Probabilistic Study of the Undrained Bearing Capacity of Strip Footings

N. Brahmi, M. Mellas, A. Mabrouki, D. Benmeddour
and M. Y. Ouahab

Abstract The undrained bearing capacity of strip footings has been extensively studied in the literature by a deterministic manner where the soil is assumed uniform. In practice, the spatial variability of random soil affects the ultimate bearing capacity of strip footings. This paper focuses on a probabilistic study using adaptive finite element limit analysis combined with random field theory to investigate the bearing capacity of strip footings subjected to vertical and centric inclined loads, on a cohesive soil with spatially variable distribution of the undrained shear strength. A parametric study is performed using OptumG2 to investigate the effects of diverse probabilistic parameters associated in the problem of undrained bearing capacity of strip footings.

Keywords Bearing capacity · Strip footing · Random field · Cohesive soil

1 Introduction

Classical methods of bearing capacity evaluation for shallow foundations consider that the soil beneath the foundation is uniform, which involves a deterministic analysis, using representative values for different soil parameters. These values are generally the averages or the most conservative values obtained from in situ or laboratory tests. These methods apply a global safety factor to take into account the uncertainties of soil properties; this factor is obtained based on experience.

The sources of uncertainties associated with geotechnical engineering practice can be classified in three main groups [1]: inherent variability or the natural

N. Brahmi (✉) · M. Mellas · A. Mabrouki (✉) · D. Benmeddour · M. Y. Ouahab
Civil Engineering Laboratory, Biskra University, BP 145, 07000 Biskra, Algeria
e-mail: narimanebrahmi@outlook.fr

A. Mabrouki
e-mail: a.mabroukii@yahoo.fr

heterogeneity, measurement errors, and model transformation uncertainties. The present study is concerned only with the spatial variability, which can be modelled as random field.

The assumption of homogeneous soil ignores some possible failure mechanisms and may lead to an overestimation of the bearing capacity of the foundations. In order to investigate the influence of randomly distributed undrained shear strength, the bearing capacity of a strip rough footing has been studied using elasto-plastic finite element analyses combined with random field theory [2]. It was found that the mean bearing capacity for spatially random soil is always lower than the deterministic bearing capacity. Also, the Random Finite Element Method (RFEM) was used to investigate the effect of the variation in the shear strength, on the bearing capacity of strip footing under combined loading [3]. The results showed that the mean bearing capacity for separate vertical, horizontal or moment loading, decreases with increasing soil variability; it was demonstrated that most of the failure envelopes are found inside the deterministic one, which means that soil variability can reduce the bearing capacity. Furthermore, the bearing capacity of buried footings was studied by using the elasto-plastic finite element method and the limit analysis method with its two bounding theorems (upper and lower bounds) [4]. It was found that the bearing capacity factors for a footing in random soils are all smaller than the corresponding bearing capacity factor in homogeneous soils.

Recently, the Random Adaptive Finite Element Limit Analysis (RAFELA) for probabilistic applications was used [5], it was found that the proposed method reduces the gap between the upper and lower bound solutions comparing with the equivalent uniform meshes. The RAFELA method has been also adopted to study the effect of spatial correlation distance in the undrained shear strength on an eccentrically loaded strip footing [6]. It was demonstrated that the greatest reduction in the bearing capacity is for pure vertical loads; moreover, the greatest reduction is obtained for the smaller correlation lengths with a critical value that represents the greatest reduction in the bearing capacity.

This paper investigates the effect of the spatial variability in the undrained shear strength of clays, on the bearing capacity of strip footings under vertical and inclined loads, using Random Adaptive Finite Element Limit Analysis (RAFELA).

2   Problem Presentation

In this paper, the finite element program OptumG2 [7] was used to study the bearing capacity of rough strip footings of width $B = 1$ m, founded on a spatially varying cohesive soil and subjected to vertical and inclined loads (Fig. 1). The soil domain of width 8B and height 3B is modelled with a perfectly plastic constitutive model and the material failure is defined by the Tresca criterion. The undrained Young's modulus and the unit weight of the soil are kept constant ($Eu = 30$ MPa and $\gamma = 16$ kN/m^3) while the undrained cohesion is spatially variable.
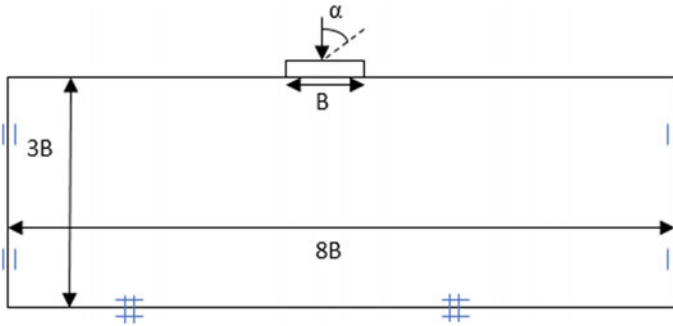
Fig. 1 Problem geometry

The footing is modelled as a rigid plate and the boundary conditions are shown in Fig. 1. The vertical sides of the model were constrained horizontally, while the base of the model was constrained in all directions.

The bearing capacity evaluation for the strip rigid footing with OptumG2 is based on subdividing the soil into a number of elements, using Finite Element Limit Analysis. Limit analysis is a powerful tool for analyzing the bearing capacity of foundations. The bounding theorems, considers the soil to be perfectly plastic and follows an associated flow rule. The bearing capacity is obtained by applying a multiplier load which is amplified until a state of incipient collapse is attained. In this study, the adaptive meshing in finite element limit analysis is used to reduce the gap between the lower and upper bounds.

Spatial variability can be characterized by a trend refers to the mean value and residual variation representing the variability about the trend. At two different points in the space, the residuals are statistically correlated to one another. This correlation is described by the autocorrelation function [6]:

$$\rho((x_1, y_1)(x_2, y_2)) = \exp\left(\frac{-|x_1 - x_2|}{\theta_x}\right) \exp\left(\frac{-|y_1 - y_2|}{\theta_y}\right) \tag{1}$$

The integral of this function represents the scale of fluctuation; this latter also called correlation length is defined as the distance over which the correlation of the properties is relatively strong.

Soil spatial variability is modelled as random field which is described by the coefficient of variation of the property (representing the percentage of dispersion in data) and by the scale of fluctuation.

In the present study, a lognormal distribution is assumed for the undrained shear strength with a mean value $\mu_{Cu} = 10$ kPa, a coefficient of variation (COV), a vertical and horizontal correlation distances which are assumed to be isotropic $CL_x = CL_y$.

Spatial variability is modelled using random field theory in conjunction with finite element limit analysis and Monte Carlo simulations.

3   Results and Discussion

3.1   Bearing Capacity Factor for Vertically Loaded Strip Footing

The limit load of strip footing on clay is evaluated through the bearing capacity factor N_c is well known to be $\pi + 2$ [8]. In the present study, this factor is calculated as follows:

$$N_C = \frac{q_u}{\mu_{Cu}} \qquad (2)$$

Where q_u: the bearing capacity, and μ_{Cu}: the mean value of the undrained shear strength.

Deterministic analyses. The bearing capacity factor was first investigated by a deterministic analysis, for a uniform soil with a value of 10 kPa for the undrained shear strength.

Table 1 shows the results for the upper and lower bound solutions using both uniform meshes and adaptive refinement, it can be seen that mesh adaptivity reduces the gap between the upper and lower bounds, the average bearing capacity factor is in a very good agreement with the exact solution [8].

Probabilistic analyses. A parametric study was investigated to explore the influence of stochastic parameters on the mean bearing capacity factor. For each analysis, four parameters are required to describe the log-normally distributed random field of Cu:

- Mean value μ_{Cu} (kPa)
- Coefficient of variation COV_{Cu} (%)
- Horizontal correlation length CL_x(m)
- Vertical correlation length CL_y (m).

Spatial correlation length is assumed to be isotropic ($CL_x = CL_y$), the dimensionless spatial correlation length is then: $\Theta = CL_x/B = CL_y/B$.

The mean value is kept constant in all analysis $\mu_{Cu} = 10$ kPa, while the coefficient of variation and the spatial correlation lengths are varied respectively as follows:

$COV_{Cu} = 10, 20, 30, 40$ and 50%.
$\Theta = 0.125, 0.25, 0.5, 1, 2, 3$ and 4.

The cumulative probability distributions for all COVs are illustrated in Fig. 2 for $\Theta = 1$. It is worthwhile noting that the bearing capacity factors in this figure are the

Table 1 Deterministic results

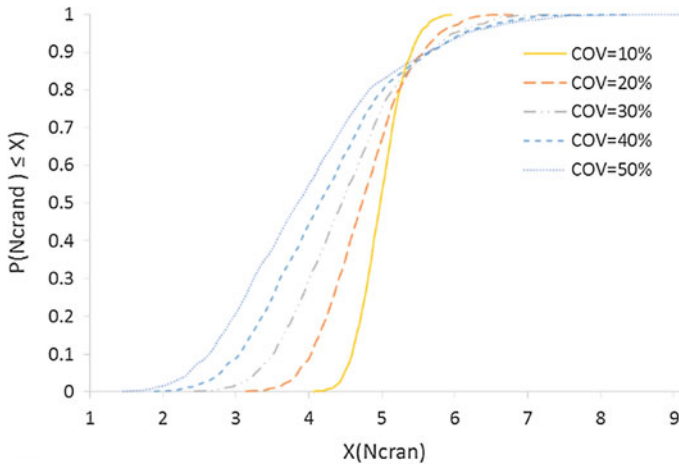| N_c	UB	LB	Average	Analytical
Uniform mesh	5.45	4.982	5.22	5.14
Adaptive mesh	5.206	5.082	5.144	

Fig. 2 Cumulative probability of bearing capacity factors N_c (for $\Theta = 1$)

averages between the upper and lower bounds. It is clear that the mean bearing capacity with all values of COV_{Cu} is less than 5.144 (the average deterministic value of N_c).

Figure 3 shows the variation of the mean bearing capacity factor normalized by the deterministic bearing capacity factor ($\mu N_c/N_{cdet}$) with COV_{Cu}. It can be seen that increasing spatial variability (COV from 10 to 50%) decreases the mean bearing capacity factor. The same result is found by Griffiths et al. [2] and Cassidy et al. [3].

The variation of the mean bearing capacity factor normalized by the deterministic bearing capacity factor ($\mu N_c/N_{cdet}$) with spatial correlation length is shown in Fig. 4. It can be seen that for all correlation lengths, the value of $\mu N_c/N_{cdet}$ is less than 1 which implies that the mean bearing capacity factor for random soils is always less than the deterministic value. Also, the mean bearing capacity factor decreases with decreasing the value of the correlation length with a critical value of $\Theta = 0.25$ that correspond to the greatest reduction of N_c. The obtained results are in a good agreement with those of Abid et al. [6] and Griffiths et al. [2].

3.2 Effect of Load Inclination on the Bearing Capacity of Strip Footing

A parametric study was carried out to explore the effect of spatial correlation distance on the bearing capacity of a strip footing, subjected to inclined loading with different load inclinations (α): $\alpha = 0°$ (pure vertical load), 5°, 10°, 15°, 20°, 30°, 60° and 90° (pure horizontal load).

The bearing capacity is expressed in terms of failure envelopes defined in VH space for different spatial correlation lengths. Failure envelopes shown in Fig. 5 are
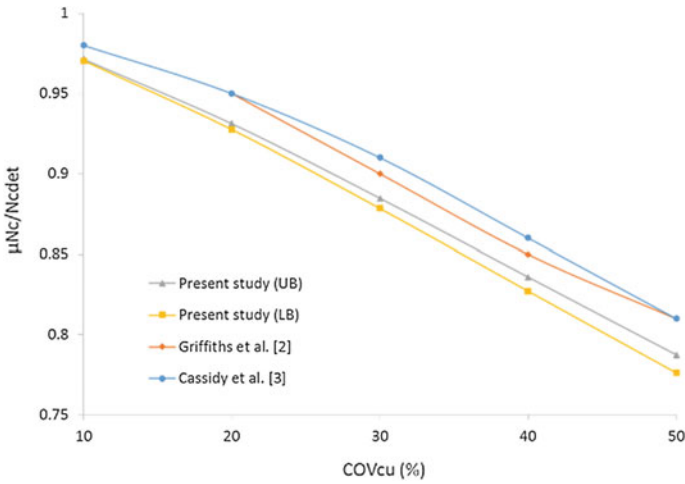
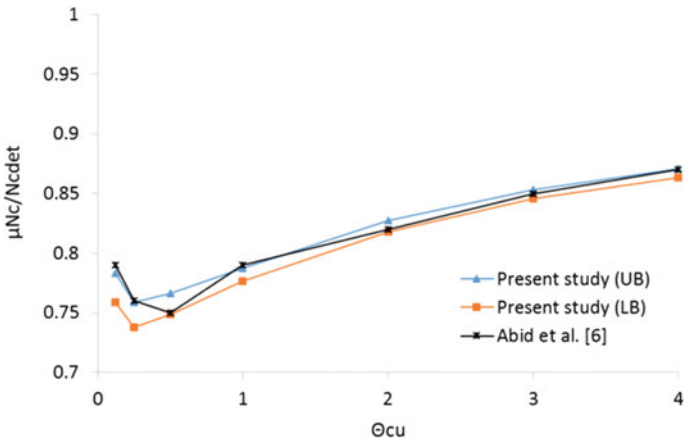**Fig. 3** Influence of varying COV_{Cu} on the mean normalized bearing capacity factor

Fig. 4 Influence of varying Θ on the mean normalized bearing capacity factor

plotted for mean normalized loads (each value of $\mu V_u/BC_u$ and $\mu H_u/BC_u$ is the average between the upper and lower bounds). It is seen that the size and shape of failure envelopes change with the variation of spatial correlation length.

Figure 6 shows the influence of spatial correlation length on the normalized combined bearing capacity (the average between upper and lower bounds), for different load inclinations. It can be seen that the decrease of load inclination, increases the reduction of the bearing capacity, as observed, the greatest reduction
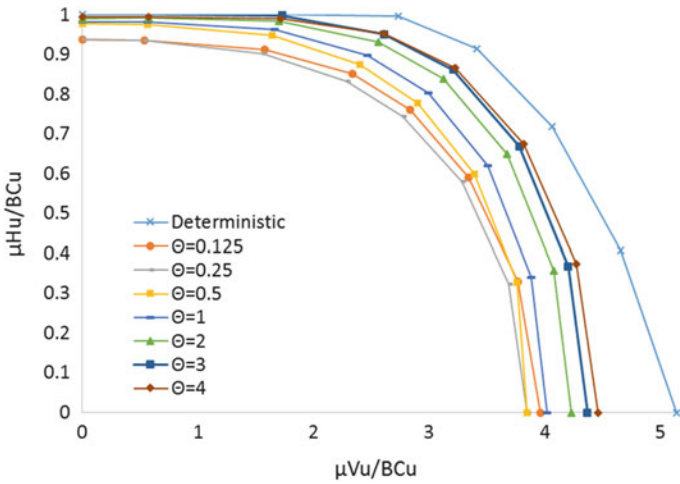
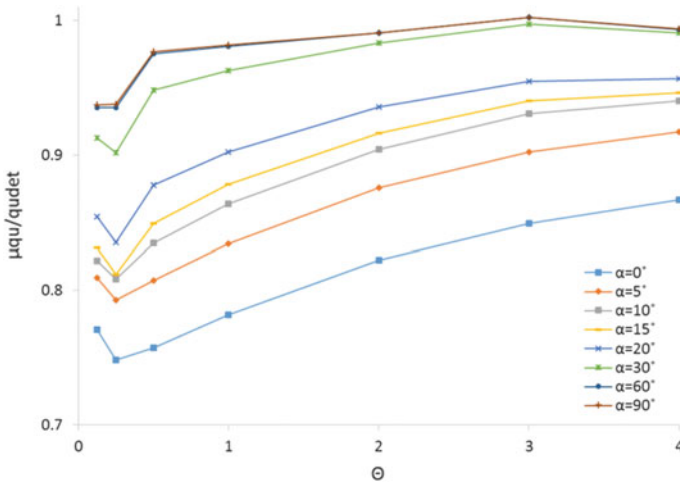**Fig. 5** Mean normalized VH failure envelope

Fig. 6 Effect of spatial correlation length on the normalized combined bearing capacity

occurs when the load is vertical, while the least reduction occurs for horizontal loads. As the spatial correlation length decreases the reduction of the bearing capacity increases (for all load inclinations). In addition, the critical spatial correlation length is 0.25.

3.3   *Failure Mechanism*

The failure mechanism associated with each spatial correlation length is shown in Fig. 7. It can be observed that the shear path follows zones where the soil is weak, resulting unsymmetrical failure mechanism especially for the greater correlation length in contrast to the symmetrical failure mode in uniform soils. This result is already observed by Li et al. [4], therefore the spatial variability in soil properties changes considerably the bearing capacity and the failure mechanism.
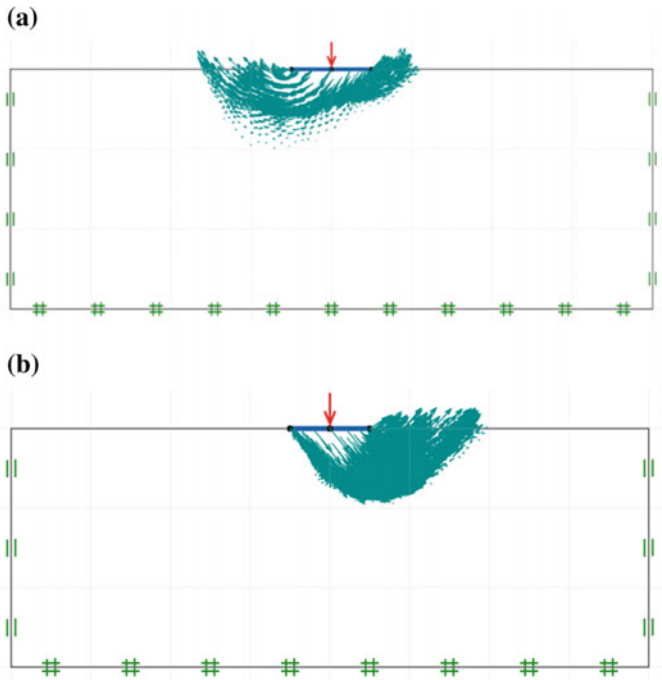

Fig. 7 Failure mechanism associated with **a** $\Theta = 0.125$ and **b** $\Theta = 4$

4 Conclusion

The influence of the probabilistic parameters (coefficient of variation and correlation length for the undrained shear strength) was investigated for the bearing capacity factor N_c of strip footings using RAFELA. It was found that random bearing capacity factors are always lower than the deterministic value for vertical loading. The mean N_c of the 1000 Monte Carlo simulations decreases with increasing COV_{Cu} and with decreasing Θ, also in all cases is lower than the deterministic value. This result demonstrates that the bearing capacity of the foundation is reduced when the soil is spatially variable; a critical value of $\Theta = 0.25$ was observed for the greatest reduction of N_c. The failure envelopes for inclined loading change with the variation of correlation length.

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