Exploring a Universal Wake Number for Finite-Height Bluff Bodies

S. Unnikrishnan and D. Sumner

Abstract The suitability of two universal wake numbers, namely Roshko's universal Strouhal number and the Griffin number, was explored for surface-mounted finite-height bluff bodies. Mean drag force coefficient, Strouhal number, and mean base pressure coefficient data for finite cylinders and finite square prisms, for various Reynolds numbers and aspect ratios, were considered. Comparison was made to established values for the two-dimensional cylinder and square prism for Reynolds numbers in the range of $10^4 < \mathrm{Re} < 10^5$. The Griffin number was found to be the most suitable universal wake number, and was reasonably successful at collapsing data for finite cylinders and finite square prisms for a wide range of aspect ratios and incidence angles, particularly if the body's aspect ratio was higher than the critical aspect ratio.

Keywords Bluff body · Wake · Vortex shedding · Finite square prism
Finite cylinder · Base pressure

1 Introduction

Vortex shedding occurs from many types of bluff bodies and the vortex street wakes of different bodies are often similar. The vortex shedding frequency is closely related to the size of the near-wake region, including the lateral spacing of the shear layers, the length and width of the near-wake recirculation zone, and the vortex formation length. For two-dimensional (2D) bluff bodies, the similarity in the vortex street wakes and the inter-relationship between the vortex shedding frequency, base pressure, and drag force, have led to the development of various

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"universal wake numbers", which can be used to characterize bluff body vortex wakes independent of the body geometry or flow regime.

The universal Strouhal number of Roshko [15], $\mathrm{St_R}$, and the universal wake number of Griffin [4], G, have been reasonably successful at scaling and collapsing vortex shedding frequency (f), base pressure (P_B), and drag force (F_D) data from a wide range of 2D bluff bodies, including stationary and oscillating cylinders and prisms [1, 4, 9], as well as groups of cylinders [16], over a wide range of Reynolds number, Re ($= U_\infty D/\nu$, where U_∞ is the freestream velocity, D is the body width, and ν is the kinematic viscosity). Collapse of the data is obtained by using theoretical or semi-empirical length and velocity scales associated with the near wake instead of the body geometry or upstream flow conditions. Both $\mathrm{St_R}$ (Eq. 1) and G (Eq. 2) are functions of the Strouhal number, St ($= fD/U_\infty$), base pressure coefficient, C_{PB} ($= 2(P_B - P_\infty)/(\rho U_\infty^2)$, where P_∞ is the freestream static pressure, and ρ is the fluid density), and mean drag coefficient, C_D ($= 2F_D/(\rho U_\infty^2 A)$, where A is the frontal area), as shown below (where $K = (1 - C_{PB})^{1/2}$).

$$\mathrm{St_R} = -\frac{\mathrm{St}}{K}\frac{C_D}{C_{PB}} \tag{1}$$

$$\mathrm{G} = \frac{\mathrm{St}C_D}{K^3} \tag{2}$$

In the present study, the suitability of $\mathrm{St_R}$ and G is explored for the flow around surface-mounted finite-height square prisms and cylinders, where the flow field is strongly three-dimensional (3D).

2 Experimental Approach

The data come from three sets of wind tunnel experiments for surface-mounted finite-height square prisms and cylinders of aspect ratio AR $= H/D = 9, 7, 5$, and 3 (where H is the body height). The C_D and St data for the finite square prisms were taken from McClean and Sumner [6] at Re $= 7.4 \times 10^4$, for incidence angles from $\alpha = 0°$ to $45°$; C_{PB} data for AR $= 3$ were obtained from near-wake seven-hole-probe measurements at Re $= 3.7 \times 10^4$ by Ogunremi and Sumner [14]; the C_{PB} data for the other aspect ratios were obtained in similar experiments (unpublished). The C_D, St and C_{PB} data for the finite cylinders, at Re $= 6 \times 10^4$, were taken from Sumner et al. [17]. The C_{PB} data were obtained at a streamwise location of $x \sim 1.2D$ downstream of the body. Since the vortex formation length, and therefore C_{PB}, vary along the height of the 3D bodies, a representative value of C_{PB} was needed to scale the data. Here, the average value of C_{PB} between $z/H = 0.1$ and 0.9 (where z is the wall-normal or vertical coordinate) was used.

3 Results and Discussion

Figure 1a shows the variation of $\mathrm{St_R}$ and G with Re for a 2D cylinder and a 2D square prism at $\alpha = 0°$ for $10^4 < \mathrm{Re} < 10^5$. For the 2D bluff bodies, the data collapse to the same value of $\mathrm{St_R} = 0.124$ (Table 1) for both the cylinder and the prism, and to comparable values of G = 0.069 and G = 0.073 for the cylinder and prism, respectively. For the 2D bodies, the two wake numbers are therefore independent of the body shape and may be considered "universal".

Also shown in Fig. 1a are data for the finite cylinder. The $\mathrm{St_R}$ data for the finite cylinders of AR = 9, 7, and 5 ($\mathrm{St_R} = 0.131$, 0.120, and 0.119, respectively) are close to the 2D cylinder value of $\mathrm{St_R} = 0.124$ (Table 1). However, for the finite cylinder of AR = 3, a higher value of $\mathrm{St_R} = 0.161$ is obtained. It is noted that the finite cylinder of AR = 3 lies below the critical aspect ratio, and has a distinct vortex wake compared to finite cylinders of higher aspect ratio [17]. In contrast to the $\mathrm{St_R}$ data for the finite cylinder, the Griffin number is relatively insensitive to AR, ranging from G = 0.051 to 0.056 for the four finite cylinders (Fig. 1a, Table 1). However, the G values for the finite cylinder are slightly lower than the 2D cylinder value of G = 0.069, a result that suggests different physical mechanisms may be influencing vortex shedding from the finite cylinder, such as variation of the separation angle and near-wake width along the cylinder height.

Figure 1a also shows data for the finite square prism at $\alpha = 0°$. Again, the results are similar to those of the finite cylinder discussed above. The finite square prism of AR = 3 lies below the critical aspect ratio [6], with a different wake structure, and hence distinct universal wake numbers of $\mathrm{St_R} = 0.109$ and G = 0.051 are obtained compared to the finite prisms of AR = 9, 7, and 5 (Table 1). Moreover, both $\mathrm{St_R}$ and G attain different values for the finite prism compared to the 2D prism, which, again, suggests different physical mechanisms (related to the free end) are influencing the vortex wake.

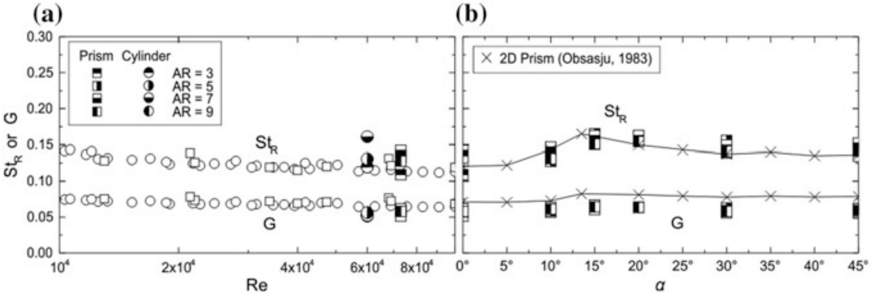

Fig. 1 Universal wake number data for finite-height square prisms and cylinders: **a** as a function of Reynolds number; **b** data for square prisms as a function of incidence angle. 2D cylinder data: ○, from Norberg [12]. 2D square prism data: □, from Bearman and Trueman [3], Bearman and Obasaju [2], Luo et al. [5], Minguez et al. [7], Nakaguchi et al. [8], Noda and Nakayama [10], Norberg [11], Obasaju [13] and Vickery [18]

Table 1 Summary of universal wake number data (for $10^4 < \mathrm{Re} < 10^5$)

Cylinders	$\mathrm{St_R}$	Std. dev.	G	Std. dev.
2D Cylinder [12], Fig. 1a	0.124	7%	0.069	5%
Finite cylinder, AR = 9	0.131		0.056	
Finite cylinder, AR = 7	0.120		0.052	
Finite cylinder, AR = 5	0.119		0.051	
Finite cylinder, AR = 3	0.161		0.053	
Square prisms (α = 0° only)	$\mathrm{St_R}$	Std. dev.	G	Std. dev.
2D square prism (various studies, Fig. 1a)	0.124	6%	0.073	4%
Finite square prism, AR = 9	0.128		0.058	
Finite square prism, AR = 7	0.142		0.061	
Finite square prism, AR = 5	0.133		0.060	
Finite square prism, AR = 3	0.109		0.051	
Square prisms (α = 0°–45°)	$\mathrm{St_R}$	Std. dev. (%)	G	Std. dev. (%)
2D Square Prism (Obasaju [13], Fig. 1b)	0.139	9	0.077	5
Finite square prism, AR = 9	0.141	7	0.060	5
Finite square prism, AR = 7	0.152	5	0.063	4
Finite square prism, AR = 5	0.149	10	0.062	4
Finite square prism, AR = 3	0.133	11	0.056	6

Figure 1b shows $\mathrm{St_R}$ and G as functions of α for both 2D and finite square prisms. Peak values of $\mathrm{St_R}$ and G at $\alpha = 13.5°$ and 15° for the 2D and finite prisms, respectively, correspond to the critical incidence angle associated with minimum C_D, maximum lift coefficient magnitude, maximum St, and greatest wake asymmetry [14]. At non-zero α (over the range $\alpha = 0°– 45°$) the average values of $\mathrm{St_R}$ and G are different than those obtained at $\alpha = 0°$, both numbers becoming slightly higher when the prism is no longer oriented at 0° (Table 1). This may be caused by asymmetry in the near wake of the prism, the distinct behaviours of the two separated shear layers, and the different wake flow patterns observed as α is varied [6]. Of the two universal numbers, the Griffin number is better at collapsing the 2D and finite prism data to common values that are sensibly independent of α. Lower values of G are again obtained for the finite prism of AR = 3, which is below the critical AR (Table 1).

4 Conclusions

This study suggests that the universal wake number concept, introduced by Roshko [15] and extended by Griffin [4] and others, may be applicable to the flow around surface-mounted finite-height cylinders and square prisms, particularly if the body's aspect ratio is greater than the critical aspect ratio, at least within the Reynolds number range of $10^4 < \mathrm{Re} < 10^5$. Of the two universal wake numbers considered here, the Griffin number (G) was more successful in collapsing finite-height bluff body data compared to Roshko's universal Strouhal number ($\mathrm{St_R}$); this result is consistent with the findings of other studies (e.g., [16]. Differences in the values of $\mathrm{St_R}$ and G between the 2D and finite-height bodies may be attributed to different physical influences on vortex shedding.

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