

Trident Convection in a Cube



M. Kodama, M. Nobuhara, K. Tatsumoto, H. Tanigawa
and K. Hirata

Abstract The authors numerically investigate the influence of the forced oscillation upon the three-dimensional thermal convection in a cubic cavity heated from one wall and chilled from its opposite wall in the gravity and zero-gravity fields without/with a forced sinusoidal oscillation. The direction of the forced oscillation is parallel to the temperature gradient direction. In addition, the direction is parallel to the direction of the gravity, in the gravity field. The authors assume incompressible fluid with a Rayleigh number $Ra = 8.0 \times 10^4$ in the gravity field without the forced oscillation or $Ra = 1.0 \times 10^4$ in the gravity field and a Prandtl number $Pr = 7.1$ (water). The forced-oscillation parameters are a vibrational Rayleigh number Ra_η and a non-dimensional forced-oscillation frequency ω . In the gravity field, $Ra_\eta = 1.0 \times 10^5$ and $\omega = 5.0 \times 10^0$, and in the zero-gravity field, $Ra_\eta = 1.1 \times 10^5$ and $\omega = 5.0 \times 10^0$. As a result, supposes reports a new flow structure in laminar and steady thermal convection, which consists of a pair of trident currents, namely, three ascending streams and three matching descending streams in a cube heated from a bottom wall and chilled from its opposite top wall. This flow is rather robust. Then, it can be observed in a stationary cube under the gravity, and can be observed in an oscillating cube under the gravity or zero gravity besides.

Keywords Rayleigh-Bénard convection · Natural convection
Convective instability

M. Kodama · M. Nobuhara · K. Tatsumoto · K. Hirata (✉)
Department of Mechanical Engineering, Doshisha University, 1-13 Tatara Miyakodani,
Kyotanabe, Kyoto, Japan
e-mail: khirata@mail.doshisha.ac.jp

H. Tanigawa
Department of Mechanical Engineering NIT, Maizuru College, Maizuru,
Kyoto 625-8511, Japan

1 Introduction

Thermal convection is a key phenomenon for heat and mass transfer and mixing. Bénard [1] considered a horizontal layer. After Bénard, a lot of researchers have studied practical problems of thermal convection.

Here, we consider thermal convection in a cavity (or a container). One of the simplest cavities is a cube. Especially for thermal convection in a cube, for example Pallares et al. [2–4] numerically showed three-dimensional flow structures in the cubic cavity for moderate Rayleigh numbers $Ra < 6.0 \times 10^4$. They reported seven different steady-flow structures; namely, four kinds of single-roll structures (referred to as S1, S2, S3 and S7), two kinds of four-roll structures (referred to as S5 and S6), and one kind of a toroidal-roll structure (referred to as S4).

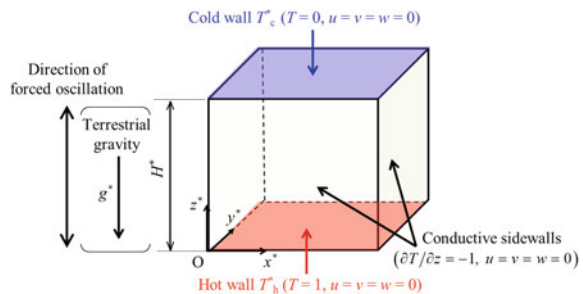
In this report, we report a new flow structure in laminar and steady thermal convection as one facet of conductive diversity. This flow structure involves three downwelling and three matching upwellings in a cube heated from below and chilled from above, as well as our past studies [5–8]. This flow structure is not transient but stable. So, it is commonly observed in various situations, such as in a stationary cube under the gravity and in an oscillating cube under the gravity or zero gravity besides.

2 System Modelling

2.1 Model and Governing Equation

Figure 1 shows the current model. The model is the flow in a cubic cavity with a length scale H^* . We analyse the thermal convection assuming incompressible flow with a constant Prandtl number $Pr = 7.1$ (water) in the non-gravitational field. A pair of opposed walls are taken to be isothermal, and the temperature of one of them (hereinafter, referred to as a cold wall) is greater than the other (hereinafter, referred

Fig. 1 Computational domain, together with coordinate system and boundary conditions



to as a hot wall). The four walls beside the cold and hot walls (hereinafter, referred to as sidewalls) are conductive.

The governing equations are the continuity equation, the dimensionless Navier-Stokes equations with the Boussinesq approximation and a dimensionless energy equation. They are as follows;

$$\nabla \cdot u = 0 \quad (1)$$

$$\frac{Du}{Dt} = -\nabla p + Pr\Delta u + Ra_\eta Pr T \sin \omega t e_z \quad (2)$$

and

$$\frac{DT}{Dt} = \Delta T \quad (3)$$

where $u = (u, v, w)$, t , p , T , e_z and ω denote velocity vector, time, pressure, temperature, the unit vector in the z direction and (angular) frequency, respectively. We the definitions of Ra_η and Pr in the following subsection. The governing equations Eqs. (1)–(3) are solved by a finite difference method based on the MAC method with the FTCS scheme using a staggered mesh system.

2.2 Governing Parameter

Non-dimensional governing parameters are as follows;

$$\begin{aligned} Ra_\eta &= \frac{\eta^* \beta^* (T_h^* - T_c^*) H^{*3}}{\nu^* \alpha^*}, \quad pr = \frac{\nu^*}{\alpha^*}, \quad \omega = \frac{\omega^* H^*}{\alpha^*} \text{ and} \\ Ra &= \frac{g^* \beta^* (T_h^* - T_c^*) H^{*3}}{\nu^* \alpha^*}, \end{aligned} \quad (4)$$

where β^* , ν^* and η^* denote thermal expansion coefficient, kinetic viscosity and acceleration amplitude, respectively. In the present study, Pr is fixed to 7.1 (water). The test ranges of Ra_η and ω are from 5.0×10^3 to 1.1×10^4 and from 1.0×10^0 to 1.0×10^3 , respectively.

As global indicators, we consider such physical quantities as a spatially-averaged kinetic energy \bar{K} . These are defined as follows.

$$\bar{K} = \frac{1}{V} \iiint \left\{ \frac{1}{2} (u^2 + v^2 + w^2) \right\} dx dy dz \quad (5)$$

3 Results and Discussion

3.1 Flow Structure $S\alpha$

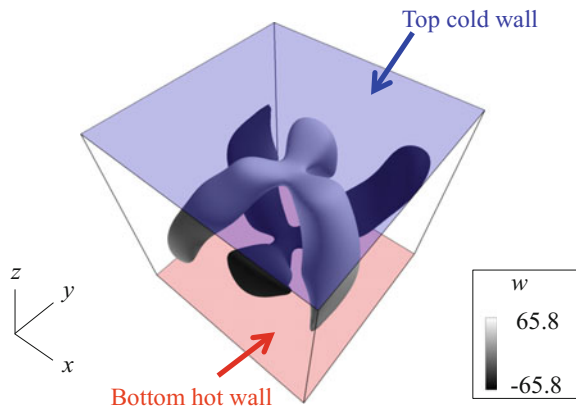
Figure 2 shows a result of the present computations for $Ra = 8.0 \times 10^4$ and $Pr = 7.1$, being based on many preliminary computations over wide ranges of system parameters. The figure is a perspective view looking downward. Three-dimensional flow structure is visualised by iso-kinetic-energy surfaces of $0.38 \times K_{\max}$, where K_{\max} is the maximum value of kinetic energy K .

Brightness of the surfaces represents the value of w which is shown as a legend on the lower right of the figure. The flow visualisation based on K is more suitable for the concerning laminar problem to grasp the whole three-dimensional structure, than other physical quantities such as velocity vector \mathbf{u} , temperature T , vorticity vector and the second invariant of the velocity-gradient tensor.

From Fig. 2, we can see a pair of trident currents on thermal convection in a cube, which consist of three ascending streams and three matching descending streams. we name this flow structure $S\alpha$. Until now, this trident convection in a cube have not been reported, mainly because of the dependency of initial condition. According to past studies [5, 6], we usually expect another flow structure with two ascending and two descending streams which is referred to as $S5$ for $Ra = 8.0 \times 10^4$ and $Pr = 7.1$. Actually, the author observes the $S5$, if we start our computation with such an initial condition as the conductive state or if we compute with very-slowly increasing/decreasing Ra from zero/infinity.

However, this initial-condition dependency does not necessarily mean weak stability of the trident convection. As an example for this, we can consider the time development of \bar{K} for $Ra = 8.0 \times 10^4$ and $Pr = 7.1$ in a stationary cube under the gravity. we regard \bar{K} as a global indicator representing the system's state. \bar{K} is defined by such a physical quantity as a spatially-averaged value of K over the whole volume of a cube. Then, as initial condition, we use the steady solution of $S2$ with a relatively-small $\bar{K} = 20.2$ for $Ra = 1.0 \times 10^4$ and $Pr = 7.1$, instead of the

Fig. 2 Perspective view (look-down view) of the flow structure visualised by iso-kinetic-energy surfaces of $0.38 \times K_{\max}$ for $Ra = 8.0 \times 10^4$ and $Pr = 7.1$. The value of w is represented by gray scale



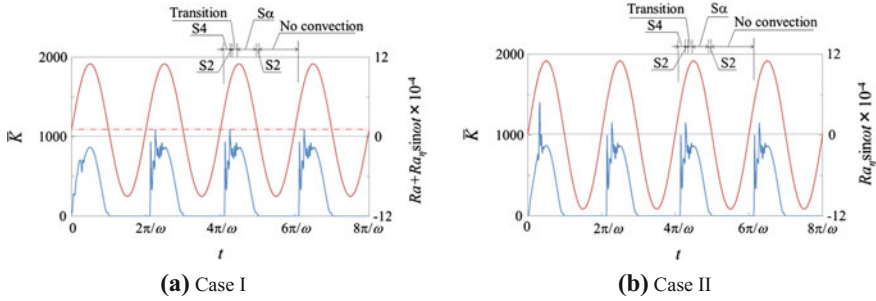


Fig. 3 Time development of \bar{K} , together with $Ra + Ra_\eta \sin \omega t$ in figure (a) and $Ra_\eta \sin \omega t$ in figure (b). In each figure, — denotes \bar{K} , and — denotes $Ra + Ra_\eta \sin \omega t$ or $Ra_\eta \sin \omega t$ - - - in figure (a) is at $Ra = 1.0 \times 10^4$ and the S5. In both the samples, the duration times of the S9 are effectively long with enough large values of \bar{K}

S5 or instead of the conductive state. Such time development of \bar{K} show that the trident convection steadily and permanently appears at $t > 0.06$, after the initial transient process where another flow structure of one ascending and one descending streams (the S2) is dominant.

As another example to show the durability of the trident convection, we consider the thermal convection in an oscillating cube instead of a stationary cube under the gravity or zero gravity. More specifically, the forced sinusoidal oscillation is parallel to the given temperature gradient direction. Figure 3a is a sample of the forcedly-oscillating cube under the gravity. Figure 3b is another sample of the forcedly-oscillating cube under zero gravity. Both the samples are rigorously periodic except for the first start-up period of computation at $t < 2\pi/\omega$, while we see high-frequency fluctuations due to higher harmonics. Then, we consider only the third period at $t = 4\pi/\omega - 6\pi/\omega$ in each figure. There exist the time-intervals where \bar{K} becomes completely zero (or in the conductive state) each forcing cycle. Convective motion always starts with the S4 in each forcing cycle. We can confirm the appearance of the trident convection S9 at $t = (4\pi + 1.01)/\omega - (4\pi + 2.07)/\omega$ in Fig. 3a and at $t = (4\pi + 1.49)/\omega - (4\pi + 2.95)/\omega$ in Fig. 3b, together with other flow structures like the S2, the S4.

4 Conclusions

To summarise, we have reported a pair of trident currents on thermal convection in a cube. This previously-unidentified flow is rather robust, and can be observed not only in a stationary cube under the gravity, but also in an oscillating cube under the gravity and zero gravity. This finding implies a diversity of three-dimensional Rayleigh-Bénard convection, even if we suppose laminar and steady flow with very simple boundary condition.

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