Graph Representation of Multiple Misconceptions

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Abstract Subject-related misconceptions take place at nearly all levels of education where the process of identifying and overcoming them is most of the time quite laborious. The majority of related research is addressing single misconceptions in particular subjects. In this paper, we propose a graphical representation of interrelations among multiple misconceptions indicated by test results. Appropriate actions based on tests outcomes implying misconceptions are also suggested.

Keywords Misconceptions · Test outcomes · Graphical representation

1 Introduction

Students knowledge and skills are regularly evaluated throughout their studies where most of the time conclusions refer to degrees to which they have learned a subject. While the amount of research on testing students learning is really enormous, it is easy to notice that approaches to identify and overcome misconceptions have received much less attention. One possible explanation to that might be that some students have to take large-enrollment courses where opportunities for the active pursuit of the unknown during a lecture appear to be somewhat difficult even while taking advantages of up to date information technologies. Another one is related to the fact that it is not always that simple to distinguish between lack of knowledge and misconceptions. In this paper, we are discussing possibilities for what can be done in case of multiple misconceptions being detected.

Bipartite graphs [3] are employed to facilitate a search for possible patterns among misconceptions. Ordered sets [2] are to be applied for ranking already identified misconceptions. The outcome is very helpful while choosing appropriate actions to enhance students learning.

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2 Orderings and Graphs

The problem of determining a consensus from a group of orderings and the problem of making statistically significant statements about orderings are addressed in [2].

A partial ordering whose indifference relation is transitive is called a *weak ordering*. All partial orderings of a set with three elements can be seen in [2].

A bipartite graph is a graph whose vertices can be divided into two disjoint sets where every edge connects a vertex in one set to a vertex in the other set, [5].

Different approaches for remediation of misconceptions in physics are presented in [4, 6]. Fostering conceptual change in different domains is discussed in [7].

3 Misconceptions

The main idea in this section is to find patterns among possible students misconceptions. Students are suggested to take a test. Their understandings of four concepts are to be evaluated by analyzing tests' outcomes. Such tests are usually expected to address problems in a unit within a subject, but the below-presented structure allows various applications. This means that if a test is addressing understanding of concepts initially represented in different subjects one can detect possible transfer of misconceptions from one subject to another. These types of transfers may easily go unnoticed when assessment of knowledge and skills is essential for grading.

In a case submitted answers indicate misconceptions, appropriate assistance will be provided in the form of personalized theoretical explanations and examples, after which the student may retake the test. Can misconceptions be related in a sense that if there is evidence for one or several misconceptions then there is a strong possibility for other particular misconceptions to be detected? In this work, we present a graphical illustration of misconceptions possibly appearing together in students responses.

To the rest of this work letters "i, j, m, n" stand for correct answers to the first, second, third, and fourth test questions where "k, o, v, w" stand for incorrect answers to these questions. Omitted answers are treated as wrong answers.

All types of responses are assigned to the vertices of the graph in Fig. 1 previously shown in [1]. Needless to say, a couple of tests responses can differ in one, two, three, or four answers (each test contains four questions). This can become very handy when a lecture is focusing on either individual or groups progress. For single students, it will assist in the process of providing personalized guidance where clustering of group responses simplifies the search for teaching materials that need modifications.

Vertices whose labels differ in one position only are connected by a single edge, Fig. 2.

Examples of response types which differ in two positions are listed below.

Any of the four groups of vertices

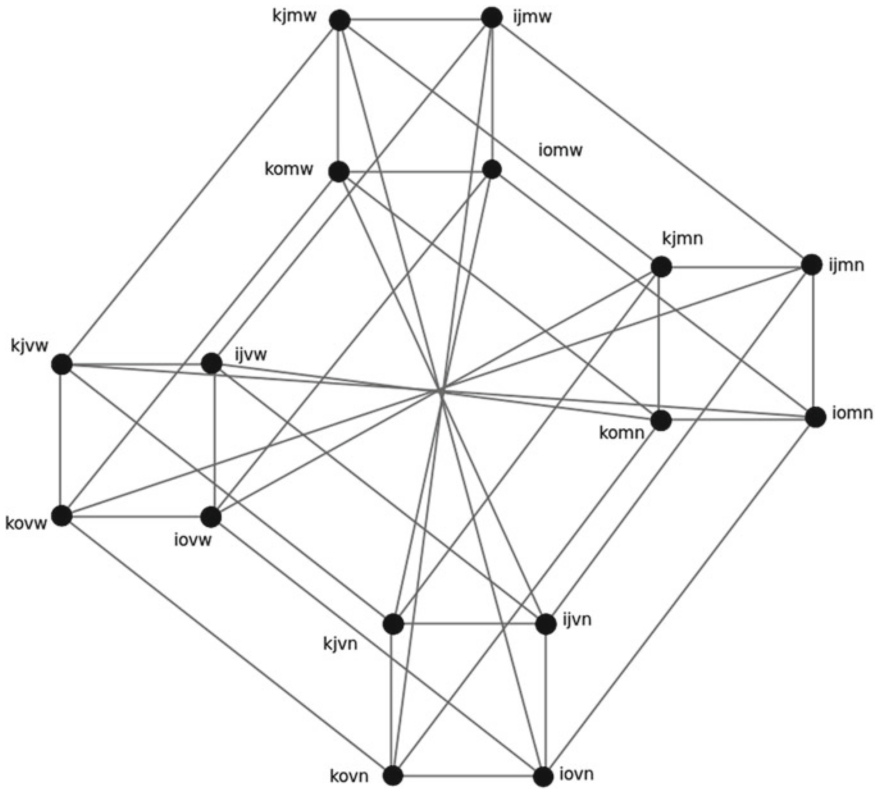


Fig. 1 Response types

- $\{k, j, m, w\}$, $\{k, j, v, w\}$, $\{k, j, v, n\}$, $\{k, j, m, n\}$ (highlighted in red, Fig. 3)
- $\{i, j, m, w\}$, $\{i, j, m, n\}$, $\{i, j, v, n\}$, $\{i, j, v, w\}$
- $\{k, o, m, w\}$, $\{k, o, m, n\}$, $\{k, o, v, n\}$, $\{k, o, v, w\}$
- $\{i, o, m, w\}$, $\{i, o, m, n\}$, $\{i, o, v, n\}$, $\{i, o, v, w\}$

contains the same response types in the first two positions.

Any of the four groups of vertices

- $\{k, j, m, w\}$, $\{k, o, m, w\}$, $\{i, o, m, w\}$, $\{i, j, m, w\}$ (highlighted in blue, Fig. 3)
- $\{k, j, v, w\}$, $\{k, o, v, w\}$, $\{i, o, v, w\}$, $\{i, j, v, w\}$
- $\{k, j, v, n\}$, $\{k, o, v, n\}$, $\{i, o, v, n\}$, $\{i, j, v, n\}$
- $\{k, j, m, n\}$, $\{k, o, m, n\}$, $\{i, o, m, n\}$, $\{i, j, m, n\}$

contains the same response types in the last two positions.

Any of the four groups of vertices

- $\{i, j, v, w\}$, $\{i, o, v, w\}$, $\{i, o, v, n\}$, $\{i, j, v, n\}$ (highlighted in green, Fig. 3)
- $\{k, j, v, w\}$, $\{k, o, v, w\}$, $\{k, o, v, n\}$, $\{k, j, v, n\}$

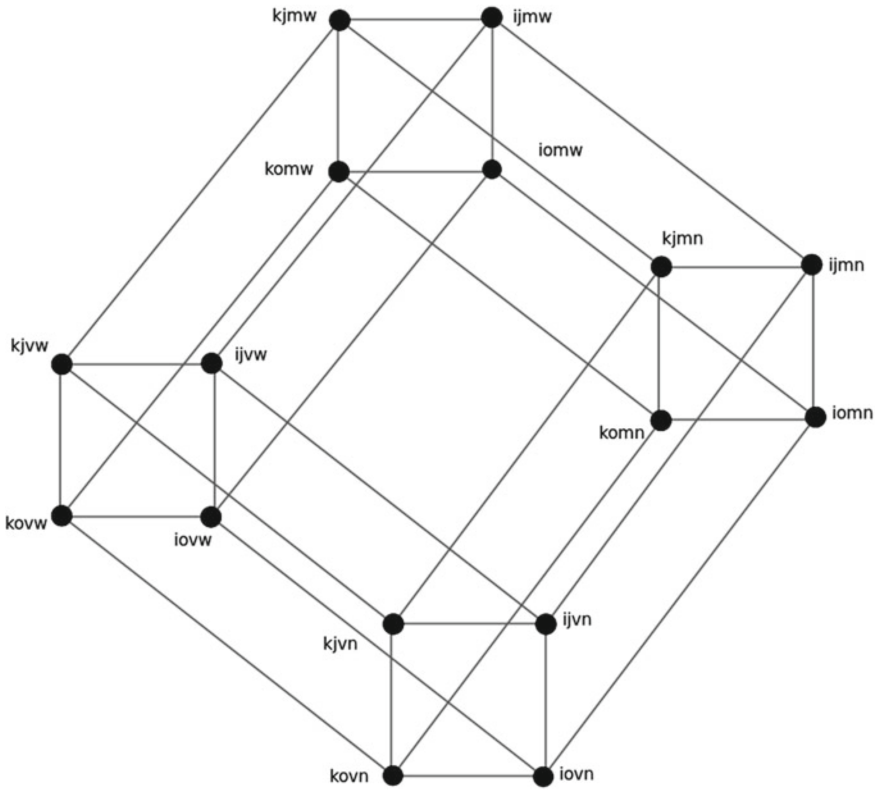


Fig. 2 Single edge connected vertices

- $\{k,j,m,w\}$, $\{k,o,m,w\}$, $\{k,o,m,n\}$, $\{k,j,m,n\}$
- $\{i,j,m,w\}$, $\{i,o,m,w\}$, $\{i,o,m,n\}$, $\{i,j,m,n\}$

contains the same response types in the first and third positions. These are only three of many other existing groups of vertices with similar properties.

The four vertices $\{i,j,v,w\}$, $\{k,j,v,w\}$, $\{k,o,m,w\}$, and $\{k,o,v,n\}$, (highlighted in read) whose labels differ in exactly three positions from vertex $\{i,j,m,n\}$ (highlighted in green), are shown in Fig. 4. The rest of such related vertices are placed in a similar configuration.

Any two vertices that do not share any response type are connected by an edge highlighted in blue in Fig. 4.

If several students answers appear to be in one of the groups described above it is beneficial to look at the particular pattern, provide appropriate assistance and follow the development. Further adjustments of teaching materials, questions' formulations, and help functions might be needed.

Let us now look into more details at how a vertex is connected to other vertices and keep in mind that all vertices are connected in the same way due to this graph

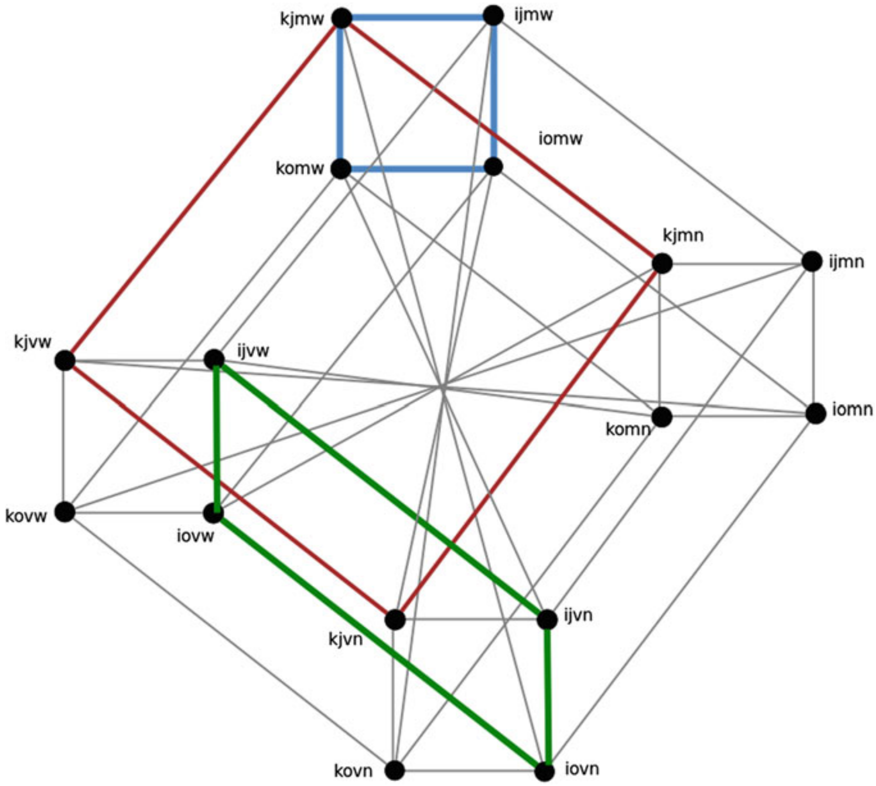


Fig. 3 Connected vertices implying similar responses

properties. A vertex $\{k, o, v, w\}$ in Fig. 5 is connected to five other vertices where: two of them (highlighted in green) are its neighbor vertices in the same square ($\{k, o, v, w\}$, $\{j, o, v, w\}$, $\{i, j, v, w\}$, $\{k, j, v, w\}$), two other vertices (highlighted in red) belong to the two neighbor squares ($\{k, o, m, w\}$, $\{i, o, m, w\}$, $\{i, j, m, w\}$, $\{k, j, m, w\}$) and ($\{k, o, v, n\}$, $\{i, o, v, n\}$, $\{i, j, v, n\}$, $\{k, j, v, n\}$), and a vertex belonging to the opposite square ($\{k, o, m, n\}$, $\{i, o, m, n\}$, $\{i, j, m, n\}$, $\{k, j, m, n\}$) where they do not share any response type (highlighted in blue).

Suppose a student is taking a test for the first time and her answers are "$\{k, o, m, w\}$". She is then suggested to go through selected learning materials and then take the test second time. All tests appear seemingly different, i.e., they are addressing the same concepts with different questions. If the answer to the second test is, Fig. 5:

- "$\{k, j, v, w\}$" or "$\{i, o, v, w\}$" (highlighted in green) then either the original presentation of the last two concepts might need adjustment and/or the related learning materials should be revised,

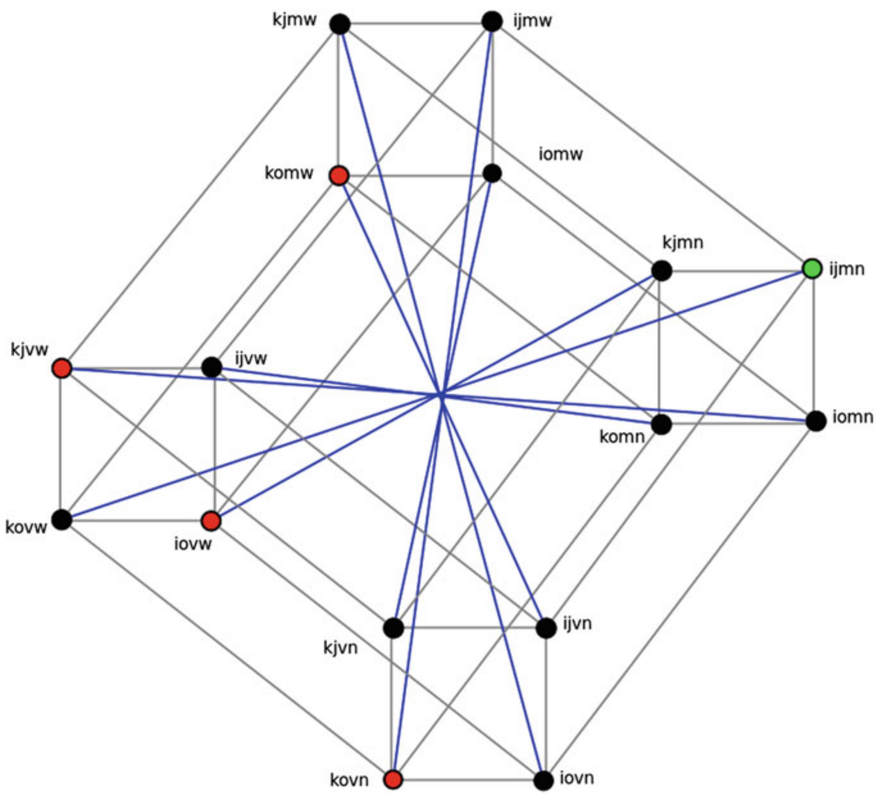


Fig. 4 Vertices whose labels differ in exactly three positions

- "$\{k, o, m, w\}$" or "$\{k, o, v, n\}$" (highlighted in red) then the original presentation of the first two concepts might need adjustment and/or the related learning materials should be revised,
- "$\{i, j, m, n\}$" (highlighted in blue) then it is of interest to find out whether the same help is appropriate for more students.

Suppose two consecutive responses from a student appear in vertices not connected by a single edge. In such cases, the provided help should follow recommendations related to single concepts being misunderstood.

Thus presented graphical illustration of students responses gives a good indication of the clarity of tests contents and usefulness of provided help.

While offered to take a test, students will also be asked to rank the concepts with respect to difficulties. Based on their responses the four concepts will be ordered and appear in one of the forms in Fig. 6. Concepts are denoted by a, b, c, d in order to be independent of local notations. They are further on placed in sets IV, III, II according to the number of concepts being compared by a student. Thus, the set IV contains all cases where all four concepts in a test are compared, in the set III only

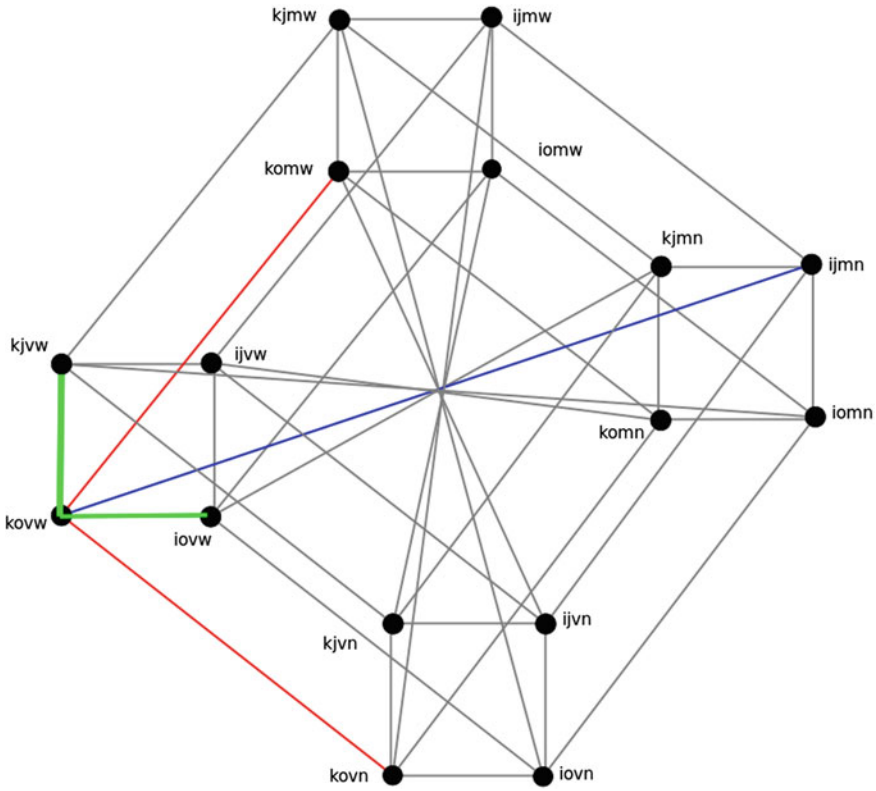


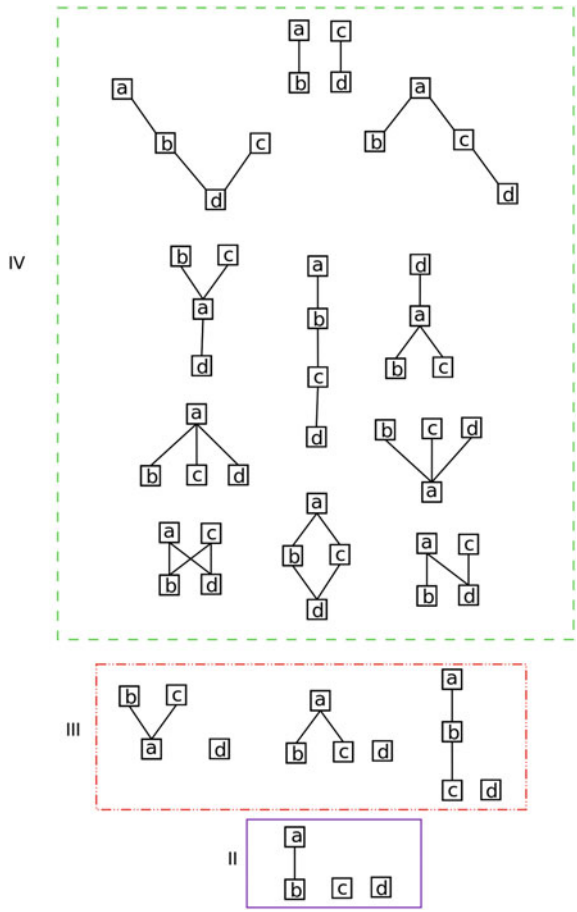
Fig. 5 Connected vertices

three of the four are compared, in the set *II* only two of the four are compared, and the set containing cases where students do not provide any opinions is trivial and therefore omitted. The top concept in any of the sets IV, III, II is supposed to be the most difficult one to understand and when concepts appear unrelated, it means they have not been compared by test takers.

Note that ranking concepts with respect to difficulties is not always related to correct responses. The purpose of ranking is to receive feedback from students and take some actions afterward to facilitate better understanding.

Remark: Distances in graphs can be also applied to facilitate automated evaluations of tests responses. Suppose a group of students takes a test only two times. This data can be used to compare distances among responses from another group of students that take the test only two times. As a result, modifications of concepts presentations might be introduced. When students take the test three times or more, both individual and group data can be used to improve knowledge transfer.

Fig. 6 Concepts ranking



4 Conclusion

Conceptual understanding can often be very demanding and energy consuming. To prevent the latter, we propose a test of only four questions addressing four concepts. Thus presented graphical illustration of students responses gives a clear indication about the quality of test's contents and usefulness of provided help. Furthermore detailed consideration of tests outcomes and concept ranking can contribute to the adjustment of teaching materials.

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