# Adaptive Control of Aircraft Wing Oscillations with Stiffness and Damping Nonlinearities in Pitching Mode

**L. Prabhu and J. Srinivas**

**Abstract** This paper presents an adaptive control strategy for aircraft wing structure based on a nonlinear aeroelastic model with plunge and pitch degrees of freedom. System nonlinearities in terms of pitching degree of freedom are accounted in stiffness and damping terms of the model. The closed-loop response of the model is studied under two cases: (i) polynomial form of nonlinearities and (ii) combined free play and polynomial form of nonlinearities. The adaptive control strategy with wing flap based on partial feedback linearization is designed to suppress the instabilities occurring at certain freestream velocities. Objective of controller is to stabilize the system within the flutter boundary. A neural network based observer is used to estimate the uncertain parameters in control law. The designed control system with neural network estimator is effective in suppressing the limit cycle oscillations considerably.

## 1 Introduction

In aeroelastic studies, the interactions of various forces such as aerodynamics, elastic, and inertia are considered using simple mathematical models. A combination of these forces leads to an aircraft instability resulting in a direct consequence of an oscillatory instability known as a flutter which eventually leads to catastrophic failure due to the loss of a system damping. Aeroelastic instability region is identified by assuming system as linear one [1], but in real practice, the nonlinearities are inevitable. Woolston [2] accounted different types of structural nonlinearities and studied the influence of the initial conditions. The LCO of an aeroelastic model with hysteresis nonlinearity was controlled by sliding mode controller and effects of time delay were studied by Xu et al. [3].

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To extend the flutter region, active flutter suppression is carried out using various control techniques. Efficiency of linear controller in a nonlinear system comes down when the system nonlinearity effects are aggressive. The fully linearized control system was designed to make the system globally stable with two flaps [4]. In several other works [5–7], adaptive controllers were employed to overcome the dynamic instabilities. The controller was designed to guarantee the stability of structurally nonlinear system with a single flap using explicit parameterization of structural nonlinearity [8, 9]. To improvise the controllability of a nonlinear system, control surfaces at trailing edge and leading edge were used [10]. Block and Strganac [11] used unsteady formulation with optimal controller and Kalman filter as an observer to enhance the flutter boundary. Effectiveness of various types of controllers such as artificial intelligence, robust and adaptive on flutter suppression was discussed in Refs. [12–14]. In earlier work, authors [15] employed a linear quadratic regulator with neural networks estimator to control the instabilities occurring in aeroelastic system.

In all the above works, parametric uncertainty is considered in pitch stiffness only. However, in few works [16], an aeroelastic system was studied with parametric uncertainties, similar kind of nonlinearities in stiffness and damping terms were used. In the present work, an adaptive feedback linearization controller is designed to control the aeroelastic model subjected to different structural nonlinearities in both stiffness and damping in pitch degree of freedom. First, the model is analyzed with polynomial nonlinearity in both stiffness and damping terms and then with free play and cubic nonlinearity in stiffness and damping terms, respectively. Finally, the neural network observer is employed as an estimator for the controller to predict the estimated uncertain parameters in the control law for further suppressing the nonlinearity effect. Additionally, the influence of initial conditions on stable region with polynomial nonlinearities and influence of free play region on system stability is presented.

## 2 Mathematics Modeling of Nonlinear Aeroelastic System

A two-dimensional aeroelastic system is illustrated by a lumped parameter model shown in Fig. 1, where the system has two degrees of freedom namely plunge translation h and the pitching rotation $\alpha$ with trailing edge surface angle $\beta$.

By defining elastic axis at E, and b as semi-chord, ba, $bx_\alpha$ as distance from airfoil mid-chord to elastic axis and distance from airfoil elastic axis to center of mass, respectively, the dynamic equation of aeroelastic system in its standard form is given by [16]:

$$\begin{bmatrix} m_T & m_w x_\alpha b \\ m_w x_\alpha b & I_\alpha \end{bmatrix} \begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha(\dot{\alpha}) \end{bmatrix} \begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha(\alpha) \end{bmatrix} \begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix}. \quad (1)$$

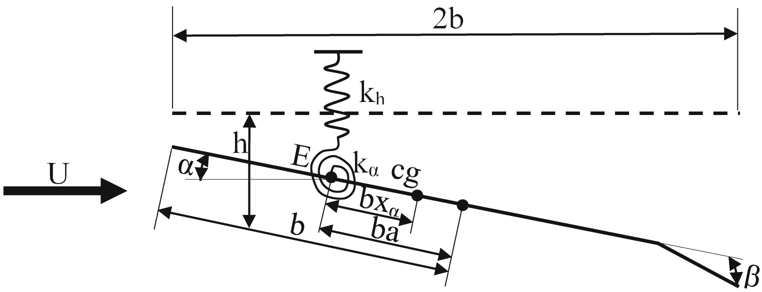


**Fig. 1** Aeroelastic model with trailing edge

Here, $m_T$, $m_w$, $c_h$, $k_h$, L, and M are respectively indicating: total mass of the system, mass of wing section, plunge damping, plunge stiffness, aerodynamic lift and moment. The L and M have nonlinear terms in stiffness and damping in pitch degree of freedom. In real practice, different forms of nonlinearities occur in combination with other. In this work, two cases are considered.

Case 1: Both nonlinear terms are approximated in polynomial form as:

$$\begin{aligned} c_\alpha(\dot{\alpha}) &= \sum_{i=1}^{m} c_i \dot{\alpha}^{i-1}. \\ k_\alpha(\alpha) &= \sum_{i=1}^{n} k_i \alpha^{i-1}. \end{aligned} \tag{2}$$

Case 2: Free play nonlinearity is considered in stiffness and polynomial form in damping. That is

$$k_\alpha(\alpha) = k_\alpha F(\alpha)/\alpha. \tag{3}$$

Here, F(α) is a function assigned to represent the free play nonlinearity given by:

$$F(\alpha) = \begin{cases} \alpha + \delta, & \text{if } \alpha < -\delta \\ 0, & \text{if } |\alpha| \le \delta \\ \alpha - \delta, & \text{if } \alpha > \delta \end{cases}. \tag{4}$$

For quasi-steady aerodynamics, lift *L* and moment *M* are given by [16]:

$$\begin{aligned} L &= \rho U^2 b s c_{l\alpha}\left(\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a\right) b \frac{\dot{\alpha}}{U}\right) + \rho U^2 b s c_{l\beta}\beta \\ M &= \rho U^2 b^2 s \bar{c}_{m\alpha}\left(\alpha + \frac{\dot{h}}{U} + \left(\frac{1}{2} - a\right) b \frac{\dot{\alpha}}{U}\right) + \rho U^2 b^2 s \bar{c}_{m\beta}\beta. \end{aligned}, \tag{5}$$

where $\bar{c}_{m\alpha}$ and $\bar{c}_{m\beta}$ represent the moment derivative coefficients per unit angle of attack and trailing edge angle, respectively.

The Eq. (1) is rewritten in state-space form as

$$\dot{X} = A(X)X + B\beta. \tag{6}$$

where *A(X)* is system matrix and *B* is a control matrix.

## 3 Adaptive Feedback Linearization Control

The adaptive control scheme reforms the controller online depending on the system performance and changes its dynamics accordingly. Two important steps of adaptive controller are

1. Online parameter estimation.
2. Control law-redesign based on Step 1.

The feedback linearization is a method to transform the nonlinear equations of motion to an equivalent linear system by deriving a suitable control law to cancel the nonlinear terms. The output function is defined as $y = \alpha = x_2$. Before designing the controller, transformation of the equations of motion is carried out.

When parameter estimations $\{\hat{c}_i \quad \hat{k}_i\}$ are unknown, then the control law of the control surface is given by [16]

$$\beta = \frac{-}{B_4}\left(G(\theta) + \sum_{i=1}^{m} \hat{c}_i M_1(\dot{\alpha}^i) + \sum_{i=1}^{n} \hat{k}_i M_2(\alpha^i) - \upsilon\right). \tag{7}$$

where $\theta = [\theta_1, \theta_2]$ is a state vector, while $M_1$ and $M_2$ are nonlinear damping and stiffness terms and $v$ is design input. Substituting the control law in $\theta$, and simplifying, it is rewritten as

$$\dot{\theta}_1 = \theta_2, \dot{\theta}_2 = \sum_{i=1}^{m} (c_i - \hat{c}_i) M_1\left(\dot{\alpha}^i\right) + \sum_{i=1}^{n} \left(k_i - \hat{k}_i\right) M_2\left(\alpha^i\right) + \upsilon. \tag{8}$$

The control input $\upsilon = -a_1\theta_1 - a_2\theta_2$ must be selected such that the resulting linear subsystem is stable when the nonlinearities are eliminated via partial feedback linearization. The update law for parameter estimation is defined as [16]

$$\left[\dot{\hat{C}} \quad \dot{\hat{K}}\right]^T = \left[\dot{\hat{C}} \quad \dot{\hat{K}}\right]^T = \theta_2 M. \tag{9}$$

### 3.1 Parameter Estimation by Neural Network

The artificial neural networks are designed based on the human neuron system and successfully used in many engineering fields. The feedforward, backpropagation (BP) network [17] is a well-known model and the inputs are passed forward from the input to output layer via one/several hidden layers. The calculated error between actual and target values are propagated back in order to update the weights. The network is constructed based on the following cost function minimization:

$$Error = \frac{1}{2}\sum_{p}(y_p^d - y_p)^2, \tag{10}$$

where $y_p$ and $y_p^d$ are the $p$th neural network output and desired (target) values. The backpropagation algorithm minimizes the above cost function with the following output weights update law:

$$w_{new} = w_{old} - \eta\frac{\partial Error}{\partial w}, \tag{11}$$

where $\eta \in (0, 1)$ is the learning rate. Likewise, hidden layer weights are also updated from the error in that cycle. After training the model, neural network can be utilized to predict the parameters. Now Eq. (8) is rewritten as

$$\theta_2 = \sum_{i=1}^{m}(c_i - \hat{y}_{ci})M_1\left(\dot{\alpha}^i\right) + \sum_{i=1}^{n}(k_i - \hat{y}_{ki})M_2\left(\alpha^i\right) + \upsilon. \tag{12}$$

where $\hat{y}_{ci}$ and $\hat{y}_{ki}$ are the parameters estimated by the neural network.

## 4 Results and Discussion

The analysis and control modules are implemented with MATLAB program. Numerical experiments are carried out to verify the performance of the controller discussed in this paper. Parameters employed in this work are taken from earlier work [16]. The flutter velocity of the aeroelastic model without considering nonlinearities is 11.57 m/s.

Polynomial nonlinearities in stiffness and damping terms are considered from [16]. The flutter boundary of the model with the polynomial nonlinearity is identified with various initial conditions and shown in Fig. 2.

The plunge “h(0)” is varied from 0 to 0.05 m, pitch “α(0)” = −0.2 to 0.2 rad, $\dot{h}(0) = 0$ and $\dot{\alpha}(0) = 0$. The flutter boundary shrinks indirectly proportional to the plunge initial condition. For further analysis, the initial conditions considered are h(0) = 0.01, α(0) = 0.1, $\dot{h}(0) = 0$ and $\dot{\alpha}(0) = 0$. As an indication of instability, flutter velocity is first obtained from nonlinear responses. The flutter velocity of the

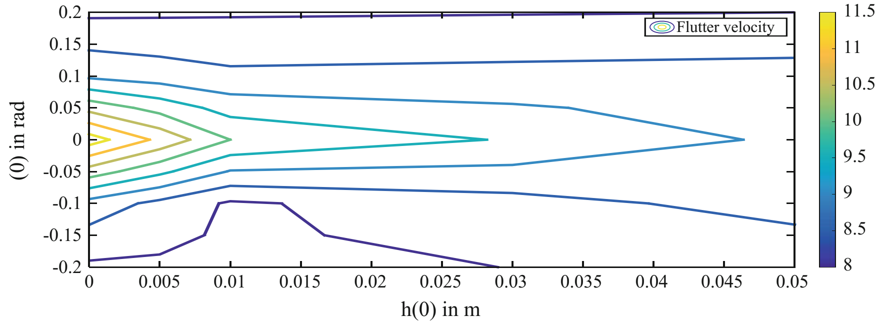


**Fig. 2** Influence of initial conditions on flutter boundary

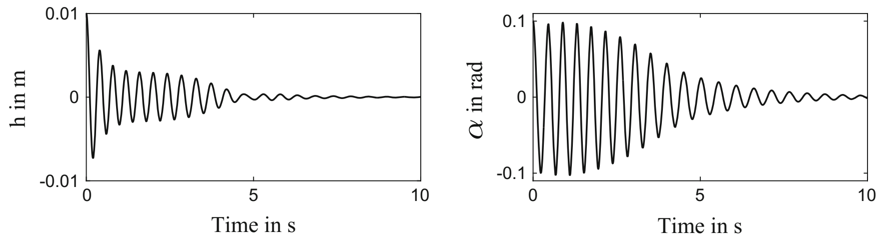


**Fig. 3** Time response plot at freestream velocity 7 m/s

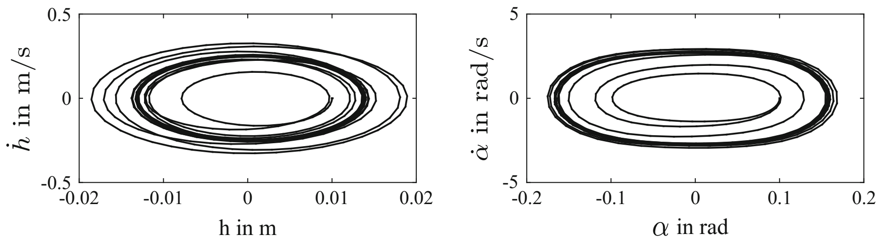


**Fig. 4** Phase plot at freestream velocity 16 m/s

system for above initial conditions is found to be 7.93 m/s and the time responses are shown in Fig. 3 for the velocity 7 m/s and clearly shows the system as stable. Further the model is simulated at a velocity 16 m/s and is shown in Fig. 4, wherein an LCO is observed due to the presence of the polynomial nonlinearity making the system to oscillate periodically instead of becoming unstable.

To study the effect of the adaptive partial feedback linearization controller, the simulations are carried out at velocity 16 m/s with nonlinearity and Fig. 5 shows the time response of plunge and pitch degree of freedom and flap deflection. It is observed that the system is stable and it shows the effectiveness of the controller in suppressing the LCO. The pitching response settles at 1.5 s comparatively quicker

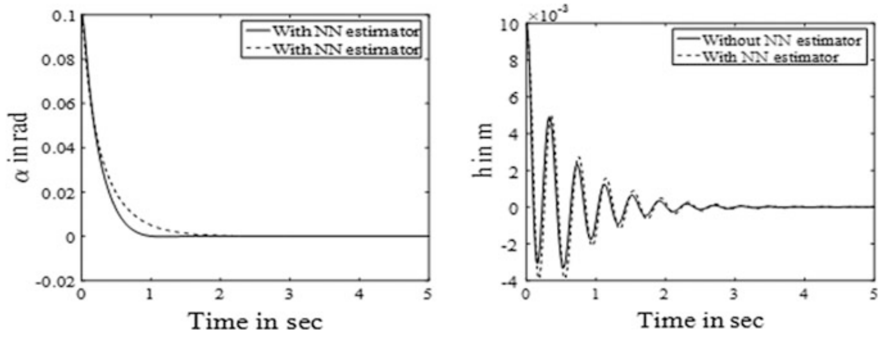


**Fig. 5** Closed-loop response at freestream velocity 16 m/s

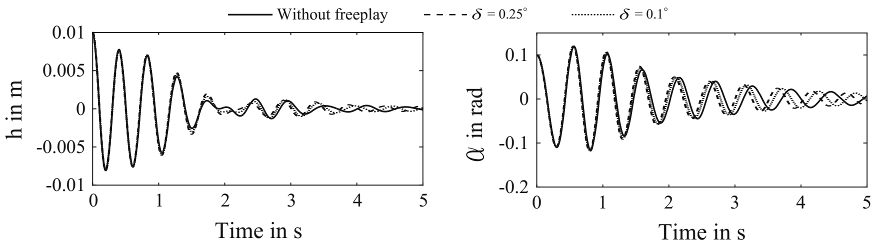


**Fig. 6** Time histories at freestream velocity 8 m/s

to plunge response which takes 3.5 s to settle. This is because of the control law which is framed based on the pitch angle as output function and with damping nonlinearity. Now, the uncertain parameters in adaptive control law are identified by the three-layer feedforward, backpropagation neural network which acts as the estimator with learning rate $\eta = 0.7$ and $\alpha$ and $\dot{\alpha}$ as inputs. The system responses with such neural network estimator are also shown in Fig. 5. However, there is no marked variation observed by incorporating neural network based estimator.

In the next study, the free play nonlinearity in stiffness and polynomial structural nonlinearity in damping in pitching degree of freedom is added to a system to study their effect on the flutter boundary. The system is simulated with the same initial conditions and free play nonlinearity is not affecting the flutter boundary but the amplitudes of the pitch and plunge responses are higher as $\delta$ increases as seen in Fig. 6. The response at post flutter velocities is divergent in this case and the time history at the freestream velocity of 12.5 m/s is shown in Fig. 7.

Figure 8 shows the system as stable at freestream velocity 16 m/s with the controller active in the system. The time taken to converge is high in pith response compared to the previous case, where both stiffness and damping nonlinearities are of polynomial type. When the neural network estimator is added, the system becomes stable.

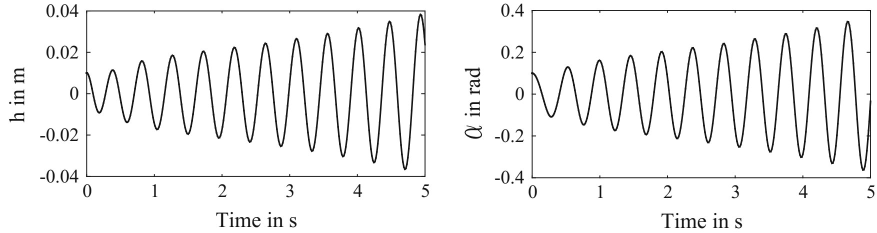


**Fig. 7** Time histories at freestream velocity 12.5 m/s

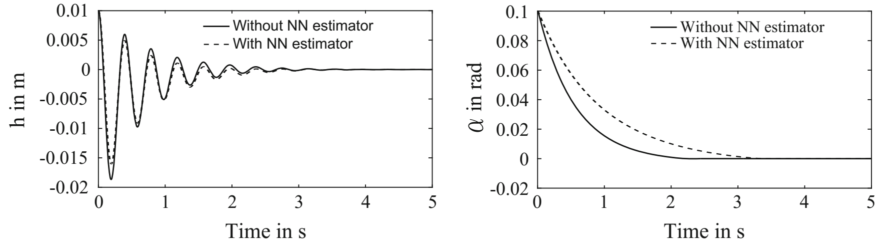


**Fig. 8** Closed-loop response at freestream velocity 16 m/s

# 5 Conclusions

In this paper, the effect of a neural network based estimator with adaptive controller was studied using two different cases on aeroelastic model with nonlinearities in pitch direction. The adaptive control law was designed based on partial feedback linearization and three-layer feedforward neural network is used as an estimator to identify the estimated uncertainty parameters in control law. The effect of initial conditions on flutter boundary with polynomial nonlinearities was studied. As the initial conditions increase, the stable region got minimized. The free play nonlinearity effect on flutter boundary is small and the response is divergent in post flutter operation. The effect of controller on unstable region was studied with and without neural network estimator in both the cases. It practicality extends the flutter boundary. For more detailed analysis, the control surface dynamics may be included in the system of equations.

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