

Extended Kalman Filter for GPS Receiver Position Estimation

N. Ashok Kumar, Ch. Suresh and G. Sasibhushana Rao

Abstract Navigation, tracking, and positioning of an object are a customary problem in many fields. Global Positioning System (GPS) is the best solution to this problem. Being GPS is wireless communication through space, the received ephemeris data are erroneous. Hence, the extraction of original ephemeris data from this erroneous data is the main hurdle. Adaptive algorithms provide better results to overcome this hurdle and also for the nonlinear type of system processing. In this paper, GPS receiver position is estimated by extended Kalman filter. A dual-frequency GPS receiver is used for input data, which is located at the Department of ECE, Andhra University, Visakhapatnam (17.73° N/83.31° E). The estimated GPS receiver position is compared with the original position coordinates to check the accuracy. Receiver clock error is also estimated. The result shows that the extended Kalman filter provides a good accuracy of the estimated results for GPS receiver positioning.

Keywords GPS • Kalman filter • Dual-frequency receiver

N. Ashok Kumar (✉) · G. Sasibhushana Rao
Department of Electronics and Communication Engineering,
Andhra University College of Engineering (A), Andhra University,
Visakhapatnam, India
e-mail: ashok0709@gmail.com

G. Sasibhushana Rao
e-mail: sasigps@gmail.com

Ch. Suresh
Department of Information Technology, Anil Neerukonda Institute of Technology
and Science, Visakhapatnam, India
e-mail: sureshchittineni@gmail.com

1 Introduction

In the present world, Global Positioning System (GPS) [1, 2] is widely used for positioning, navigation, and timing. GPS receiver location can be obtained as two-dimensional coordinates and also as three-dimensional coordinates by trilateration principle. To determine the position of the receiver, one must have to know the GPS satellite's orbital position in space and distance between satellites to the receiver. The GPS receiver position can be calculated with the help of four or more satellites. In this paper, more than four satellites are considered in each epoch, and then, extended Kalman filter [2–4] is applied to estimate GPS receiver position [5] along with the receiver clock error. Section 2 covers GPS and its basic equations for finding receiver position. Section 3 covers all of extended Kalman filter. In Sect. 4, extended Kalman filter is applied to find the GPS receiver position. The paper ends with conclusions in the last section.

2 Global Positioning System

The Global Positioning System (GPS) is a real-time radio navigation system, which provides, 24 h a day, accurate GPS receiver position. GPS positioning tasks are satellite signal acquisition, tracking, and positioning. Satellite signal acquires approximated estimates of signal parameters. Tracking is to keep track of these parameters as they change over time. After tracking is done, GPS navigation data will be obtained and pseudoranges are calculated, and then, the receiver position can be computed. The GPS receiver position estimated with the help of four satellites is as shown in Fig. 1.

More than four satellites can also be used to find a receiver position. The basic observed pseudorange equation is given by

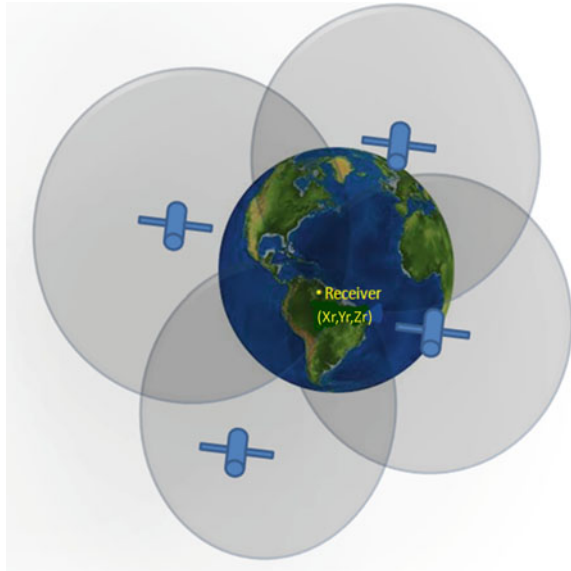
$$\rho_r^{obs} = \rho^{Gr} + c(\Delta t^{sat} - \Delta t) + \Delta_{ion} + \Delta_{trop} + \xi_{mr} \quad (1)$$

where Δt is the receiver clock offset, Δt^{sat} is the satellite clock offset, Δ_{ion} is the delay imparted by ionosphere, Δ_{trop} is the delay imparted by the troposphere, ξ_{mr} is the effects of multipath and receiver measurement noise, c is the velocity of light, and ρ^{Gr} is the geometric range between a particular satellite and receiver. From ephemeris data, position of the satellite ($x^{sat}, y^{sat}, z^{sat}$) can be computed. ρ^{Gr} is the geometric range between satellite and receiver, given by

$$\rho^{Gr} = \sqrt{(x^{sat} - x_r)^2 + (y^{sat} - y_r)^2 + (z^{sat} - z_r)^2} \quad (2)$$

where (x_r, y_r, z_r) is the position of the receiver. Except the receiver clock error, remaining error terms in Eq. (1) are ignored for simplification. Therefore, Eq. (1) will be given as,

Fig. 1 GPS receiver 3D-positioning using four satellites



$$P_r^{obs} = \sqrt{(x^{sat} - x_r)^2 + (y^{sat} - y_r)^2 + (z^{sat} - z_r)^2} + clk_r \tag{3}$$

where clk_r is the receiver clock error expressed in the distance (equal to $-c \cdot \Delta t$). In Eq. (3), there are four unknown parameters x_r, y_r, z_r , and clk_r . To determine these unknown parameters, we need four range equations from four different satellites. Therefore,

$$\begin{aligned} P_r^{obs}_1 &= \left((x^{sat1} - x_r)^2 + (y^{sat1} - y_r)^2 + (z^{sat1} - z_r)^2 \right)^{1/2} + clk_r \\ P_r^{obs}_2 &= \left((x^{sat2} - x_r)^2 + (y^{sat2} - y_r)^2 + (z^{sat2} - z_r)^2 \right)^{1/2} + clk_r \\ P_r^{obs}_3 &= \left((x^{sat3} - x_r)^2 + (y^{sat3} - y_r)^2 + (z^{sat3} - z_r)^2 \right)^{1/2} + clk_r \\ P_r^{obs}_4 &= \left((x^{sat4} - x_r)^2 + (y^{sat4} - y_r)^2 + (z^{sat4} - z_r)^2 \right)^{1/2} + clk_r \end{aligned} \tag{4}$$

Above equations are nonlinear simultaneous equations. To get four unknown parameters, first linearize the nonlinear Eq. (3) and then solve. Differentiating Eq. (3) results

$$\Omega P_r^{obs} = \frac{-(x^{sat} - x_r)\Omega x_r - (y^{sat} - y_r)\Omega y_r - (z^{sat} - z_r)\Omega z_r}{\sqrt{(x^{sat} - x_r)^2 + (y^{sat} - y_r)^2 + (z^{sat} - z_r)^2}} + \Omega clk_r \tag{5}$$

Consider Ωx_r , Ωy_r , Ωz_r , and Ωclk_r as the unknown parameters in the above equation. x_r, y_r, z_r , and clk_r are treated as known values by taking some initial values for these quantities. These initial values are substituted in Eq. (5) which yields the solution for $\Omega x_r, \Omega y_r, \Omega z_r$, and Ωclk_r . These are used to update the initial x_r, y_r, z_r , and clk_r to find another new set of solutions for $\Omega x_r, \Omega y_r, \Omega z_r$, and Ωclk_r . This new set is used again to update x_r, y_r, z_r , and clk_r . This recursive process continues until the values of the parameters $\Omega x_r, \Omega y_r, \Omega z_r$, and Ωclk_r are small and within a certain predefined limit. The desired solution is obtained when the parameters $\Omega x_r, \Omega y_r, \Omega z_r$, and Ωclk_r reached this predefined limit. Then, the final GPS receiver coordinates are x_r, y_r, z_r , and clk_r . This is an iteration method. In this paper, more than four satellites are considered for GPS receiver position calculation even though four satellites are enough. For example, if we consider nine satellites for GPS receiver position estimation, then Eq. (5) can be composed as a

Matrix form, which is expressed as

$$\begin{bmatrix} \Omega p_{r1}^{obs} \\ \vdots \\ \Omega p_{r9}^{obs} \end{bmatrix} = \begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \xi_{91} & \xi_{92} & \xi_{93} & 1 \end{bmatrix} \begin{bmatrix} \Omega x_r \\ \Omega y_r \\ \Omega z_r \\ \Omega clk_r \end{bmatrix} \quad (6)$$

where the elements ξ_{i1} , ξ_{i2} , and ξ_{i3} are given by

$$\xi_{i1} = -\frac{(x^{sat} - x_r)}{p_r^{obs} - clk_r}, \xi_{i2} = -\frac{(y^{sat} - y_r)}{p_r^{obs} - clk_r}, \xi_{i3} = -\frac{(z^{sat} - z_r)}{p_r^{obs} - clk_r} \quad (7)$$

3 Extended Kalman Filter

The extended Kalman filter (EKF) [2–4, 6–8] is the nonlinear genre of the Kalman filter [9, 10] which linearizes about an estimate of the current mean and covariance. The state transition and observation models for the extended Kalman filter are taken as

$$\chi_t = f(\chi_{t-1}) + \omega_{t-1} \quad (8)$$

$$z_t = h(\chi_t) + v_t \quad (9)$$

where ω_{t-1} is the process noise with zero mean and covariance Q_t , and v_t is the observation noise with zero mean and covariance R_t .

The functions $f(\chi_{t-1})$ and $h(\chi_t)$ are used to compute the predicted state from the previous estimate and predicted measurement from the predicted state, respectively. Instead of applying $f(\chi_{t-1})$ and $h(\chi_t)$ to the covariance directly, a Jacobian matrix is applied which is evaluated with current predicted states at each time step. This process essentially linearizes the nonlinear function around the current estimate.

Discrete-time extended Kalman filter's prediction (time update) and correction (measurement update) equations are given by,

Prediction (time update):

- i. Predicted state estimate

$$\hat{\chi}_{t|t-1} = f(\hat{\chi}_{t-1|t-1}) \tag{10}$$

- ii. Predicted covariance estimate

$$P_{t|t-1} = F_{t-1}P_{t-1|t-1}F_{t-1}^T + Q_{t-1} \tag{11}$$

Correction (measurement update):

- i. Kalman gain

$$K_t = P_{t|t-1}H_t^T(H_tP_{t|t-1}H_t^T + R_t)^{-1} \tag{12}$$

- ii. Updated state estimate

$$\hat{\chi}_{t|t} = \hat{\chi}_{t|t-1} + K_t(z_t - h(\hat{\chi}_{t|t-1})) \tag{13}$$

- iii. Updated covariance estimate

$$P_{t|t} = (I - K_tH_t)P_{t|t-1} \tag{14}$$

Where the Jacobian for state transition and observation matrices are defined as

$$F_{t-1} = \left. \frac{\partial f}{\partial \chi} \right|_{\hat{\chi}_{t-1|t-1}} \tag{15}$$

$$H_t = \left. \frac{\partial h}{\partial \chi} \right|_{\hat{\chi}_{t|t-1}} \tag{16}$$

The Jacobian H_t is taken from Eqs. (6) and (7) as

$$H_t = \begin{bmatrix} \xi_{11} & \xi_{12} & \xi_{13} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \xi_{91} & \xi_{92} & \xi_{93} & 1 \end{bmatrix} \tag{17}$$

The above method is extended Kalman filter (EKF) of first order. Higher-order EKFs can be composed by retaining more terms of the Taylor series expansions.

4 Results and Discussion

Real-time GPS data, recorded for 24 h, has been taken, at the Department of Electronics and Communication Engineering, Andhra University, Visakhapatnam (latitude 17.73° N/longitude 83.31° E), from a dual-frequency GPS receiver. In each epoch, data include more than four different satellites' position and the pseudorange between satellites to GPS receiver. In this paper, extended Kalman filter is applied to this data. Assume that initial receiver position coordinates are (0 0 0). The resultant GPS receiver position coordinates obtained, from the filtering operation, are compared with the original GPS receiver position coordinates to show how much they are close to the original position. This shows the accuracy provided by the application of extended Kalman filter. These results are presented in Table 1.

In Table 1, x-position error (Xe), y-position error (Ye), z-position error (Ze) in meters, and receiver clock error (Rclk) in nanosecond (10^{-9}) are presented. From the results in Table 1, it is observed that error in coordinates and receiver clock error are slightly varied for every 2 h due to neglecting other error sources in GPS pseudorange expression. The mean values of x-position error, y-position error, and z-position error for 24 h data are 48.47 m, 100.63 m, and 34.6 m, respectively. These mean values are the error in three-dimensional coordinates while comparing with the original GPS receiver position coordinates. The mean value of receiver clock error estimated is 100 ns, due to neglecting ionosphere, troposphere, and multipath errors.

In Table 2, x-position variance, y-position variance, z-position variance, and receiver clock variance in distance, i.e., σ_x^2 , σ_y^2 , σ_z^2 , and σ_{clk}^2 respectively, are presented. From Table 2, variances obtained are very low which ensures the deviations

Table 1 Position error estimation

Extended Kalman filter				
GPS time in hours of the day	Xe (m)	Ye (m)	Ze (m)	Rclk (ns)
00–02	50.98	105.89	39.93	58
02–04	49.59	108.63	33.37	74
04–06	42.20	98.29	30.79	118
06–08	41.83	83.59	32.06	150
08–10	45.82	83.37	34.02	152
10–12	51.16	98.10	39.71	120
12–14	50.90	103.07	35.50	105
14–16	51.37	107.50	34.08	95
16–18	47.39	106.48	35.72	90
18–20	51.69	107.76	34.51	83
20–22	48.84	101.60	31.83	89
22–24	50.35	117.94	43.60	33

Table 2 Variance estimation

Extended Kalman filter				
GPS time in hours of the day	σ_x^2	σ_y^2	σ_z^2	σ_{clk}^2
00–02	0.130	0.188	0.126	0.127
02–04	0.128	0.211	0.133	0.139
04–06	0.127	0.204	0.130	0.136
06–08	0.128	0.198	0.139	0.138
08–10	0.124	0.192	0.134	0.129
10–12	0.133	0.204	0.132	0.132
12–14	0.131	0.193	0.129	0.132
14–16	0.133	0.186	0.127	0.131
16–18	0.139	0.193	0.130	0.136
18–20	0.135	0.185	0.127	0.128
20–22	0.134	0.203	0.133	0.143
22–24	0.131	0.189	0.130	0.135

are low from its mean values. From Tables 1 and 2 results, it is observed that extended Kalman filter provides good accuracy for estimating GPS receiver position.

5 Conclusions

Extended Kalman filter approximates nonlinear function. Here, linearizing the nonlinear functions helps to use linear Kalman filter for the estimation of GPS receiver position with good accuracy. For GPS receiver position estimation, extended Kalman filter provides less variance and good accuracy in position estimation and also provides receiver clock error of 100 ns; it is because of neglecting other sources which influences the clock error. However, the mean values for the error in GPS receiver position while comparing with the original GPS receiver position coordinates are 48.47 m, 100.63 m, and 34.6 m, respectively. Therefore, extended Kalman filter is better algorithm for GPS receiver position estimation and also for nonlinear type of estimation problems.

References

1. Rao, G.S.: Global Navigation Satellite Systems. McGraw Hill Education Private limited (2010). ISBN (13): 978-0-07-070029-1
2. Grewal, M.S., Andrews, A.P., Bartone, C.G.: Global Navigation Satellite Systems, Inertial Navigation, and Integration, 3rd edn. (2013)

3. Gordan, N., Pitt, M.: A comparison of sample based filters and the Extended Kalman Filter for the bearings only tracking problem. In: Signal Processing Conference (EUSIPO) (1998)
4. Burl, J.: A reduced order Extended Kalman Filter for sequential images containing a moving object. *IEEE Trans. Image Process.* **2**(3), 285–295 (1993)
5. Wang, Y.: Position estimation using Extended Kalman Filter and RTS-smoother in a GPS receiver. In: IEEE Conference, 5th International Congress on Image and Signal Processing, (CISP 2012), pp. 1718–1721
6. Simon, D.: *Optimal State Estimation*. John Wiley & Sons, Inc. (2006)
7. Song, Y., Grizzle, J.W.: The Extended Kalman Filter as a local asymptotic observer for discrete-time nonlinear systems. *J. Math. Syst. Estim. Control* **5**(1), 59–78 (1995)
8. Yin, Jianjun, Ming, Gu: The expanded state space Kalman filter for GPS navigation. *Inf. Technol. J.* **10**, 2091–2097 (2010)
9. Welch, G., Bishop, G.: *An Introduction to the Kalman Filter*. University of North Carolina, 5 April 2004
10. Tolman, B.W.: GPS precise absolute positioning via Kalman filtering. In: 21st International Technical Meeting of the Satellite Division of the Institute of Navigation, pp. 1640–1650 (2008)