Interpretations of Relationships Among Knowledge Assessment Tests Outcomes

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Abstract. Recently developed knowledge assessment tests provide students and educators with information about degrees to which certain skills are mastered, terms and concepts are understood, and to which extend abilities to solve predefined problems efficiently and in a timely fashion without difficulty are demonstrated. In case of partially correct or incorrect answers, students are suggested appropriate theory, examples, and possibilities to take new tests addressing those specific issues. Provided help is primarily related to a problem a student has not solved, a question not being answered correctly, or an answer has been omitted. In this work, we attempt to unveil hidden correlations among correct and wrong answers from students in a test.

Keywords: Tests · Knowledge · Dependencies

1 Introduction

Digitalization of knowledge assessment is an ongoing process. Earlier works were dedicated to electronic delivering av tests for both formal and self-assessment purposes as well as offering automated and to some extend personalized help in case of failure [3,7,16].

After being taken, most tests provide students and educators with information about degrees to which certain skills are learnt, terms and concepts are understood, and abilities to solve predefined problems are exhibited. In case of partially correct or incorrect answers, they are suggested appropriate theory, examples, and eventually are offered to take a new test with similar questions. Such help is primarily related to the problems a student has not solved or respectively to the questions she has not answered correctly or simply has omitted to answer. This work is an attempt to use a somewhat different approach to finding explanations for students inabilities to provide correct responses to test questions.

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2 Related Work

The n-dimensional cube Q_n is the graph for which $V(Q_n)$ is the set of all 0–1 sequences of length n, with two vertices being adjacent if and only if they differ in exactly one position [1]. The authors investigate the problem of factoring Q_n as $Q_n = H \times K_2$. According to [6,14], Q_n can be presented as a direct product $Q_n = H \times K_2$ where H is a hypercube of dimension $n-1$ and K_2 is a complete graph. A bipartite graph is a graph whose vertices can be divided into two disjoint sets where every edge connects a vertex in one set to a vertex in the other set [11]. Bipartite graphs can be factored as a product of two graphs one of which is a hypercube and the other is a complete graph [1]. For further reading on bipartite graphs, we refer to f. ex. [5,12,13]. Bipartite graphs have special value for running of iterative enhancement processes [4]. They have been employed in algorithm design [15,18] and used in social network analysis [22]. Bipartite graphs were previously employed for solving assignment and transportation problems [2,20].

Let P be a non-empty ordered set. If $sup\{x, y\}$ and $inf\{x, y\}$ exist for all $x, y \in P$, then P is called a *lattice* [10]. In a lattice illustrating partial ordering of knowledge values, the logical conjunction is identified with the meet operation and the logical disjunction with the join operation.

Association rules are used to analyze data for frequent if/then patterns and to identify the most important relationships [19].

Knowledge assessment has been of interest to many authors [8,17,21]. Learning preferences describe person's predispositions in receiving, processing, and delivering information. Learning styles are considered to be one of or a mixture of sensory, intuitive, visual, verbal, active, reflective, sequential, and global type [9].

3 Interpretation of Tests Outcomes

One of the foremost tasks of knowledge assessment is discovering what is causing students inability to provide correct answers to test questions. Thus, if all or the majority of wrong answers indicate misunderstanding, misconception, and misinterpretation of terms, then a revision of corresponding learning materials seems to be an appropriate action. When wrong answers are submitted in a following test on the same topic, it can be an idea to consider students learning preferences and provide them with assistance in an appropriate form. Amount of wrong answers might also be reduced by reformulation of answer alternatives, since if the latter are unclear or ambiguous, this can mislead students.

We are now looking at a test with four questions where the correct answers are denoted with 'e, f, g, l' and wrong answers are denoted with 'p, q, r, s'. The latter can be due to misconception, inability to apply a combination of skills, insufficient factual or explicit knowledge, or both, etc. Note that all concerns about protected exam environment, permutation of questions, number of correct answers following a question, partially correct answers, and so on are not in the scope of this work, and students have not been directly involved in this study.

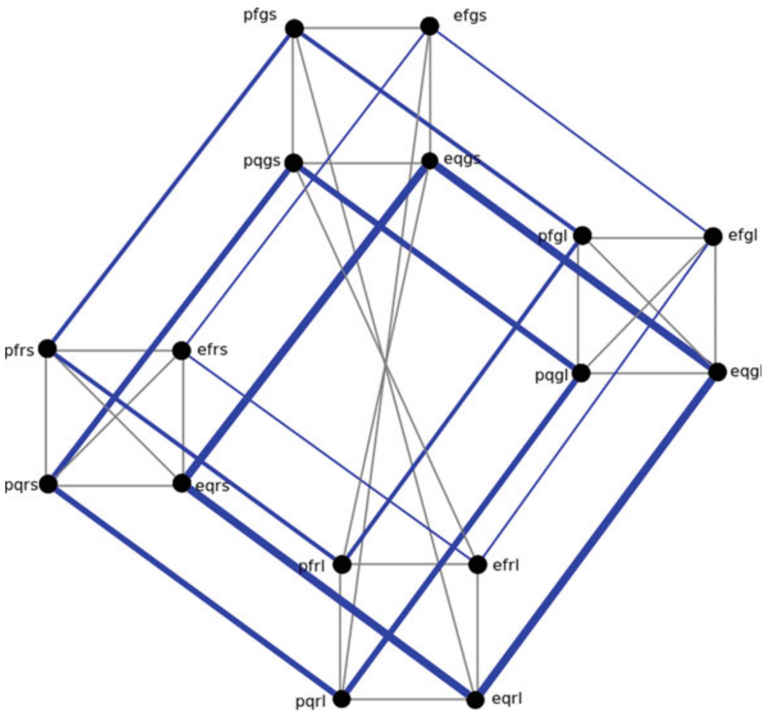

Fig. 1. Answers concerning the first and second questions

Students' responses to test questions are ordered in sixteen quadruples where the first position is reserved for an answer to the first question, the second position is reserved for an answer to the second question, and so on. This way the first position is occupied by either e or p, the second position is occupied by either f or q, the third position is occupied by either g or r, the forth position is occupied by either l or s, and any two quadruples differ in at least one position. This particular property allows use of n-dimensional cubes. A quadruple is assigned to a vertex of the graph shown in Fig. 2 originally presented in [1] as graph H.

A number of possibilities can be considered while searching for dependencies among students answers to questions in such a test.

Any of the four squares in Fig. 1 (edges are highlighted in blue with different thickness to simplify visual recognition) relates responses that coincide in the first two positions in a quadruple:

- $\{p, f, g, s\}$, $\{p, f, g, l\}$, $\{p, f, r, l\}$, $\{p, f, r, s\}$ the answers to the first question are wrong and to the second question are correct
- $\{e, f, g, s\}$, $\{e, f, g, l\}$, $\{e, f, r, l\}$, $\{e, f, r, s\}$ the answers to the first and second questions are both correct

- $\{p, q, g, s\}$, $\{p, q, g, l\}$, $\{p, q, r, l\}$, $\{p, q, r, s\}$ the answers to the first and second questions are both wrong
- $\{e, q, g, s\}$, $\{e, q, g, l\}$, $\{e, q, r, l\}$, $\{e, q, r, s\}$ the answers to the first question are correct and to the second question are wrong

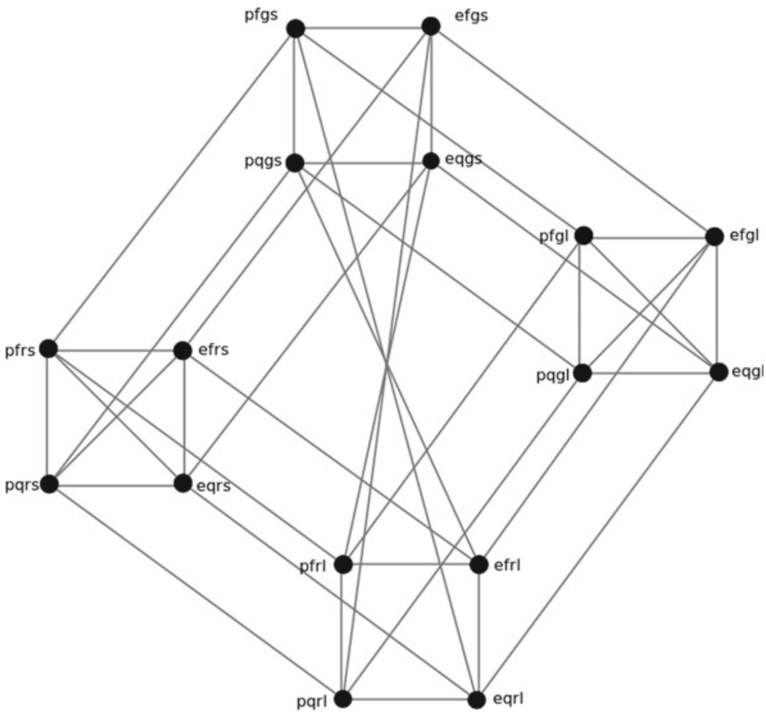

Fig. 2. Answers concerning the third and fourth questions

Any of the four squares in Fig. 2 (vertices indicated with black circles) relates responses that coincide in the last two positions in a quadruple:

- $\{p, f, g, s\}$, $\{e, f, g, s\}$, $\{e, q, g, s\}$, $\{p, q, g, s\}$ the answer to the third question is correct and to the fourth is wrong
- $\{p, f, g, l\}$, $\{e, f, g, l\}$, $\{e, q, g, l\}$, $\{p, q, g, l\}$ the answers to the third and fourth questions are both correct
- $\{p, f, r, l\}$, $\{e, f, r, l\}$, $\{e, q, r, l\}$, $\{p, q, r, l\}$ the answer to the third question is wrong and to the fourth is correct
- $\{p, f, r, s\}$, $\{e, f, r, s\}$, $\{e, q, r, s\}$, $\{p, q, r, s\}$ the answers to the third and fourth questions are both wrong.

Similar parallelograms relate responses to any other couples of questions, e.g., $\{p, f, g, s\}$, $\{p, q, g, s\}$, $\{p, q, r, s\}$, $\{p, f, r, s\}$ the answers to the first and fourth questions are both wrong, Fig. 3.

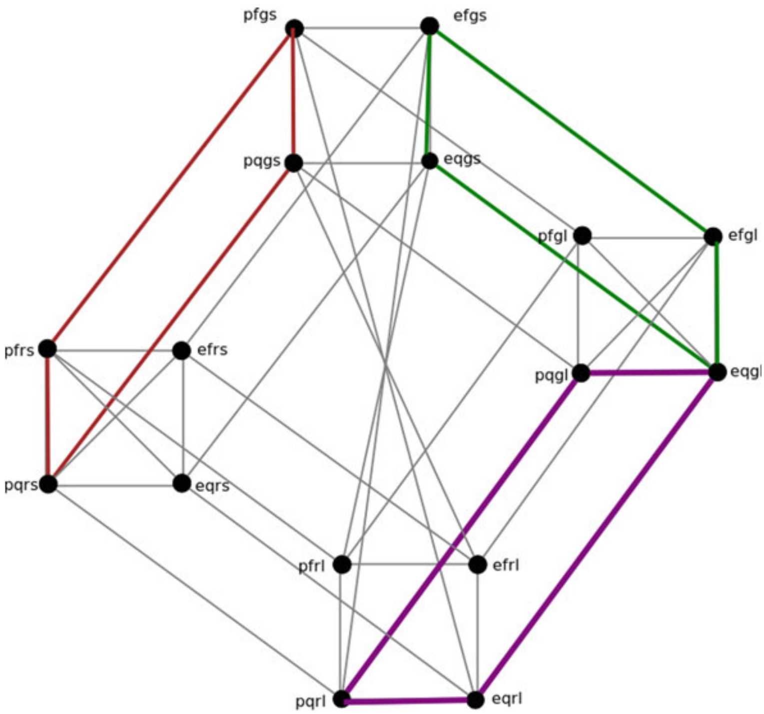

Fig. 3. Answers concerning the first and second questions

Suppose we are looking for possible explanations to why a particular question is answered wrong, let us take the first one. To manage that, we can find all cases with the highest similarity level of responses, i.e., all quadruples that differ in one position only and contain the same response to the first question, (highlited in green) in Fig. 4. In order to manage that, we should consider the three vertices connected by a single edge to the vertex we have in mind: For example, $\{p, f, g, s\}$ is connected to $\{p, f, r, s\}$, $\{p, q, g, s\}$, and $\{p, f, g, l\}$. Focusing on the groups of students placed in these four vertices may provide useful information about which types of difficulties they experience and what can be done to help them overcome these difficulties. Similar inquiries can be executed beginning at any vertex in Fig. 4.

Another gripping question is related to responses placed in quadruples that differ in exactly two positions. This will be illustrated by the following example. Let us begin with quadruple $\{e, f, r, s\}$. The quadruples that differ in exactly two positions from $\{e, f, r, s\}$ can be divided into two sets:

(1) $\{e, f, r, s\}$, $\{e, q, r, l\}$, $\{p, f, r, l\}$
(2) $\{e, f, r, s\}$, $\{e, q, g, s\}$, $\{p, f, g, s\}$.

Note that there is one more quadruple $\{e, f, g, l\}$ that differs in exactly two positions from $\{e, f, r, s\}$. This is a trivial case where all provided answers are correct and no helpful conclusions can be made.

In set (1), all students provided wrong answer to the third question and answered correctly to two of the remaining questions. In set (2), all students provided wrong answer to the fourth question and answered correctly to two of the remaining questions. Such ways of looking at students responses might throw different light on where the learning problems are. It is important to follow changes in the number of students placed in different vertices since this can give indications about what can be done to improve students knowledge and understanding of new concepts.

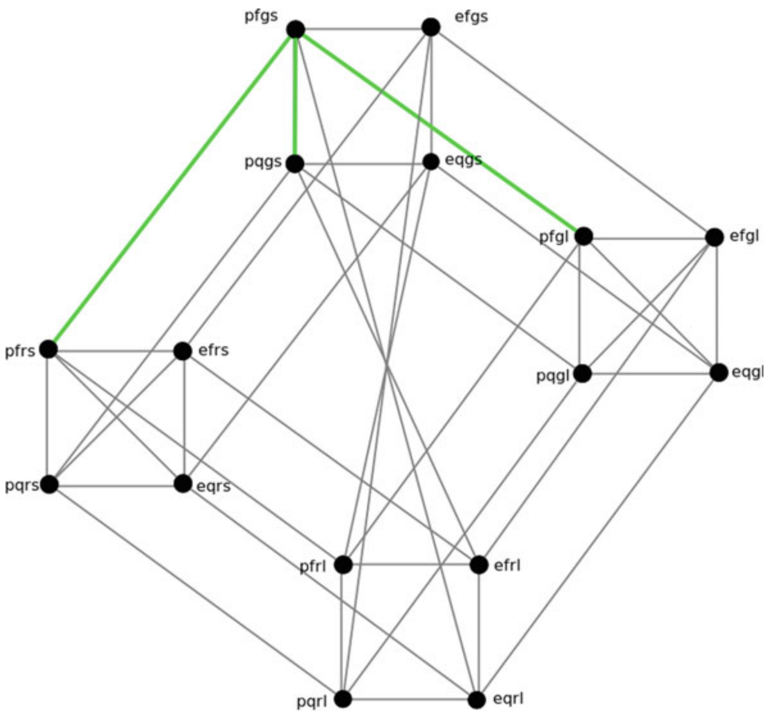

Fig. 4. Answers concerning the first and second questions

Remark: In terms of graph theory, corresponding vertices will be at distance two. Calculations modulo 2 will simplify digitalization of this approach; see Fig. 5. Correct answers are denoted by '1' and wrong answers are denoted by '0'. In these notations, the two above discussed sets are

(1) {1100}, {1001}, {0101}
(2) {1100}, {1010}, {0110}.

The paths from quadruple {1100} to any of the remaining four quadruples are highlighted in different colors, in addition to one edge with double line illustrating two paths. The n-dimensional cube Q_n structure can be used to reflect on tests outcomes with larger number of questions while any number of students can be placed in a vertex. Responses to any four of a test's questions can also be placed in quadruples and thus take advantage of the above discussed reasoning possibilities. Conclusions can be further on used to make predictions of students performance and provide automated personalized help when there is a need for it.

Students background should also have influence on the process of forming patterns. This includes level of previous education, number of years since it has been completed, relevant work experience, results from other exams and or tests, in order to provide individual help. It is important to know what they have been working with when they pass or fail and how this is connected to the other answers in a test.

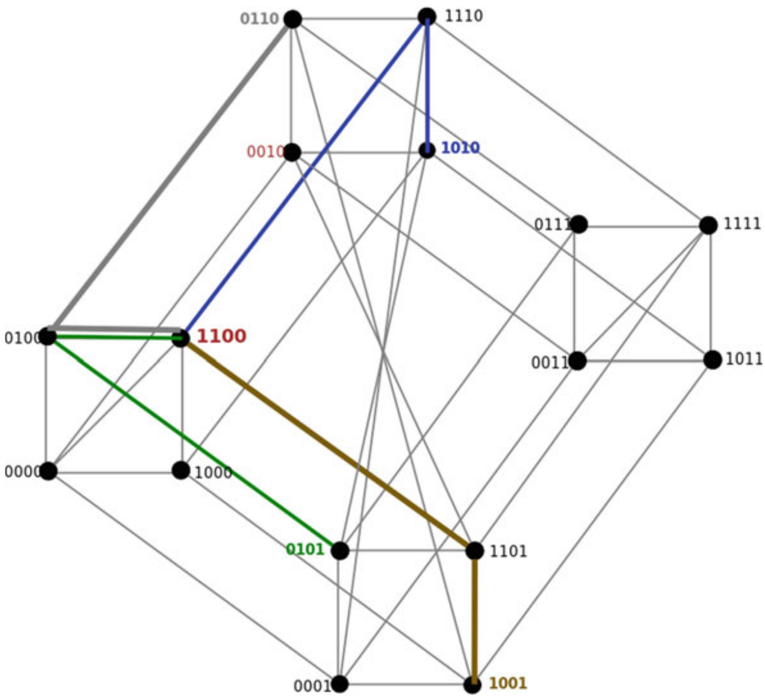

Fig. 5. Examples of quadruples at distance two

Formal concept analysis can be employed to discover more dependencies among answers and possible misconceptions of students placed in a single vertex, in a set of vertices that differ in exactly one position, and in a set of vertices that differ in exactly two positions. This way educators can consider reasonings based on smaller amount of more specific attributes as well as reasonings based on a larger amount of attributes that can point to connections that are otherwise difficult to notice. Association rules applied to the same dataset may indicate important tendencies.

4 Conclusion

Evaluation of students knowledge is an ongoing research that requires further work. Our belief is that involvement of well-established mathematical theories and structures contributes for employing a systematic approach that can be used in different areas. Visualization of tests outcomes may improve understanding of main reasons for insufficient learning since both students and educators can benefit from discovering new patterns in students responses.

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