97]	−0.621*, [−0.16, −0.01]	0.03*, [0.01, 0.92]

* statistically signifiacnat at 0.05 level of significant

countries. But for the country i.e. India S&P CNX NIFTY, USA DOWJONES, USA S&P-500 and UK FTSE-100 are also statistically significant at 5% level of significance from the sample of developed countries. The smallest is −0.0009 for India BSE SENSEX, and the highest value is −0.621 for UK FTSE 100. So we can conclude that there is a strong and significant leverage effect present in India S&P CNX NIFTY, USA DOWJONES, USA S&P-500 and UK FTSE-100 from the developed countries stock markets.

The posterior means of the volatility of volatility coefficient $\hat{\sigma}$ are within the range of 0.03 (UK FTSE-100) and 0.22 (USA S&P-500).

From Table 4, it is observed that the approximation of the volatility persistence coefficient ϕ is in among 0.802 Japan (NIKKI-225) and 0.956 Malaysia (KLSE). The results indicate that there is high volatility persistence in these stocks markets. The upper intervals values of the 95% credibility intervals are less than 0.99 except

Table 4 Estimation results of the posterior means of parameters for emerging countries

Countries	$\hat{\phi}$	$\hat{\rho}$	$\hat{\sigma}$
France (CAC-40)	0.911, [0.71, 0.92]	0.111*, [−0.35, – 0.22]	0.54*, [0.13, 0.76]
Spain (IBEX 35)	0.921, [0.90, 0.97]	−0.214*, [−0.24, −0.16]	0.46*, [0.19, 0.44]
Malaysia (KLSE)	0.956, [0.97, 0.99]	−0.251, [−0.42, −0.33]	0.56*, [0.42, 0.49]
Japan (NIKKI-225)	0.802, [0.92, 0.94]	−0.006*, [−0.14, −0.04]	0.68* [0.05, 0.52]
Singapore (STRAIT TIMES)	0.947, [0.96, 0.99]	−0.544, [−0.14, −0.35]	0.72*, [0.28, 0.47]
Taiwan (TAIWAN WEIGHTED)	0.910, [0.90, 0.95]	−0.744, [−0.20, −0.45]	0.47*, [0.07, 0.82]

* statistically signifiacnat at 0.05 level of significant

Malaysia (KLSE) and Singapore (STRAIT TIMES). These results are consistence with the results suggested by Asai and Angelo Unite (2010).

The posterior means of the coefficient $\overset{\Lambda}{\rho}$ indicate the correlation among shocks to return at time t, and shocks to volatility at time t + 1 are negative for all the countries. But for the countries like (CAC-40), Spain (IBEX 35) and Japan (NIKKI-225) are also statistically significant at 5% level of significance from the sample of emerging countries. The smallest is −0.006 for Japan (NIKKI-225), and the highest value is −0.744 for Taiwan (TAIWAN WEIGHTED). So we can conclude that there is a strong and significant leverage effect present in France (CAC-40), Spain (IBEX 35) and Japan (NIKKI-225) from the emerging countries stock markets. The posterior means of the volatility of volatility coefficient $\overset{\Lambda}{\sigma}$ are within the range of 0.46 Spain (IBEX 35) and 0.72 Singapore (STRAIT TIMES). The results also suggested that there is the higher variability present in emerging countries stock market as compared to the developed countries stock markets.

4 Conclusions

This research is to study the stock markets volatility exists in different developed and emerging stock markets by using irregular stochastic volatility (ASV) and Markov Chain Monte Carlo (MCMC) models. The datasets involved daily closing prices indices of developed and emerging markets. In developed countries, there are six indices, i.e. India (S&P CNX NIFTY and BSE SENSEX), USA (DOWJONES, NASDAQ-100 and S&P-500) and UK (FTSE-100) and six emerging countries stock returns, i.e. France (CAC-40), Spain (IBEX 35), Malaysia (KLSE), Japan (NIKKI-225), Singapore (STRAIT TIMES) and Taiwan (TAIWAN WEIGHTED) for the period 01/01/2010–31/12/2016, a total 1769 observations.

The empirical evidence provided that in the developed stock markets high volatility persistence was present in USA NASDAQ-100 and India BSE SENSEX and in the emerging stock markets high volatility was present in Malaysia and Singapore. The strong and significant leverage effect present in India S&P CNX NIFTY, USA DOWJONES, USA S&P-500 and UK FTSE-100 from the developed countries stock markets and the leverage effect present in France (CAC-40), Spain (IBEX 35) and Japan (NIKKI-225) from the emerging countries stock markets. The results also suggested that there is a higher variability present in emerging countries stock market as compared to the developed countries stock markets. The results help the investors to invest in developed and emerging countries stock markets.

The approximation for the developed markets and volatility persistence coefficient φ is in among 0.861 USA NASDAQ-100 and 0.956 India BSE SENSEX. The results indicate that there is high volatility determination in these stocks markets. The upper intervals values of 95% credibility intervals are less than 0.99 except India BSE SENSEX and USA NASDAQ-100. These results are consistence with the results suggested by Jacquier et al. (1994, 2004).

The approximation for the emerging countries and volatility determination coefficient ϕ is in among 0.802 Japan (NIKKI-225) and 0.956 Malaysia (KLSE). The results indicate that there is high volatility determination in these stocks markets. The upper intervals values of 95% credibility intervals are less than 0.99 except Malaysia (KLSE) and Singapore (STRAIT TIMES). These results are consistence with the results suggested by Asai and Unite (2010).

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