2D Surface Water Quality Model: A Forecasting Tool for Accidental Pollution in Urban River—Application to the Var River, France

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1 Introduction

Nowadays, with the growth of population and industrialization, the environment is increasingly exposed to several dangers. This led policy makers and authorities to introduce limitations of water extraction and exploitation [1]. Indeed, water users have to refer to the amount of water that the river can supply, either quantitatively and qualitatively, taking into account the environmental protection. Obviously, maintaining an acceptable quality in the river can assure the correct resource exploitation. This is why authorities should provide suitable monitoring systems.

A typical incorrect use of water resources is the uncontrolled discharge of sewage into rivers and streams. Especially, the main problematic raised by this

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phenomenon is how to get information about the pollution presence, and what are its variation. Of course, the water quality in a river depends on the quantity of water in which the pollutants are contained. So, water flow, level, and velocity are the variables which can determine the pollutant behavior. This is why 2D surface water quality models are proper tools to represent the behaviors of pollutants in water environment. The methodology is to introduce the pollutant concentration in the hydrodynamic model as a function of times and space, and it follows the approach of the advection–dispersion transport [2–4].

The progress of computer science and mathematical procedures has introduced tools able to help scientists and professionals to ensure water quality protection. Obviously, these tools are useful to have control of water quality in the river, lake, and the aquifer. According to Cox [5]: "*a water quality model can mean anything from a single empirical relationship through a set of mass balance equations, to a complex software piece*". Three types of water quality models can be identified:

- the physical model consists of a reproduction of reality at different scales;
- the analogical models are based on a formal identity of the mathematical expressions that interpret different phenomena;
- the mathematical models interpret the reality by means of the numerical values that can be adopted to quantify the various phenomena and their components.

A review of the surface water quality models has been made with the objectives to explain the development of these models at three stages and analyze the suitability, the precisions, and the methods among the different models [6]. Since 1925, the water quality models have been improved. From 1925 to 1965, water quality models focused on the interactions among different components in river systems affected by living and industrial point source pollution like transmission, sediment oxygen, and algal photosynthesis. From 1965 to 1995, water quality models have been improved with multidimensions: 2D and 3D models. Furthermore, nonlinear relationship was used. For example, QUAL [7, 8], MIKE 11 [9], and WASP [10, 11] were different water quality models developed during this period. After 1995, nutrients and toxic chemical materials have been included in the model framework because of increasing of organic compounds, heavy metals, and nitrogen compounds. Cao and Zhang [12] classified the water quality models depending on the water body, the model-establishing methods, the water quality coefficients and components, the properties, the spatial dimensions, and the reaction kinetics. Today, software like Mike 21, Mike 21 FM, or Telemac2D are useful tools to prevent the pollution transport. And, they are used in different ways: hydrodynamic–ecological way [13], trajectory, and residence time of biochemical pollutant [14], oil spills [15] etc. However, these software tools are controlled by different methods and present probably different results.

In France, 23 oil spill accidents occurred from 1940 to 2014 with five accidents happened in rivers or lakes (source: Center of Documentation, Research and Experimentation on Accidental Water Pollution (CEDRE)). In average, the spilled volume varies from 5 to 20 m^3. This is the main potential source of pollution for the

Var river in its lower part. Indeed, with the urbanization and the different bridges which cross the river, the local municipalities would like to prevent a potential oil spill event in the river. On the other hand, several industrial areas can be a potential source point of pollution (biochemical, chemical, heavy metals). Hence, the lower Var river valley and its aquifer, which is one of the main drinking water resources, are vulnerable to an accidental pollution and local municipalities would like to provide a monitoring system to prevent this. This leads to use 2D modeling tools like Mike 21, Mike 21 FM and Telemac2D, and to compare the results. In this paper, we compare three software tools of 2D surface water quality process modeling. First of all, the methodology is described and scenarios are presented. Then, the models have been set up in order to simulate pollutant transport. To conclude, the results are compared with each other to understand the differences between the modeling approaches.

2 Materials and Methods

To achieve the comparison between pollutant transport models using 2D-SWEs-based numerical codes, two scenarios of accidental pollution in the Var river are simulated with Mike 21, Mike 21 FM, and Telemac2D. As a first step, the hydrodynamic model is set up and calibrated in Mike 21. As a second step, the Mike 21FM and Telemac2D models are based on the same numerical parameters as the calibrated model. As a last step, the advection–dispersion model is used to simulate the two accidental pollutions in each software. The results are compared by propagation time, concentration of pollutant, and polluted surface.

2.1 Study Area

The Lower Var valley river is located in the southeast of France. This 22 km long section connects the mountainous area to the Mediterranean Sea [16]. It drains water of a catchment of 2893 km^2. The yearly average discharge is 50 m^3/s, while the highest measured instantaneous discharge during flood peak can reach 3750 m^3/s (the 1994 flood event, [17]). At this location, several pumping stations provide water for 600,000 people (cf. Fig. 1). In addition, this groundwater is also a source of water for agricultural and industrial activities. The annual estimated pumping volume is around 50 million of m^3. Therefore, groundwater pollution is a main issue in the Lower Var river valley because the unconfined aquifer is one of the main freshwater resources.

In this context, the alluvial aquifer faces the threat of groundwater pollution. This freshwater source is extremely vulnerable due to its strong exchanges with the river in terms of quantity and quality. Not only can the pollution comes from direct industrial contamination [18] but also from an accidental contamination in the river.

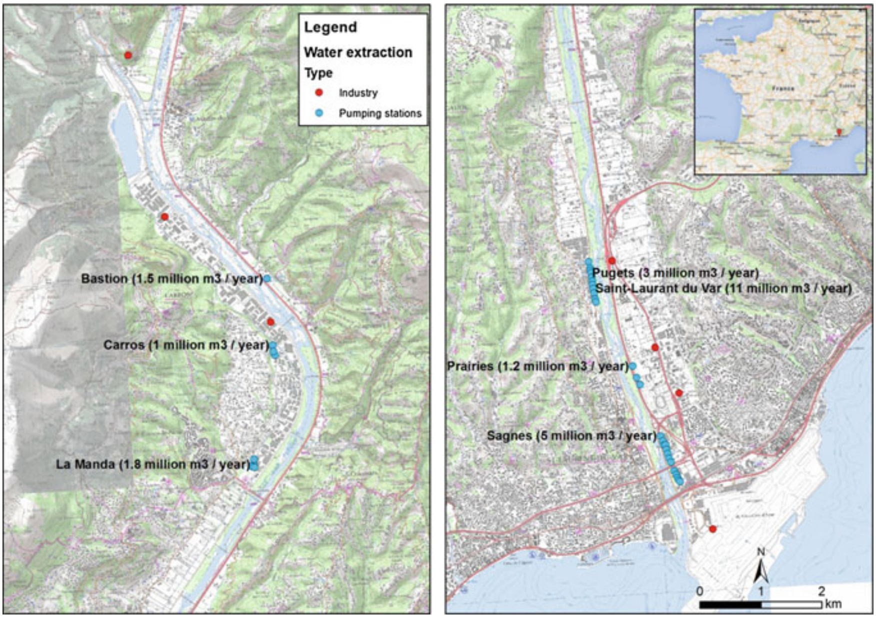

Fig. 1 Locations of the water extractions in the lower Var valley (*Source* Nice municipality): on the left side appears the north part of the study area and on the right side appears its south part

Therefore, the local municipalities have to anticipate the accidental pollution in the river which is connected with the aquifer.

In the lower Var valley river, two types of accidental pollution can occur: industrial rejection or truck accident on a bridge. In order to predict the transfer of chemical pollutant, the 2D hydraulic model is an appropriate tool. Actually, numerical models are able to represent hydrodynamics of the river and the pollutant effects on the surface water.

2.2 Scenarios of Pollutant Transport

Several industrial areas are located in the lower Var valley (cf. Fig. 2). In addition, four bridges cross the river from the Broc lake to the airport. Each of these zones is a potential pollutant source of the Var river and thus can contaminate the pumping stations in the downstream part.

Regarding the industrial activities, different types of pollutions have been identified like the chemical oxygen demand, hydrocarbon, heavy metals, etc.

Moreover, an oil spill accident from a truck accident can occur on one of the four bridges which cross the Var river. Finally, among these several potential pollutants in the lower Var valley, two types of pollutant transfers have been simulated: a

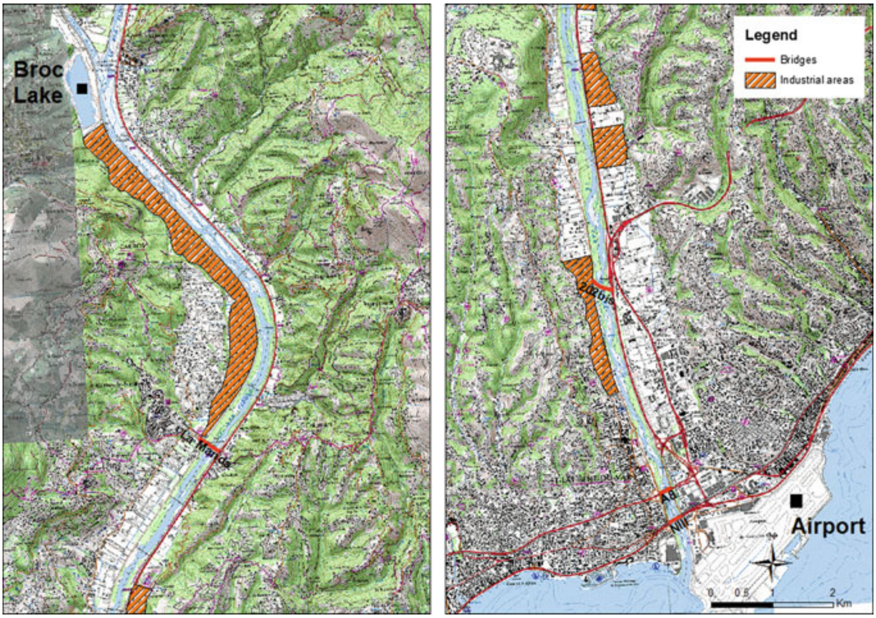

Fig. 2 Locations of industrial areas and bridges which are a potential accidental source of pollution

nonconservative pollutant (COD) and a conservative pollutant (Cadmium (Cd)). A nonconservative pollutant simulation takes into account the degradation process of the chemical entity. In other words, a decaying equation is used to describe the degradation process for the pollutant in the water. In this paper, two types of accidental pollution have been simulated using three different tools (Mike 21, Mike 21 FM, and Telemac2D). The first scenario is the COD transfer in the river from a pharmaceutical factory in the most northern industrial area. The second scenario is the cadmium transfer in the river from a painting factory. To do that, two scenarios were simulated with Mike 21, Mike 21 FM, and Telemac2D. Then, the comparison has been done at the closest pumping stations from the point source.

2.3 2D Modeling Tools

2.3.1 Hydrodynamic Module

On the first hand, the 2D model provides hydrodynamic aspect in a water body with an average in the vertical direction. On the other hand, 3D model is similar to 2D model from basic principles but allows to describe the flow in a water column that can be useful for some types of pollutant. However, it is costly and time-consuming. In the Var river context, 2D models have been applied: Mike 21, Mike 21 FM, and Telemac2D.

Simulation of the hydrodynamics part has been carried out solving the 2D shallow water equations of mass and momentum (2D-SWEs). As known, the flow can be modeled by Navier–Stokes equations and their most used simplifications are the 2D-SWEs, which are valid under the following conditions [19, 20]:

- water is incompressible and homogeneous;
- velocity components in the vertical direction are negligible;
- pressure distribution is hydrostatic in the vertical direction;
- bottom slope is small;
- friction terms (viscosity, bottom, and free surface friction) can be simulated by empirical expressions of steady flow.

2D shallow water equations, also called Saint-Venant equations [21], are described by the following mass and momentum conservations:

$$\begin{cases} \partial_t h + \partial_x(hu) + \partial_y(hv) = P - I \\ \partial_t(hu) + \partial_y(huv) + \partial_x\left(hu^2 + g\frac{h^2}{2}\right) = gh\left(S_{0x} - S_{fx}\right) \\ \partial_t(hv) + \partial_x(huv) + \partial_y\left(hv^2 + g\frac{h^2}{2}\right) = gh\left(S_{0y} - S_{fy}\right) \end{cases}$$
$$\text{with } S_{0x} = -\partial_x z(x,y) \text{ and } S_{0y} = -\partial_y z(x,y)$$

with,

$h(t,x,y)$	water depth [m],
$(u,v)(t,x,y)$	velocities, respectively, in x- and y-direction [m/s],
g	gravitational constant [m/s^2],
$P(t,x,y)$	rainfall intensity [m/s],
$I(t,x,y)$	infiltration rate [m/s],
$\vec{S}_f = \left(S_{fx}, S_{fy}\right)$	roughness term (depending on the roughness law),
$z(x,y)$	topography [m].

To solve these equations, different methods are applied in Mike 21, Mike 21 FM, and Telemac2D. Bear in mind that spatial discretization is specific to each software. **Mike 21** solves the 2D-SWEs with the Alternating Direction Implicit method (ADI), which is a finite difference method [22]. The domain is designed as a structured mesh. Then, the solution is performed with a finite volume method in **Mike 21 FM** [23]. The user can choose the time integration between two options: lower order method which is the first order explicit (Roe scheme) or higher order method which is the second order, Runge–Kutta method. In both cases, the convection flux is computed with a Roe scheme [24] and a TVD slope method is used to minimize oscillations [25] and to perform a second-order accuracy in space. The geometry can be designed with triangular or quadrangular cells. Finally,

Telemac2D proposes several numerical methods, but here the finite element technique was used [26]. Spatial integrations are controlled by the SUPG (Streamline Upwind Petrov/Galerkin) method. Unstructured mesh is used in Telemac2D.

2.3.2 Advection–Dispersion Module

In order to describe the pollutant behavior, physical and chemical processes are taken into account in 2D models. Two types of pollutants have been used in this study: conservative and nonconservative. First of all, the pollutant is described by its concentration in terms of mass per unit volume. After that, two types of transporting are considered to represent the pollutant transport. The first one is the advection transport [2]: it is the first mechanism, which transports the pollutant in the water body. It depends on water velocity; this is why hydrodynamic part is important. The second one is the dispersion transport, determined with Fickian law: it is the evolution of the concentration even without any motion. To do this, pollutant flux and the concentration gradient are introduced in the equations. In the three models, the advection–dispersion equation is used to perform the pollutant simulation:

$$\partial_t(hC) + \nabla \cdot (\mathrm{h}C\vec{u}) = \nabla \cdot \left(K_c \overrightarrow{\nabla C}\right) + C_{sce}S_{ce}$$

where,

$C(x, y, t)$	the concentration [kg/m^3],
$\vec{u}$	the velocity field [m/s],
K_c	the diffusion coefficient [m^2/s],
C_{sce}	the initial concentration [kg/m^3],
S_{ce}	the flux of the pollutant [m/s].

Mike 21 solves the 2D tracer equation by using an explicit scheme named QUICKEST [27]. Then, the solution is performed thanks to a finite volume method in **Mike 21 FM** [28]. The user is able to choose the time integration like the hydrodynamic module in order to get more accurate results. In both cases, TVD-MUSCL limiter method is used to minimize oscillations [25, 29]. Finally, **Telemac2D** is based on the method of characteristics and the upwind SUPG scheme to solve the tracer equation [26].

On the other hand, the components which decay linearly in time can be defined in each model. Many processes can be approximated by linear decay, which is generally described by:

$$\frac{\partial c}{\partial t} = -kc$$

where, c is the specific concentration and k is the decay constant. In DHI software tools the decay coefficient is specified, per second, by the user. However, in Telemac2D this coefficient is defined as:

$$k = \frac{2.3}{T_{90}}$$

where, T_{90} is the time to consume 90% of the initial mass. This parameter is defined by the user.

3 2D Surface Water Quality Model

3.1 HD Parameters

First of all, the hydrodynamic model has been set up to generate the hydrodynamic situation in the lower Var river valley. As mentioned before, three numerical tools have been used to simulate the chosen scenarios chosen: Mike 21, Mike 21 FM, and Telemac2D. The first difference appears in the discretization used to describe the domain in each software (cf. Fig. 3). The same resolution has been chosen for each HD model: 20 m resolution for the river bed, and 100 m for the floodplain. To generate the geometry, the same DEM has been used to interpolate the mesh. This DEM has been given by the municipality with a resolution of 1 m and an accuracy of 50 cm and it has been resampled, with ArcGIS tools, with a 20 m resolution.

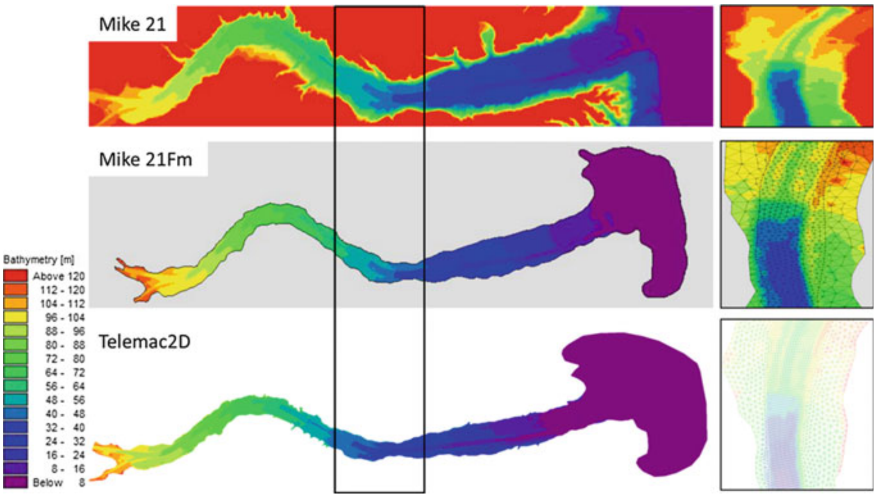

Fig. 3 Mesh description for Mike 21, Mike 21 FM, and Telemac2D

In order to calibrate the HD model, Strickler coefficient has been changed on the river bed. The final roughness coefficient is 45 $m^{1/3}/s$ for the river bed, 40 $m^{1/3}/s$ for trees and brush, 70 $m^{1/3}/s$ for the urban areas, and 40 $m^{1/3}/s$ for agricultural areas. The numerical models have been calibrated thanks to a flood event in November 2014 (cf. Fig. 4).

In order to evaluate the reliability of each model, the Nash–Sutcliffe coefficient has been computed. It allows to evaluate the efficiency of the 2D free surface flow models [30]. The Nash–Sutcliffe coefficient is described as follows:

$$E = 1 - \frac{\sum_{t=1}^{T}\left(H_{\mathrm{Obs}}^{t} - H_{\mathrm{Sim}}^{t}\right)^{2}}{\sum_{t=1}^{T}\left(H_{\mathrm{Obs}}^{t} - \overline{H_{\mathrm{obs}}}\right)^{2}}$$

where,

H_{Obs}^{t} water depth observed at t time (m),
H_{Sim}^{t} water depth simulated at t time (m),
$\overline{H_{\mathrm{obs}}}$ the average observed water depth (m)

The Nash–Sutcliffe coefficient has been calculated for each model (cf. Table 1).

With a coefficient between 0.6 and 1, the model is able to represent the reality. Here, the three models are efficient models for the hydrodynamics in the Lower Var valley.

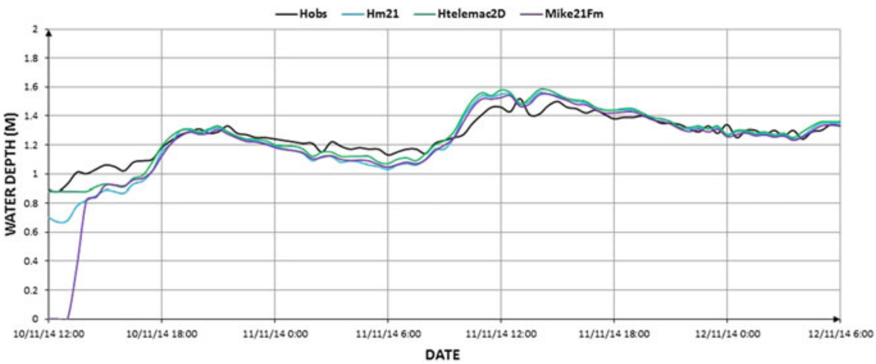

Fig. 4 HD results compared with the observed water depth at Napoléon III bridge close to the airport

Table 1 Nash–Stucliffe coefficient evaluating efficiency of Mike 21, Mike 21FM, and Telemac2D models

Model	Mike 21	Mike 21 FM	Telemac2D
Nash–Sutcliffe	0.75	0.65	0.65

3.2 AD Parameters

HD module is the basis for advection–dispersion (AD) simulation. Once HD models are calibrated, the pollutant transport can be simulated using different parameters. Here, two scenarios have been chosen to represent the accidental pollution in the Var river: a nonconservative pollutant (COD pollution), and a conservative pollutant (Cadmium pollution). To describe the chemical aspect in the water, two parameters have to be defined: the decaying coefficient for the non-conservative pollutant, and the dispersion coefficient for both pollutions.

The decaying coefficient is defined by the experimental study [31], where it is varying between 0.006 and 0.6 day^{-1}. Here, for both scenarios, the decaying coefficient is taken at 0.4 day^{-1}.

The dispersion coefficient is divided into two terms: longitudinal dispersion and transverse dispersion. The second one is defined as ten times bigger than the first one. Therefore, computing longitudinal dispersion is enough to obtain the two dispersion coefficients. The longitudinal coefficient is described by Socolofsky [32]:

$$D_L = 5.93hu^*$$

where, h is the water depth (m) and u^* is the velocity (m/s). First of all, Mike 21 and Mike 21 FM have been compared for the two scenarios. Second, Mike 21 and Telemac2D have been compared for the two scenarios. The following tables describe AD simulation parameters for each software (cf. Table 2).

4 Results and Discussions

4.1 Cadmium Simulation: Conservative Pollutant

Regarding the issue about the water resource in the lower Var valley, the comparison point of pollutant concentration is located at the closest pumping stations of the point source of Cd. The first affected pumping station by a Cd pollution, in the

Table 2 AD parameters used in Mike 21 and Mike 21 FM

Pollutant	COD	Cd
Type	Decaying pollutant	Conservative pollutant
Discharge (m^3/s) in continuity	10	5
Concentration (mg/l)	1000	200
Decaying coefficient (day^{-1})	0.4	No
Transversal dispersion (m^2/s)	1.7	1.7
Longitudinal dispersion (m^2/s)	17	17

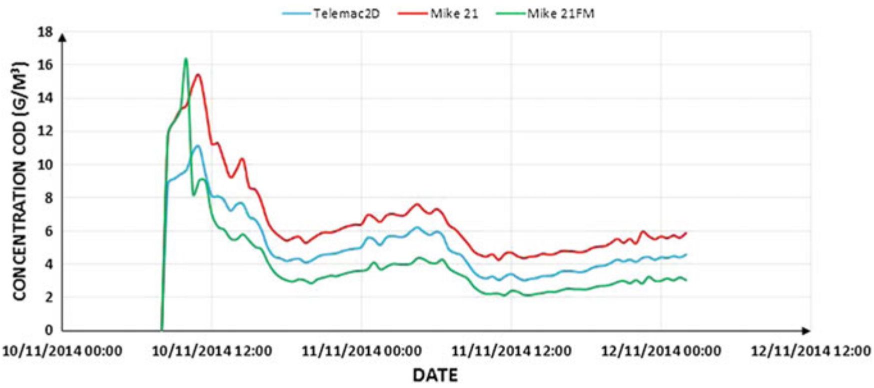

Fig. 5 Concentration of Cd closest to the La Manda station for each software

scenario 2, is the la Manda station. The concentration of the pollutant is compared for each software (cf. Fig. 5).

The trend of each model is similar. Regarding the graphics, Mike21 gives the highest concentration. Here, the mesh discretization and the numerical scheme can influence a punctual concentration. The propagation time of the pollutant is more significant in Telemac2D. This difference can be explained by the treatment of advective transport and maybe a finer mesh probably will improve results (cf. Fig. 6).

4.2 COD Simulation: Nonconservative Pollutant

Regarding the issue about the water resource in the lower Var valley, the comparison point of pollutant concentration is located at the closest pumping stations of the point source of COD. The first affected pumping station by a COD pollution, in the scenario 1, is the Bastion station. The concentration of the pollutant is compared for each software (cf. Fig. 7).

The concentration computed by Telemac2D is far less than the concentrations computed by Mike 21 and Mike 21 FM (more than 110% difference). The trend of AD results is similar to Mike 21 and Mike 21 FM. The relative difference between both is 15%. COD simulation takes into account the degradation of the pollutant. This degradation seems to be treated by a different method in Mike 21/Mike 21 FM and Telemac2D. The transfer time of the pollutant is also different for Telemac2D (cf. Fig. 8).

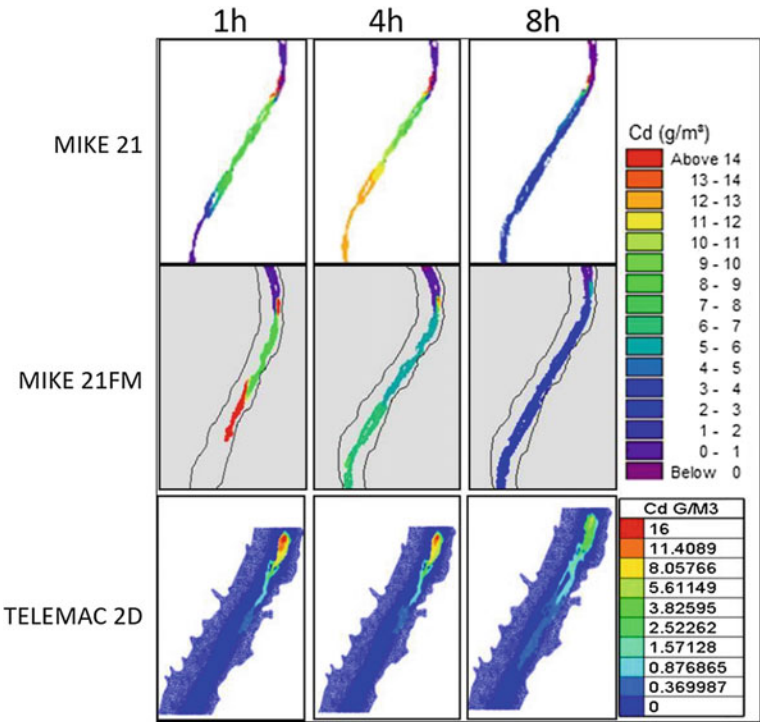

Fig. 6 2D representation of Cd simulation with Mike 21, Mike 21 FM, and Telemac2D (after 1, 4, and 48 h)

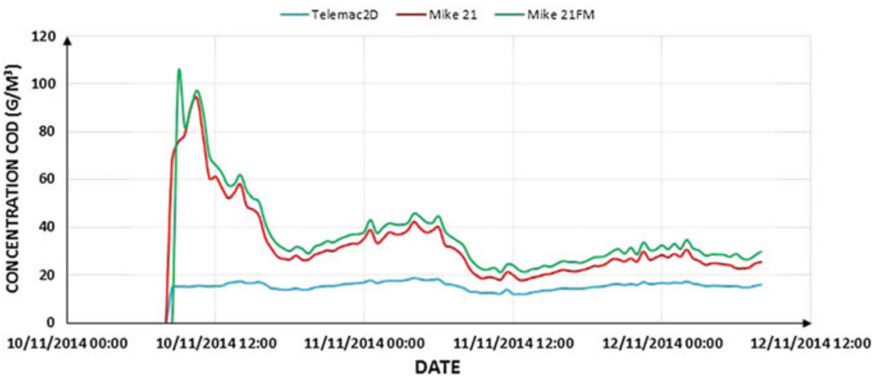

Fig. 7 Concentration of COD closest to the Bastion station for each software

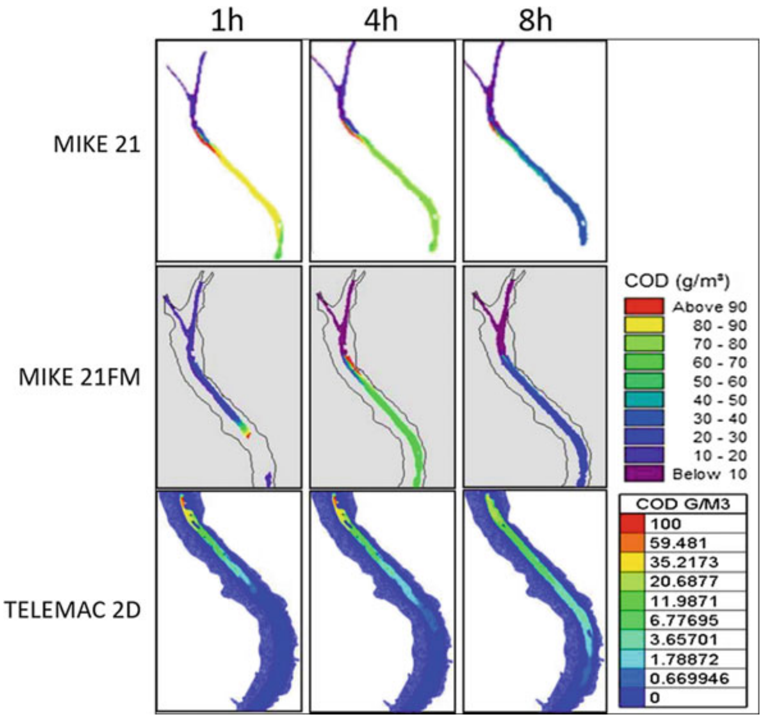

Fig. 8 2D representation of COD simulation with Mike 21, Mike 21 FM, and Telemac2D (after 1, 4, and 48 h)

5 Conclusion

Nowadays, different modeling tools are used to simulate the 2D free surface flow, and each one has its proper advection–dispersion treatment. Here, Mike 21, Mike 21 FM, and Telemac2D solve the SWE with different numerical methods: respectively, finite difference, finite volume, and finite element. Furthermore, the geometry is described by two types of meshes: unstructured and structured cells. By modeling two accidental pollution in the Var river, the different models show different results. Indeed, the simple treatment of advection in Telemac2D affects the concentration in comparison with Mike 21 and Mike 21 FM. Moreover, the propagation time of pollutant is relatively different and could be explained by the grid size. It may be complementary to analyze the grid convergence before these comparisons. This idea is to investigate the spreading of an initial tracer injection in a 1D channel where an analytical solution exists, and find mesh resolution required to come close to the analytical solution. All things considered, Telemac2D appears as a more diffusive model because of its finite element method.

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