A Stability Analysis of Inverted Pendulum System Using Fractional-Order MIT Rule of MARC Controller

Deep Mukherjee, Palash Kundu and Apurba Ghosh

Abstract In this paper, modification of MIT rule of MARC (Model Adaptive Reference Controller) using fractional derivative concept has been proposed for an integer-order-inverted pendulum system which is highly unstable. Here, the G-L fractional derivative method has been proposed to design fractional-order MIT rule of MARC controller. This controller has been tuned by adaptive gain and an additional degree of freedom to the stable angular displacement of the pendulum and to track the reference model better with respect to time domain specifications. Next, this stability of inverted pendulum using fractional-order MIT rule has been analyzed with normal integer-order MIT rule.

Keywords Inverted pendulum · MARC · MIT rule · Fractional-order MIT rule

1 Introduction

A case study has been shown to implement model adaptive reference control of inverted pendulum system which is one of the most challenging tasks of control engineering, using normal MIT rule followed by fractional-order MIT rule and to compare their performances on stabilization of angular movement of the inverted pendulum. The MIT rule was designed in Massachusetts Institute of technology.

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Several researchers proposed various schemes to analyze the performance of inverted pendulum using different control techniques. Mohammad Ali [1] proposed the control of inverted pendulum cart system by use of PID controller. Reza [2] proposed the design of PID controller for inverted pendulum using a genetic algorithm. Adrian-Duka [3] proposed MARC using lyapunov theory and fuzzy model reference control for an inverted pendulum. The adaptive controller has been chosen as it is more effective than fixed gain PID controller to handle difficult situations.

In our work, the design of model adaptive reference control for inverted pendulum has been suggested by normal MIT rule [4] based on adaptation gain but this rule by itself does not improve stability and adaptive controller designed using MIT rule is very sensitive to the amplitudes of the signals. So the adaptive gain is generally kept small to make stable. Recently nowadays, fractional order [5] provides an effective means of capturing the approximate nature of the real world. So, MIT rule has been modified as fractional-order MIT rule with an extra degree of freedom which has been varied with fixed adaptation gain to test the characteristics of error between the reference model and plant and to analyze the nature of performance of pendulum using fractional-order rule over integer-order MIT rule.

1.1 Inverted Pendulum

The inverted pendulum is among one of the difficult systems to control in the field of control system [6] and has been taken as a benchmark for analyzing control strategies. An inverted pendulum is a nonlinear dynamic system including a stable equilibrium point, when the pendulum is at pending position and a stable equilibrium point when pendulum is at upright position. In our work, the inverted pendulum has been assumed which has been shown in in Fig. 1.

The pendulum is a weight suspended from a pivot, so that it can swing freely. When a pendulum is displaced sideways from its resting, equilibrium position, it is

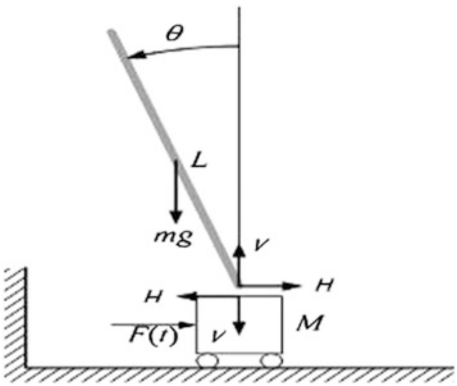

Fig. 1 Rotational inverted pendulum system that shows a free body diagram

subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position at $\theta = 0$. When released, the restoring force combined with the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth. In the above figure, let d_1 = length of pendulum, c = frictional constant, m = mass of pendulum, g = acceleration, d_0 = half length at centre, and T = tension.

The equation of motion for a pendulum is

$$J\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} - mgd_0\cos\theta = (d_1)t. \tag{1}$$

Take the Laplace transform as

$$\frac{\theta(s)}{T(s)} = \frac{d_1}{Js^2 + cs - mgd_0}. \tag{2}$$

The parameters are given as

a. J (Inertia) = 0.2453 N/S
b. C (Frictional Constant) = 0
c. m (Mass) = 900 g
d. g (Acceleration) = 9.81
e. d_1 (Length of Pendulum) = 0.102 m
f. d_0 (Half length at center) = 0.945 cm.

Substituting the values of parameters for a real-time process, the overall system transfer function is

$$\frac{\theta(s)}{T(s)} = \frac{0.102}{0.2453s^2 - 0.049}. \tag{3}$$

1.2 MARC (Model Adaptive Reference Controller)

Model adaptive reference control [7] has been chosen to design the adaptive controller, which is dependent on the adaptive gain by altering its output of real plant tracks the output of a reference model having same reference input and the adaptation law uses the error between the process and the model output. The basic block diagram of model adaptive reference control is shown in Fig. 2.

The transfer function of the reference model has been taken as follows:

$$T.F = \frac{9}{s^2 + 6s + 9}. \tag{4}$$

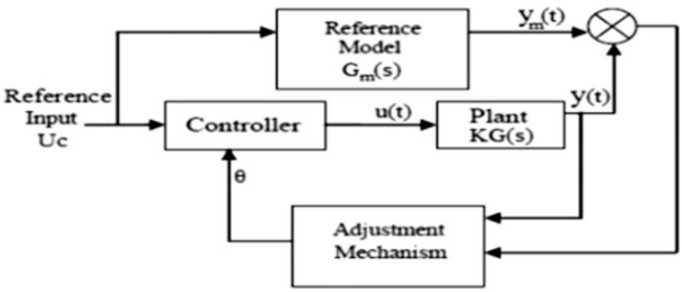

Fig. 2 Block diagram of model adaptive reference controller (MARC)

The inverted pendulum system has been taken as a plant to track the reference model.

1.3 MIT Rule

Adaptive gain plays a vital role to design MIT rule [8] and this adaptive gain is incrementally adjusted to minimize the error between the plant and model. The gradient process has been considered to design MIT rule. The cost function is defined as as follows:

$$J(\theta) = \frac{e^2}{2}, \tag{5}$$

where θ is the adjustable parameter and e is the error between the plant and model. θ is kept in the direction of the negative gradient of J. The normal MIT rule expression has been shown as follows:

$$\frac{d\theta}{dt} = -\gamma e \frac{de}{d\theta} \tag{6}$$

$$e = y(t) - y_m(t) \tag{7}$$

$$e = G_p u - G_m u_c \tag{8}$$

$$u = \theta_1 u_c - \theta_2 y_{plant} \tag{9}$$

γ is called as an adaptive gain and its range lies between 0.3 and 5 to track the model better. The reference model has been considered very close to plant.

$$\frac{de}{d\theta_1} = \frac{a_{1m}s + a_{0m}}{s^2 + a_{1m}s + a_{0m}} u_c \tag{10}$$

$$\frac{de}{d\theta_2} = -\frac{a_{1m}s + a_{0m}}{s^2 + a_{1m}s + a_{0m}} y_{plant} \tag{11}$$

1.4 Fractional-Order MIT Rule

The controller is very sensitive to the changes in the amplitude of the reference input using MIT rule and to overcome this problem an advanced method [9] of MIT rule has been introduced for parameter adjustment by to develop the control law. In our work, G-L fractional derivative [10] has been approached on error signal following the normal MIT rule expression and the new equation becomes as shown in below.

$$\frac{d\theta}{dt} = -\gamma e \frac{d^\alpha e}{d\theta^\alpha} \tag{12}$$

where the rate of change of parameter θ depends on both adaptation gain and the derivative order alpha. So, G-L fractional derivative [11, 12] has been defined as

$$D^\alpha f(t) = \lim_{h \to 0} \frac{1}{h^\alpha} \sum_{k=0}^{n} (-1)^k \binom{n}{k} f(t - kh), \tag{13}$$

where h is defined as step size.

Now assuming, $D^\alpha f(t) \approx D_h^\alpha f(t)$ it has been obtained as

$$D_h^\alpha f(t) = h^{-\alpha} \sum_{j=0}^{k} (-1)^j \binom{\alpha}{j} f(kh - jh). \tag{14}$$

Now $\binom{\alpha}{j}$ can be approximated as

$$\frac{\alpha!}{j!(\alpha - j)!} = \frac{\Gamma(\alpha + 1)}{\Gamma(j+1)\Gamma(\alpha - j + 1)}, \tag{15}$$

where Γ is defined as gamma function. Now, this gamma function [13] has been applied on the error signal.

$$\frac{d\theta}{dt} = -\gamma\left(\frac{d^{\alpha}}{d\theta^{\alpha}}e\right)y_m \tag{16}$$

where γ is adaptive gain, e is error between model and plant, y_m is reference output, and α is an additional degree of freedom. Using this mathematical expression fractional-order rule has been designed in Simulink.

2 Result and Analysis

Fractional-order MIT rule has been approached to study the nature of performance with one extra degree of freedom alpha [14], which has been varied with fractional order less than one keeping fixed adaptive gain value. The Simulink model is shown in Fig. 3.

The nature of the response of inverted pendulum using fractional-order MIT rule is shown in Fig. 4.

Now from Table 1, it has been analyzed that keeping the gain fixed as 0.4 and varying alpha between 0.5 and 0.75, the reference model has been tracked better using only the value of alpha with 0.5 with respect to rise time followed by settling time and peak overshoot as performance metrics (Table 2).

Now to compare with fractional-order technique design of normal MIT rule of MARC controller has been shown in Fig. 5.

The nature of the response of inverted pendulum with integer-order MIT rule is shown in Fig. 6 (Table 3).

So, from the above table, it has been studied that using the gain value 0.4 reference model has been tracked better with respect to performance metrics rise

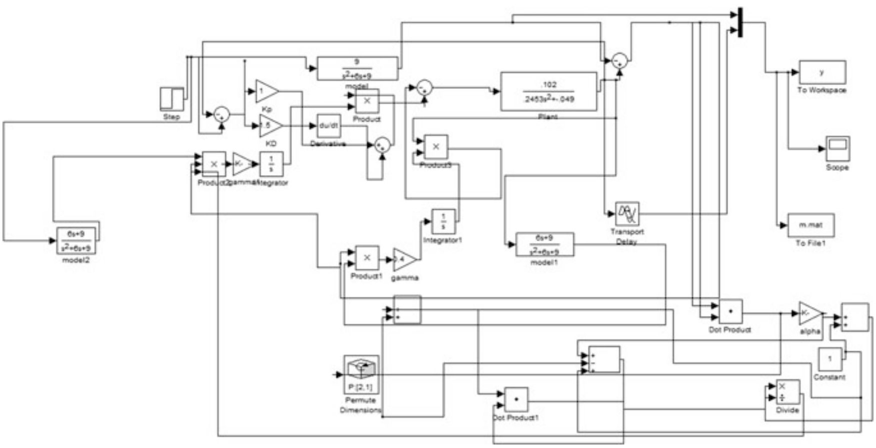

Fig. 3 Simulink model of fractional-order MIT rule

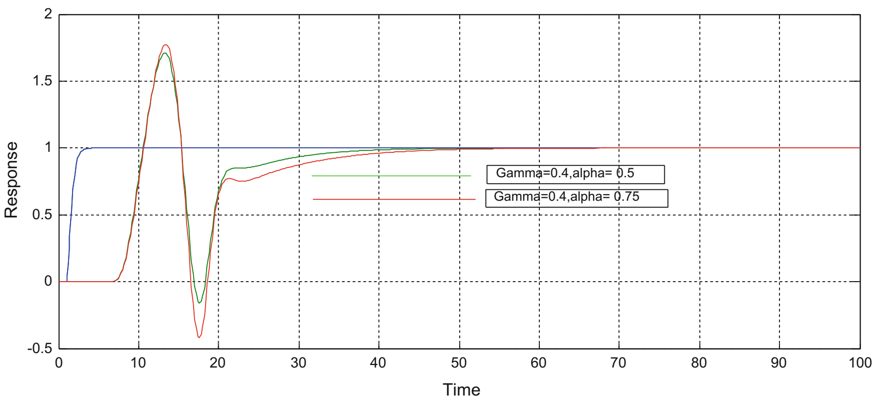

Fig. 4 Blue response shows reference output, red and green response shows plant output using fractional-order MIT rule

Table 1 Performance indices for plant using fractional-order MIT rule with fixed gain

Alpha	Gamma	T_r (s)	T_s (s)	$\%M_p$
0.5	0.4	5.40	32.60	1.71
0.75	0.4	5.40	40.00	1.77

Table 2 Performance indices for reference model

T_r (s)	T_s (s)	$\%M_p$
2.40	2.80	1.00

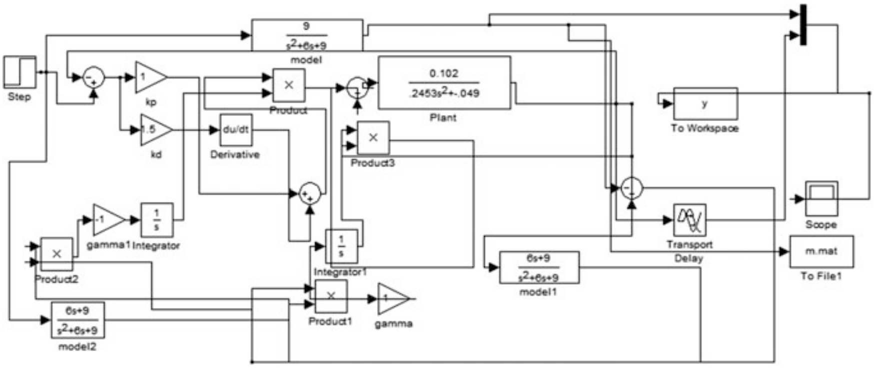

Fig. 5 Simulink model of integer-order MIT rule

time, settling time, and overshoot. But, keeping the fixed adaptive gain fractional-order rule is applied with one extra degree of freedom with value 0.5 reference model has been tracked with desired performance criteria where peak

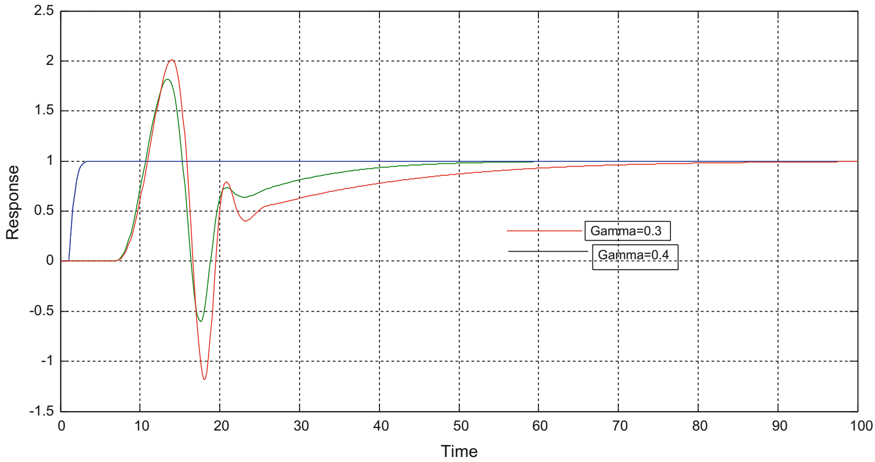

Fig. 6 Blue response shown model output, red and green response shows plant output with gain 0.3, 0.4 using integer-order MIT rule

Table 3 Performance indices for plant using normal MIT rule

Gamma	T_r (s)	T_s (s)	$\%M_p$
0.3	5.80	75.00	2.01
0.4	5.60	45.00	1.81

overshoot followed by rise time and settling time are less than the value of performance metrics using normal MIT rule.

3 Conclusion

A stability of an inverted pendulum has been analyzed approaching fractional-order MIT rule and normal MIT rule on model adaptive reference controller to track a stable reference model. Using normal MIT rule to track the reference model adaptive gain has been changed and with better adaptive gain as 0.4 the desired performance of plant has been achieved with rise time 5.60 s followed by settling time 45 s and overshoot 1.81. Next keeping fixed the adaptive gain as 0.4 fractional-order MIT rule has been applied on the current plant and varying additional degrees of freedom as 0.5 followed by 0.75, it has been studied that performance of plant has been achieved better with less rise time, settling time and overshoot than normal MIT rule. Using fractional-order rule the reference model has been tracked better than normal MIT rule with better stability.

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