On the Thresholds of Vortex Identification Methods

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Abstract Several vortex identification methods along with a newly proposed Ω method are examined in the Burgers vortex and the Sullivan vortex. Of particular interest is the physical meaning of the parameter, especially the thresholds. While all the methods are capable of capturing precise vortex boundaries in these two analytical vortices, only the parameter Ω seems to have a clear physical meaning, i.e. to what extent the local fluid particles tend to rotate like a rigid-body. Therefore, the parameter Ω might be helpful and informative when utilized to investigate the interaction of vortical structures.

Keywords Threshold · Vortex identification · Ω method

1 Introduction

The ambiguity introduced by choosing a threshold when employing vortex identification methods like Q, λ_2, Δ and λ_{ci} [2, 8] corrupts the objectivity of vortex dynamics study. For example, Smith et al. [6] hold the idea that a myriad of hairpin type vortices dominate wall adjacent region of near-wall turbulence and low-speed streaks are generated by the passage of hairpin vortices. This hierarchy of hairpin vortices and vortical packets are also believed to play a significant role in fully-developed turbulence [1] and during transition [4, 7]. However, other researchers like Schlatter et al. [5] denies the dominance of hairpin vortices in fully developed turbulent boundary layers. This disagreement might result from the difference of chosen thresholds utilized by different researchers. Thus, to identify the vortical structures without an ambiguous threshold is of great importance in the study of turbulence generation and sustenance mechanism.

Recently, a new Ω vortex identification method proposed by Liu et al. [3] is claimed to be able to capture vortical structures with a uniform threshold in

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various flows. The parameter Ω is expected to be 1 in regions with rigid-body rotation and 0 in uniform flows. Along with other criteria mentioned above, these vortex identification methods are applied to several exact vortex solutions and analysis on the thresholds is given.

2 The Burgers Vortex

In a Burgers vortex, the velocity components written in cylindrical coordinates is

$$\begin{aligned} V_r &= -ar \\ V_\theta &= \frac{\Gamma_0}{2\pi r}[1 - e^{-\frac{ar^2}{2\nu}}] \\ V_z &= 2az \end{aligned} \tag{1}$$

where Γ_0 is the circulation, a the axisymmetric strain rate, and ν the kinematic viscosity. Apply the vortex identification methods to the Burgers vortex, we get

$$\mathrm{Q} = a^2(\mathrm{Re}^2\eta(\tilde{r}) - 3) \tag{2}$$

$$\Delta = \frac{a^6\mathrm{Re}^2\eta(\tilde{r})}{27}[9 + \mathrm{Re}^2\eta(\tilde{r})] \tag{3}$$

$$\lambda_2 = a^2(1 - \mathrm{Re}^2\eta(\tilde{r})) \tag{4}$$

$$\Omega = \frac{0.5}{1 - 2\mathrm{Q}/(a\mathrm{Re}e^{-\tilde{r}^2/2})^2} \tag{5}$$

where $\mathrm{Re} = \Gamma_0/2\pi\nu, \tilde{r} = r\sqrt{a/\nu}$, and the auxiliary function $\eta(\tilde{r})$ is defined as

$$\eta(\tilde{r}) = \frac{1}{\tilde{r}^4}((1 + \tilde{r}^2)e^{-\tilde{r}^2/2} - 1)(1 - e^{-\tilde{r}^2/2})$$

Apply the criteria to a Burgers vortex with $a = 1s^{-1}$, $\nu = 0.02m^2/s$ and $\Gamma_0 = 5m^2/s$ (thus $\mathrm{Re} = 39.79$), and the criterion values distribution along radial direction is shown in Fig. 1, and the identified locations of vortex boundaries by these criteria are listed in Table 1.

The red dashed lines in Fig. 1 indicate the thresholds of corresponding methods based on its original ideas. However, thresholds with magnitude much larger than zero (like 1000 or 10000) are used in practical applications. On the other hand, the Ω method is able to use a uniform threshold that a bit larger than 0.5 (like 0.52) to identify vortices in various applications [3].

As expected, the parameter Ω equals 1 on the center of the Burgers vortex where rigid-body rotation happens, and approaches 0 as r becomes larger from Fig. 1. The variation of Ω is smooth and logical. Thus, for a Burgers vortex the parameter Ω can

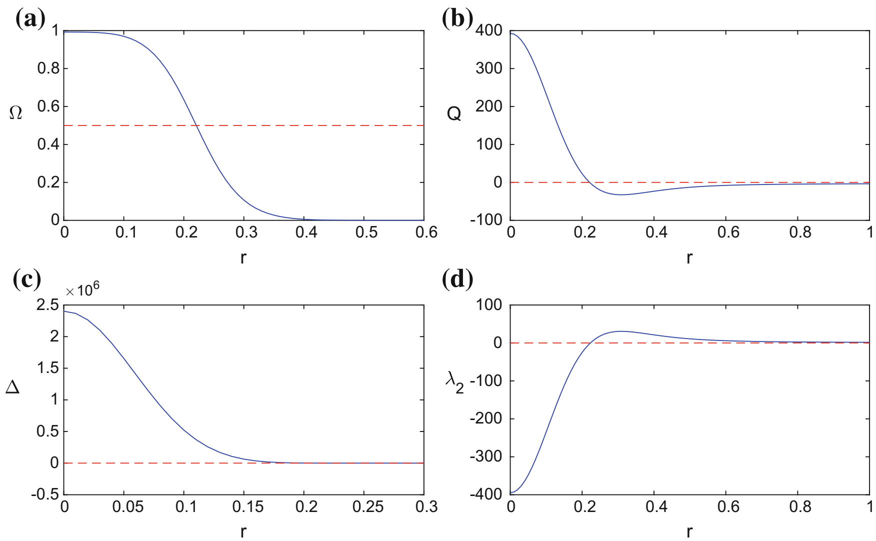

Fig. 1 Vortex criteria test by a Burgers vortex

Table 1 Calculated radius of the Burgers vortex by different criteria

	$r(V_\theta max)$	Ω	Q	Δ	λ_2
r_0	0.2241	0.2208	0.2208	0.2358	0.2230

be a local measure that to what extent the fluid motion tend to be rigid-body rotation. However, the ranges of parameters in other methods are not normalized. Although a larger parameter represents stronger swirling strength, the absolute physical meaning of the parameters is unclear. In addition, Q, Δ and λ_2 tend to be zero as r becomes larger while the the thresholds based on the original ideas of these criteria are zero. This might cause problems when the Burgers vortex domain of interest is too large. The identified boundary radii are given in Table 1. $r(V_\theta max)$ is the radius where the maximum circumferential velocity locates. And the radii of the Burgers vortex identified by Ω and Q methods are identical which results from the clear relationship between the two parameters expressed by Eq. 5. The radii predicted by Δ and λ_2 methods are a little larger, but also smaller than $r(V_\theta max)$. One particular problem with the Δ method is that the gradient at its threshold is near zero, which will cause the iso-surface becomes rough in visualization softwares.

3 The Sullivan Vortex

The Sullivan vortex is a two-celled vortex aimed to describe the flow in an intense tornado with a central downdraft. Its mathematical form is

$$
\begin{aligned}
V_r &= -ar + \frac{6\nu}{r}\left[1 - \exp\left(-\frac{ar^2}{2\nu}\right)\right] \\
V_\theta &= \frac{\Gamma_0}{2\pi r}\frac{H(ar^2/2\nu)}{H(\infty)} \\
V_z &= 2az\left[1 - 3\exp\left(-\frac{ar^2}{2\nu}\right)\right]
\end{aligned}
\tag{6}
$$

where $H(\eta) = \int_0^\eta \exp\left(-s + 3\int_0^s \frac{1-e^{-\tau}}{\tau}d\tau\right)ds$ and thus $H(\infty) = 37.905$. Apply the criteria to a Sullivan vortex with $a = 1\,\mathrm{s}^{-1}$, $\nu = 0.02\,\mathrm{m}^2/\mathrm{s}$ and $\Gamma_0 = 5\,\mathrm{m}^2/\mathrm{s}$, the parameters of the methods along radial direction r are shown in Fig. 2 and the radii of identified inner and outer vortex boundaries are shown in Table 2.

Despite the difference in the radii as shown in Table 2, the methods are all capable of identifying the inner cell and outer cell vortex. The physical meaning of parameter Q, Δ and λ_2 is still unclear, especially for λ_2 method that the three eigenvalues represented by black dotted line (λ_1), blue line (λ_2) and green starred line (λ_3) in Fig. 2. It can be seen that the first and second eigenvalues connect at two locations,

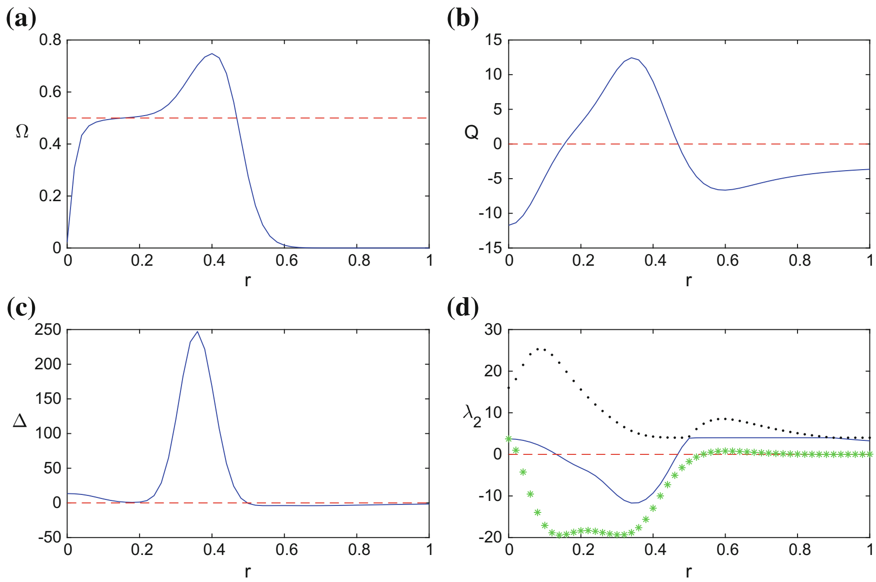

Fig. 2 Vortex criteria test by a Sullivan vortex

Table 2 Calculated radius of the Sullivan vortex by different criteria

	Ω	Q	Δ	λ_2
r_{0in}	0.1542	0.1542	0.1807	0.1323
r_{0out}	0.4689	0.4689	0.4969	0.4678

which means the eigenvalues' magnitude alternates. Thus the physical meaning of λ_2 is unclear. On the other hand, The parameter Ω still ranges from 0 to 1. The maximum Ω in this Sullivan vortex is around 0.7477, which relates to the chosen a, ν and Γ_0. Therefore, for the Sullivan vortex the parameter Ω can still be a local measure of the level fluids rotate like rigid-body.

4 Conclusions

The Burgers vortex and Sullivan vortex are examined with vortex identification methods including Ω, Q, Δ and λ_2 methods. It is concluded for the Burgers vortex and the Sullivan vortex, the parameter Ω could be a indication of to what extent the local fluid particles tend to rotate like a rigid-body, while parameters in other methods could not. Therefore, the Ω parameter might be helpful and informative in investigating vortex interactions in wall-bounded transitional and turbulent flows.

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