Chapter 12
Making Connections to Realize Learning Potential in Early Childhood Mathematics

Aubrey H. Wang and James P. Byrnes

Abstract In this chapter, we discussed how the Opportunity-Propensity (O-P) framework could be used to conceptualize and test the effects that children's (0–8 years) background, mathematics learning opportunities, self-regulation, and prior achievement have on mathematics learning. In prior studies of the O-P framework, we identified and verified the predictive role of antecedent factors such as family socioeconomic status, parent educational expectations for their children, age, birth weight, gender, and ethnicity. With respect to opportunity factors, we identified and confirmed the predictive role of several aspects of instruction. With respect to propensity factors, we have identified and confirmed the role of prior knowledge, motivation, and self-regulation. We encourage our international researchers to build on this work in order to create the most accurate predictive model of early children mathematics achievement. This way, we can collaborate in guiding early mathematics policymakers, practitioners, and professional and advocacy organizations by providing them with a framework on early mathematics achievement to scaffold understanding, generate and test hypotheses, and adapt targeted interventions to address context- and cultural-specific problems.

Keywords Opportunity-Propensity framework · Opportunity factors
Propensity factors · Structural equation modeling · Mathematics learning
Early childhood

A. H. Wang (✉)
Saint Joseph's University, Philadelphia, USA
e-mail: awang@sju.edu

J. P. Byrnes
Temple University, Philadelphia, USA
e-mail: jpbyrnes@temple.edu

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12.1 Background

In the 1990s and early 2000s, retrospective publications began to appear, suggesting that most studies conducted by educational researchers were not particularly scientific nor focused on solving real educational problems in the USA (Kaestle, 1993; Shavelson & Towne, 2002; Walters & Lareau, 2009). Although this claim was disputed by some (e.g., Walters & Lareau, 2009), it ultimately affected the conduct of subsequent research due to two key factors. The first was a report of the National Research Council (NRC) of the USA that focused on what it means for educational research to be scientific (Shavelson & Towne, 2002). Special issues of journals and texts were devoted to reacting to this influential report.

The second source of change was the fact that the Institute of Education Sciences (the primary grant funding agency for educational research in the USA) adopted many of the assumptions of the NRC publication and created a new paradigm for making decisions about the kinds of studies that it would fund. This paradigm assumed that scientific designs in education can be arrayed along a progression ranging from (1) designs that simply reveal phenomena or outcomes that need to be explained (exploratory studies such as surveys) to (2) designs that reveal the correlates or predictors of these outcomes as a means of building theories or explanations of these outcomes (e.g., multivariate longitudinal studies) to (3) finally experimental designs that create interventions that target the causal factors identified by the constructed theories (theory testing designs). The goal is to be always progressing toward creating theories and theory-based interventions in this new view.

12.2 Our Approach to Theory Building

Our approach to theory building relies on several strategies to efficiently identify the predictive factors that could serve as the core constructs of a multivariate theory that can explain achievement disparities between individuals, groups, or schools. The first strategy is to build a comprehensive, multivariate theory out of smaller theoretical accounts that have consistent empirical support but appeal to only one or two factors. With a few notable exceptions, most educational fields, including the field of educational psychology and mathematics education, are unfortunately divided into specialist camps of researchers who primarily focus on one construct (e.g., motivation) and not on the constructs that are important to researchers in other camps (e.g., study strategies).

To fully account for achievement disparities, it would seem that theorists need to begin to incorporate constructs from these multiple camps into a single, more comprehensive account. The second strategy is a derivative consequence of taking a more comprehensive, multivariate approach: When more predictors are included in a model in a longitudinal study, one can see which predictors account for the most variance in the outcome variable (therefore, more predictive) and also see whether some predictors turn out to be spurious because prior less comprehensive studies failed to include the right mix of powerful, authentic predictors in their designs.

The third strategy is to test the comprehensive model using an existing, national database that (a) includes information on a number of potential predictors of achievement outcomes and (b) followed a large number of participants longitudinally. This strategy is obviously more efficient and cost-effective than the more common approach in which individual groups of researchers collect less comprehensive data from single schools. Grant agencies in the USA recognized the utility of funding these large-scale multivariate studies in order to facilitate the discovery and theory-building process. Certainly, this belief is shared by government agencies and early education researchers around the world as they have funded large-scale multivariate studies such as the Longitudinal Study of Australian Children, the Québec Longitudinal Study of Child Development for the same purpose. The fourth strategy is to use advanced quantitative methods such as structural equation modeling to improve our measurement, hypotheses, and relationships between predictors and outcomes.

In the last section of this chapter, we propose a fifth strategy to increase the generalizability of the model by asking our international colleagues to replicate and extend the model using similar longitudinal studies drawn from different national populations of children and their communities. We believe this proposed strategy will generate strong evidence to support the generalizability of our theory across national contexts in the following ways: (1) Understand how well the model fits data collected from different populations of children; (2) examine which core variables remain as consistent predictors and which become spurious; and (3) investigate new variables that can be added to further strengthen the model. Our long-term goal is to create a web that would house the accumulated research on the OP model of achievement (Byrnes, 2003; Byrnes & Miller, 2007; Byrnes & Wasik, 2009; Jones & Byrnes, 2006; Wang, Shen, & Byrnes, 2013); highlight the simplicity and flexibility of the O-P model for use in research, policymaking, and program implementation; and connect the growing number of O-P researchers and users around the world.

12.3 Opportunity-Propensity Model

The present study benefits from the previous theory-building efforts of researchers who have conducted both secondary analyses and prospective smaller studies in order to test and the O-P model of achievement (Byrnes, 2003; Byrnes & Miller, 2007; Byrnes & Wasik, 2009; Jones & Byrnes, 2006; Wang et al., 2013). The multivariate O-P model is an attempt to synthesize existing theoretical models of psychological constructs that have been individually found to be associated with literacy, science, and math achievement. In all studies, academic achievement was measured by standardized assessments. For the studies on early mathematics achievement (e.g., Byrnes & Wasik, 2009; Wang et al., 2013), the items were designed by experts in early childhood development to measure children's conceptual knowledge, procedural knowledge, and problem-solving skills. Test items related to number sense, number properties, operations, geometry and spatial sense, data analysis, statistics, probability, patterns, and algebra, and functions.

The psychological constructs include domain-specific (prior) knowledge (Hailikari et al., 2008; Murphy & Alexander, 2002), motivation (Wigfield & Cambria, 2010), intelligence (Soares et al., 2015), and self-regulation (Schunk & Zimmerman, 2013). But the model also extends beyond these psychological accounts by attempting to integrate this work with scholarship in other fields such as early childhood education, mathematics education, and education policy where researchers have likewise been interested in achievement but focused on constructs not ordinarily investigated by educational psychologists such as formal and informal mathematics, effective teaching methods, and the notions of educational opportunity and income-based educational disparities. Furthermore, we are using advanced quantitative methods such as structural equation modeling for theory building.

12.4 Opportunity and Propensity Conditions

The model building began by asking the core question, "Why do some children attain higher levels of achievement on end-of-year tests than other children?" The answer that was ultimately generated through an intensive, synthesis-oriented examination of the literatures in educational psychology, teacher education, and education policy was that (a) higher scoring children were presented with more high-quality opportunities to learn the content on these tests than lower scoring children and (b) higher scoring children were more willing and able to learn this content when it was presented.

The former requirement was dubbed the "opportunity condition," and the latter was dubbed the "propensity condition" (e.g., Byrnes & Miller, 2007). Both conditions have to be fulfilled in order to obtain high achievement. Typically, educational psychologists tend to focus more on propensity factors than opportunity factors (because they focus on characteristics of learners more than characteristics of high-quality instruction), and mathematics education and education policy researchers tend to focus on opportunity factors more than propensity factors (because they focus on high-quality instruction and learning opportunities more than characteristics of learners).

One merely needs to read standard textbooks in educational psychology, mathematics education, or education policy to see the relative weight given to opportunities and propensities in these distinct fields. The model-building efforts of O-P theorists conducted since 2005 were oriented toward identifying variables that should be included in the two categories of opportunity factors and propensity factors. That is, many claims have been made about instructional and child factors that predict achievement, but it has not all been clear which would continue to predict after all (or most) are included in the same multivariate longitudinal study.

This process of distinguishing between authentic predictors and spurious predictors is an essential first step in building an accurate multivariate theory and eliminating factors from consideration. So far, secondary analyses of the National Assessment of Educational Progress (NAEP), National Educational Longitudinal Study of 1988 (NELS:88), and Early Childhood Longitudinal Study-Kindergarten (ECLS-K) and Early Childhood Longitudinal Study-Birth (ECLS-B) databases have shown that the propensity factors that have remained significant predictors after multiple controls include prior domain-specific knowledge, domain-specific interest/competence, and self-regulation.

The opportunity factors that have remained significant after controls include content exposure (indexed by courses taken and teacher reports of content coverage) and teacher-reported style of presentation (traditional, reform, and balanced approaches in literacy, science, and math). As we report later, there are also factors in a third category called antecedent factors, but their role can best be understood after describing propensity factors and opportunity factors in a little more detail.

12.5 Propensity Factors

As noted above, disparate groups of researchers in educational and developmental psychology examined factors such as preexisting knowledge, motivation, and self-regulation in relative isolation from each other. OP theorists have endeavored to integrate these separate strands of research into a single model. The first step in doing so was to identify characteristics of learners that are consistent predictors of achievement by reviewing the literature (e.g., Byrnes & Miller, 2007). The second step was to understand how and why these variables might explain different levels of knowledge growth in children across an academic year. In what follows, we briefly discuss the conclusions of this second step.

Before doing so, however, we should note that “achievement” is typically equated with scores on high-stakes standardized tests in many of the studies that we report. In the USA and elsewhere, teachers are held accountable for presenting specific content that is assessed on these standardized tests. Our goal was to develop a theory that explains why some children seem to acquire more of this content than other children in order to help inform intervention and policy efforts. But the theory could explain the acquisition of any kind of knowledge or skill (e.g., in music, visual arts) not just those assessed on standardized tests. We also recognize that standardized tests also privilege certain cultural groups over others and also often sometime require procedural skill more than conceptual understanding, but large-scale data sets such as the National Assessment of Educational Progress (NAEP) and Early Childhood Longitudinal Study-Kindergarten sample (ECLS-K) require computational skill, conceptual understanding, and problem-solving.

Prior Achievement. The strongest predictor of later achievement in a domain (e.g., math) is prior achievement in that domain, followed by self-regulation and motivation (Byrnes, 2011; Byrnes & Miller, 2007; Byrnes & Wasik, 2009; DiPerna et al., 2005; McClelland et al., 2007).

IQ. General ability (e.g., IQ) also predicts but tends to explain less than 10% of the variance of the dependent variable or is less predictive when prior knowledge, self-regulation, and motivation are controlled (Sternberg et al., 2001).

Motivation and Self-Regulation. Research has shown that children's motivation (including their goals, interests, and self-efficacy) and self-regulation skills and academic achievement are significantly related. Consistent evidence suggests that greater levels of attention, task persistence, and active participation have strong associations with standardized test scores and teacher-rated achievement that is independent of initial cognitive ability and prior basic skills (Alexander et al., 1993; DiPerna et al., 2005; Duncan et al., 2007; Hindman et al., 2010).

12.6 Opportunity Factors

As for the explanatory basis of the opportunity factors confirmed to date through our extensive review of the literature to identify consistent predictors, the role of content exposure is relatively self-evident: Students cannot be expected to show mastery of content that was not presented by their teachers. Thus, measures of whether content was exposed should predict achievement, especially when these measures are accurate and precise. Even though the year preceding kindergarten has been found to be extremely important in mathematics development (Clements & Sarama, 2009; NAEYC, 2002), results from the few observational studies of prekindergarten teachers and programs in the USA suggest that in general, very little mathematics is normally presented during the prekindergarten years (Early et al., 2005; Graham et al., 1997; Lamy et al., 2004; Clements & Sarama, 2007, 2008; Tudge & Doucet, 2004). This finding was revealed by evaluation studies of 14 US-based prekindergarten (for 3- and 4-year-olds) curriculums including Bright Beginnings, Creative Curriculum, and others which found that most of these programs were built on literacy goals with minimal time devoted to mathematics (Farran et al., 2007).

In response, a number of researchers conducted experimental studies to examine the effects of structured early mathematics curriculum on early mathematics knowledge and skill of prekindergarten children from low-income families (Chard et al., 2008; Clarke et al., 2011; Clements & Sarama, 2007, 2008; Clements et al., 2011; Starkey et al., 2004). As for the relative weight given to mathematics content and reading skills in early childhood programs in countries other than the USA, we are unaware of comparable studies. In international studies of older children (e.g., PISA, TIMSS), nations do differ in content exposure and there is a corresponding difference in achievement levels.

12.6.1 Early Mathematics Curriculum

Wang, Firmender, Power, and Byrnes (2016) recently conducted meta-analysis of 29 experimental and quasi-experimental studies of early mathematics programs for prekindergarten (for 3- and 4-year-olds) and kindergarten (age 5–6) environments. They found an overall moderate-to-large average effect size (Cohen's $d = 0.62$, range of 0.50–0.75) across these 29 early mathematics programs. The 10 included studies that evaluated four mathematics curricula, Building Blocks Curriculum, Early Learning in Mathematics, Experimental Mathematics Curriculum, Pre-K Mathematics Curriculum, had a moderate-to-large average effect size (Cohen's $d = 0.63$, range of 0.44–0.82).

The mode minutes of mathematics exposure across these curricula was 63 min per week and 1450 total minutes across the whole curriculum (Wang et al., 2016). For example, the Building Blocks' instructional approach is finding the mathematics in, and developing mathematics from, children's activity (Clements & Sarama, 2007). Children are guided to extend and mathematize (i.e., explicate, articulate, and describe) their everyday activities, from block building to art to songs to puzzles, in mathematical language. Thus, the processes of communicating and reasoning, and mathematizing are continually developed through discussions.

Activities include whole group (about 10 min per day), small group (10–15 min once per week for each child, working in groups of four with the teacher), and centers (including a computer center, 5–10 min twice a week for each child). The curriculum includes 30 weeks of instruction; teachers completed from 24 to 30 weeks. Teachers ask students to solve problems or tasks and then ask such questions as "How do you know?", "Why?", and "Can you tell how you figured that out?" More detailed descriptions of Building Blocks are available (Clements & Sarama, 2007, 2008; Clements et al., 2011).

12.6.2 Supplemental Mathematics-Related Activities

In the Wang et al. (2016) meta-analysis, studies that were coded as supplemental mathematics-related activities ($n = 19$), or interventions that were implemented in addition to the regular mathematics curriculum, were found to also have a moderate to large average effect size (Cohen's $d = 0.63$, range of 0.45–0.81). The mode minutes of mathematics exposure across these studies was 90 min per week and 720 min in total (Wang et al., 2016). For example, Ramani and Siegler (2008, 2011) engaged young children to play linear board games as a way to develop their numerical representations. An experimenter met individually with each preschooler for five 15- to 20-min sessions within a three-week period. Sessions were held in either their classroom or an unoccupied room nearby. The experimenter used The Great Race linear board game. It included a board that was 52 cm wide and 24 cm high with the name of the game printed at the top of the board. Below the name were 10 equal-sized squares of different colors arranged in a horizontal array.

Each square contained one number, with the numerical magnitudes increasing from left to right. Experimenters asked children to play a game that developed their knowledge and skills in counting, number line estimation, magnitude comparison, numeral identification, and arithmetic. Experimenters asked such questions as: "If this is where 0 goes (pointing) and this is where 10 goes (pointing), where does N (e.g., 5) go?", "Can you choose the bigger number between these two numbers?", "Can you name the numeral on this card?", and "Suppose you have N oranges and I give you M more; how many oranges would you have then?" More detailed descriptions of the linear board game are available (Ramani & Siegler, 2008, 2011; Siegler & Ramani, 2009).

Moreover, there are emerging evidence suggesting that children from low-income families have differential exposure to even the typical mathematics learning opportunities during the early years (Wang, 2010) and that low-income kindergarten children's exposure to analytic and reasoning mathematics activities is significantly related to their mathematics test scores (Georges, 2009; Wang, 2010). For instance, Georges (2009) found that while instruction explained only four percent of the variance in mathematics test scores attributable to classrooms for all kindergarten students, the portion of variance attributable to classrooms was larger (or more predictive) for students in high-poverty classrooms than in low-poverty classrooms. She further found that for high-poverty classrooms, activities that built on students' analytic and reasoning abilities and worksheet-related activities were positively related to students' test scores in all subtests (Georges, 2009). However, Wang (2010) found that there were statistically significant variations in the frequency in which children in poverty engaged in analytic and reasoning activities, suggesting that this group of children has differential exposure to typical, early mathematics learning opportunities.

12.7 Antecedent Factors

As noted above, variables in the two categories of opportunity factors and propensity factors do not account for all of the variables that have been found to be predictive of individual and group differences in early mathematics achievement. Recall that opportunity and propensity factors provide answers to the initial question, "Why do some children demonstrate higher levels of conceptual and procedural knowledge of mathematics than other children?" (answer: The former were provided with more opportunities to learn mathematics and were more prone to take advantage of these opportunities).

The third set of factors emerged when a second-tier question is asked: Why are some children presented with more opportunities to learn in early childhood and enter these opportunities more willing and able to learn math? Analysis of literatures beyond educational psychology and teacher education revealed that there are aspects of children's home lives and sociocultural demographics that have also been found to be predictive of achievement and need to be incorporated into whatever comprehensive, synthetic model is ultimately constructed.

Because factors in the third occur earlier in time than opportunity and propensity factors and help explain their emergence, O-P theories call them antecedent factors. They include age, birth weight, parental expectations, maternal education, and family income.

12.7.1 Age

Age has a well-established relationship to children's mathematical competence (Jordan et al., 2006; Hindman et al., 2010; Ransdell & Hecht, 2003). Age is appropriately construed as an antecedent factor because it can explain why some children were exposed to more opportunities than others (e.g., older children usually have been exposed to more opportunities than younger children) and why some children might be more prone to take advantage of these opportunities (e.g., an increase in cognitive skills due to brain maturation).

Generally, very young children begin with basic mathematics competence that over time develops into more complex mathematical skills. This is especially true during the preschool ages, when children's mathematical competence develops from recognizing small groups of objects to counting the number of objects in correct order and, eventually, developing early arithmetic skills (Geary, 2007). Furthermore, age is an educationally relevant variable since there is an 11- to 12-month variation in children's chronological age during each academic year of schooling (Dowker, 2008).

12.7.2 Birth Weight

Low birth weight (born less than 2,499 g), which is prevalent among low-income children (Collins & David, 1990), has been associated with greater likelihood of increased cognitive delay at 2 years of age (Hillemeier et al., 2010), increased frequency of impaired language functioning (Sajaniemi et al., 2001), lower scores on standardized measures of academic achievement (Bowen et al., 2002), and greater likelihood to fall behind academically (Bowen et al., 2002).

12.7.3 Parental Expectations

Higher parental expectations for children have been associated with greater likelihood of selection of more core academic courses (Catsambis, 2001), better school attendance (Kurdek & Sinclair, 1988), and stronger academic performance (Fehrmann et al., 1987; Rutchick et al., 2009). Even with socioeconomic status controlled, parental expectations have been found to explain significant variance in opportunities, propensities, and achievement (Byrnes & Miller, 2007; Byrnes & Wasik, 2009).

Parental expectations have been found to influence child expectations (Rutchick et al., 2009) and motivation (Wood et al., 2011), both of which are associated with better academic performance.

12.7.4 Maternal Education

Maternal education has been found to have direct and indirect, positive relationships with achievement (Davis-Kean, 2005; Eccles, 2005) and explains much of the variance in cognitive outcomes for low- income prekindergarten children (Perry & Fantuzzo, 2010). Maternal education has been associated with greater exposure to literacy and numeracy skills at home (Lung et al., 2011), more educational opportunities in local communities (Furstenberg et al., 1999), and higher level of schooling expected of their children (Byrnes & Wasik, 2009; Davis-Kean, 2005), all three of which are predictive of stronger academic outcomes. Additionally, maternal education has been found to indirectly affect a child's academic achievement to the extent that it influences family income and residence (Coleman, 1987; Furstenberg et al., 1999).

12.7.5 Family Income

Research in the USA has clearly documented the existence of achievement differences by family income. A seminal research that empirically examined a nationally representative sample of kindergarten children found substantial differences in children's test scores by family income, with children in the highest income group scoring 60% above the scores of children from the lowest income group as they begin kindergarten (Lee & Burkam, 2002). A more recent study utilizing data from random assignment studies on welfare and poverty programs in the USA found that a $1,000 increase in annual income increases young children's achievement by 5–6% of a standard deviation (Duncan et al., 2011).

12.8 Using Structural Equation Modeling for Theory Building

Structural equation modeling (SEM) provides mathematics education and early childhood researchers and practitioners with the ability to estimate multiple cross-dependency relationships and capture unobserved variables in such relationships while controlling for measurement error (Bollen, 2011). SEM, also known as causal modeling, simultaneous equation modeling, covariance and mean structure modeling, is used to confirm the developed theoretical framework and offers multiple ways to evaluate the validity of the model (Bollen, 2011). Typically, SEM proceeds in two stages. The first stage is the estimation of the measurement model, which represents a set of observable variables as multiple indicators of a smaller set of latent variables (dimensions of opportunity, propensity, and antecedent factors), and the theoretical model, which describes dependency relations between the latent variables, and which is grounded in theory (hypotheses). The second stage is the estimation of the structural model, which is a combination of the measurement and path models.

For instance, in our 2013 study (Wang et al., 2013), we tested the O-P model on mathematics achievement of a subsample of 350 African American, 350 Hispanic American, and 300 White children who were below poverty line from the restricted use Early Childhood Longitudinal Study-Birth (ECLS-B) database. As a first step, we used measurement models to consolidate the multi-dimensional latent constructs of antecedent, opportunity, and propensity. Our antecedent factor was indicated by birth weight, early cognition, age, parent expectation, and mother's years of education.

Opportunity was indicated by the latent constructs of teacher-initiated activities for learning basic mathematics skills and teacher-initiated, integrated learning activities. Propensity was indicated by the latent constructs of prekindergarten parent rating and prekindergarten provider rating, as well as preexisting cognition. Prekindergarten parent and provider ratings were indicated by items from previous research. A confirmatory factor analysis, as part of the measurement model (see Fig. 12.1), was conducted to confirm the latent factors of opportunity and propensity. The five items forming teacher-initiated activities for learning basic mathematics skills included teacher-reported frequency of counting out loud, using geometric manipulative, using counting manipulative, doing calendar activities, and discussing shapes and patterns. The five items forming teacher-initiated, integrated learning activities included teacher-reported frequency of doing mathematics games, music with math concept, creative movement with math, rules, cups and spoons, and telling time. A path model was used to test the effects of antecedent, opportunity, and propensity on prekindergarten mathematics achievement (see Fig. 12.1). The final set of parameter estimates and standard errors were obtained by combining the five sets of estimates using arithmetic rules outlined by Rubin (1987).

Second, in order to validate the multi-dimensional latent constructs of antecedent, opportunity, and propensity factors, first-order confirmatory factor analysis

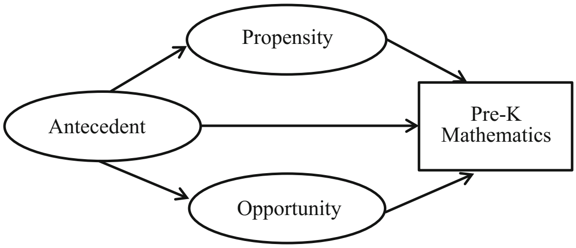

Fig. 12.1 Path model. Adapted from Wang, Shen and Byrnes (2013), with permission from Elsevier

(CFA) was conducted for the measurement model of antecedent, and second-order CFA was conducted for the measurement models of opportunity and propensity. The evaluation of the factor loadings showed that the observed indicators had high factor loadings on their common factors, indicating that they adequately reflected their underlying latent variables. All indicators in the model had statistically significant factor loadings (see Fig. 12.2), confirming the existence of significant associations among measured indicators and their latent constructs.

Third, in order to test if antecedent, opportunity, and propensity were important educational constructs, structural equation modeling (SEM) was performed. In our case, we examined whether the hypothesized O-P structural model provided a reasonable fit to the data by examining the goodness of fit between the hypothesized model and our data (Wang et al., 2013). A number of goodness-of-fit statistics were used, including the chi-square statistic or the Likelihood Ratio Test which examined the closeness of fit between the unrestricted sample covariance matrix and the restricted covariance matrix; the Comparative Fit Index (CFI) which evaluated the gain in improved fit between the null model and alternative model; and the Root Mean Square Error of Approximation (RMSEA) and its confidence interval which examined how well the hypothesized model would fit the population covariance matrix if it was available (Kline, 2011). Our results indicated a good fit. In everyday terms, the model you are trying to fit (see Fig. 12.1 for example) is evaluated by seeing if the relationships described in the model (the arrows pointing from one variable represented by an oval to another variable) seem to be present in the data itself. If there is a good fit, the theory or model is confirmed.

Fourth, we evaluated the direct effects, and as our fifth verification process, we evaluated the indirect and total effects of our model by evaluating the unstandardized and standardized validity coefficients for evaluation of sign, statistical and substantive significance.

Our last step was to evaluate the unique validity variance. In a prior study, Byrnes and Wasik (2009) found that their set of opportunity, propensity, and antecedent factors explained 63–71% of the variance of the outcome variable for a subsample of 17,401 US children (51% were female, 57% were White, 17% were Hispanic, 14% were Black, 6% Asian, and 6% were other) from ELCSK who had data on entry to kindergarten (age 5–6), end of kindergarten, grade one (age 6–7), and grade three (age 8–9). Stronger models explain more variance of the outcome variable. If the set of predictors do not really explain why some children learned

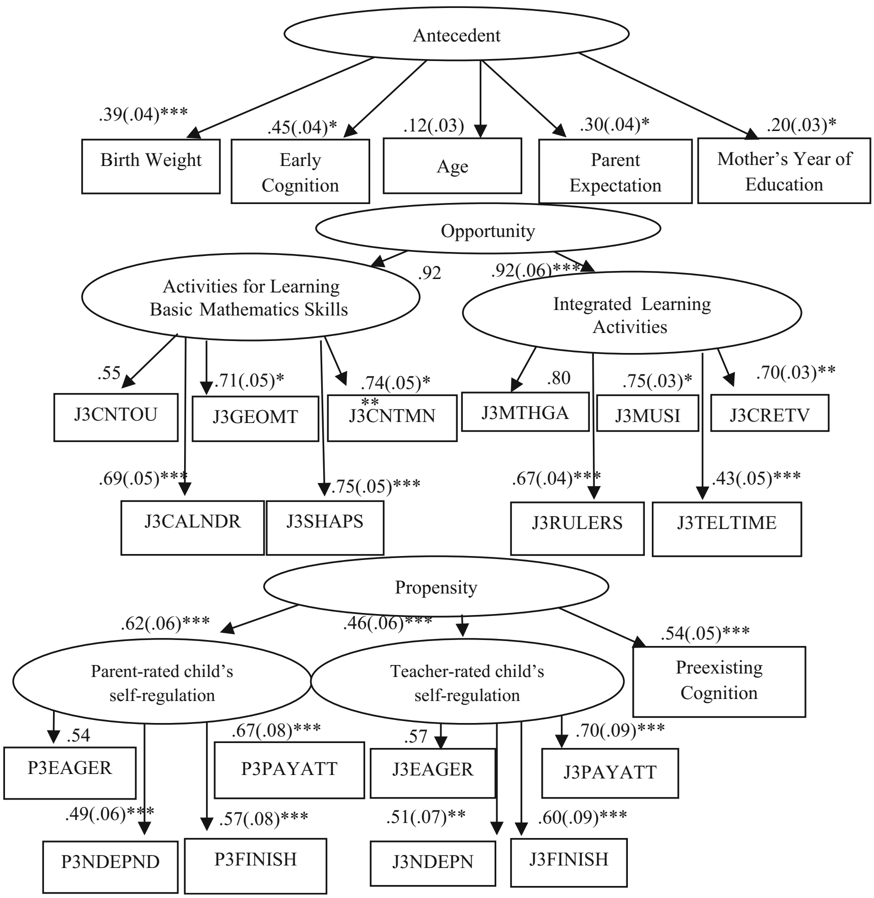

Fig. 12.2 Standardized factor loadings in the measurement models. Standard errors are reported in parentheses. ** $p < 0.01$; *** $p < 0.001$. Adapted Wang, Shen and Byrnes (2013), with permission from Elsevier

more than other children, 0% of the variance is explained. If all of the correct predictors are included, 100% of the variance is explained. So the O-P model seems to be gathering many of the correct, authentic predictors by explaining 63–71% of the variance in end-of-year mathematics scores. More specifically, at kindergarten (age 5–6), the propensity variables explained 69% of variance, opportunity variables explained less than 1% of variance, and antecedent explained less than 1% of variance. At grade one (age 6–7), propensity variables explained 46% of variance, opportunity explained less than 1% variance, and antecedent explained 1% of the variance. At grade three (age 8–9), propensity accounted for 46% variance, opportunity accounted for less than 1% variance, and antecedent accounted for 2% of variance.

In the Wang et al. (2013) study, the set of opportunity, propensity, and antecedent factors examined explained 34% of the variance. More specifically, the antecedent variables accounted for 21% of the variance; antecedent and propensity variables accounted for 31% of the variance; and antecedent, opportunity, and propensity variables accounted for 34% of the variance. The antecedent factor was positively associated with the opportunity and propensity factors. The opportunity factor was positively related to prekindergarten mathematics achievement. The total effect of the antecedent factor on prekindergarten mathematics achievement was also significant.

12.9 Testing and Extending the O-P Model on International Populations of Young Children

One particularly promising way to strengthen the generalizability of the O-P model is to test and extend the model on other populations of young children, using comparable multivariate longitudinal studies conducted by our international colleagues. While these researchers have yet to test and confirm the O-P model, their work can provide clues as to potential additional variables that we have not tested to date, and also see if there are variations in the predictive value of certain variables across cultures. Such variations can provide potentially useful information for theory revision and generalization. In our examination of the work of our international colleagues, we found our colleagues used different and promising authentic predictors for their opportunity, propensity, and antecedent variables (Table 12.1).

We highlight these promising authentic predictors below and invite our international colleagues to replicate and extend the OP model using their existing data sets from their context. Together, we can advance the field by generating evidence of convergent and divergent validity for the different factors of the theory, improving our measurement of these factors, strengthening our predictions, guiding the development of effective interventions, and supporting the successful application of theory to practice.

12.9.1 Opportunity Indicators Examined by International Colleagues

12.9.1.1 Home Numeracy Environment

Manolitsis et al. (2013) tested whether formal home numeracy environment will predict mathematics fluency in grade one on a sample of 82 Greek children (53 males and 29 females, mean age = 64.67 months, SD = 3.26, at the first time of measurement) from Heraklion, a typical urban city in Greece. The children were

Table 12.1 Promising authentic predictors examined by international colleagues

Study name	Population	Antecedent	Opportunity	Propensity
Alloway and Passolunghi (2011)	206 typically developing Italian children aged 7 and 8 (109 boys) recruited from four mainstream schools located in the northwest of Italy. The majority of parents came from professional homes that were predominantly middle class but included families from across the social spectrum. For the statistical analyses, participants were divided into two age-groups: 7-year-olds (n = 100; M = 88 months, SD = 3.5 months, 50 boys) and 8-year-olds (n = 106; M = 103 months, SD = 3.6 months, 46 boys). None was receiving special education services or had documented brain injury, or behavioral problems. None of the assessed children belonged to families with sociocultural disadvantage	Age	Not measured	General ability as measured by the vocabulary subtest of PMA (Thurstone & Thurstone, 1968). Working memory as assessed by 12 tests from the Automated Working Memory Assessment (AWMA, Alloway, 2007)
Aunio, Heiskari, Van Luit, and Vuorio (2015)	235 ordinary Finnish children (111 girls and 124 boys) around 6-years-old on average (in months, M = 74.55;	Not measured	Typical instruction offered during kindergarten from September until April	Prior numeracy skills, measured in September and January during kindergarten

(continued)

Table 12.1 (continued)

Study name	Population	Antecedent	Opportunity	Propensity
	SD = 3.50) who were a subsample of the Finnish Early Numeracy Test (ENT). The children in this study attended 18 ordinary Finnish kindergartens in the capital region. Children with a multilingual background (i.e., immigrant children) or those with special educational needs were excluded from the sample. The current data were collected at three measurement times (September, January, and April 2003) during 1 kindergarten year			
Carmichael, MacDonald, and McFarland-Piazza (2014)	2450 Australian kindergarten children (aged between 8.25 and 10; 53% male; 2% indigenous status) from the third wave of Longitudinal Study of Australian Children (LSAC) who completed their numeracy component of the Australian National Assessment Program—Literacy and Numeracy (NAPLAN) test for the first time	Parent education, family SES, home environment, parental warmth, parental hostility	Teacher qualification (education, years of teaching the grade), teacher self-reported math learning environment, children's attendance, Student–Teacher Relationship Scale, parent provision of academic support at home, parent and school interaction, parent–teacher interaction	IQ measured by the Matrix Reasoning Test, a component of the Wechsler Intelligence Scale for Children (WISC–IV) (Wechsler, 2003); child attitude toward school and child attitude toward math and number work at school

(continued)

Table 12.1 (continued)

Study name	Population	Antecedent	Opportunity	Propensity
Fitzpatrick and Pagani (2012)	1824 children (approx. 50% females with average age of 29 months) from the Québec Longitudinal Study of Child Development (QLSCD) with working memory data at 29 and 41 months	Child age, temperament, sleep, weight-for-gestational age, months breastfed, family functioning and configuration, and maternal age were entered and removed as did not significantly contribute to the variance in the model. Socioeconomic status and sex were retained	Not measured	Working memory as measured by the Imitation Sorting Task (Alp, 1994) assessed at 29 and 41 months by trained examiners
Manolitsis, Georgiou, and Tziraki (2013)	82 Greek children (53 males and 29 females; mean age = 64.67 months, SD = 3.26, at the first time of measurement) from Heraklion, a typical urban city in Greece, were followed from kindergarten to grade 1. The children were randomly selected from six kindergarten schools (serving children aged 5–6), which were, in turn, selected with a stratified randomized approach in order to represent a range of demographics. The children were native speakers of Greek, Caucasian, and 57% had attended prekindergarten (serving children aged 4–5)	Mother's education	Home numeracy environment (adapted in Greek from LeFevre et al. (2009) where parent self-report frequency of teaching child to identify numbers, count objects, sort objects, count, simple calculations)	Basic math concept (assessed by four tasks adapted from the Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2003). Verbal counting (child asked to count from 1 up to 50, or the higher number they could)

(continued)

Table 12.1 (continued)

Study name	Population	Antecedent	Opportunity	Propensity
Praet, and Desoete (2014)	132 Dutch-speaking children (53% girls) from five kindergartens serving children from families with working and middle-class-socioeconomic backgrounds	Not measured	Not measured	IQ measured by Wechsler Preschool and Primary Scale of Intelligence or the WPPSIIII-NL. Oral language skills measured by the Clinical Evaluation of Language Fundamentals or the CELF-4Nl (Kort et al., 2008; Semel et al., 2008). Estimations using number words and mathematics (in three formats, Arabic digits, dots, and number)
Wu and Chiang (2015)	19,499 children (53% were boys, 8% were preterm) who completed the 6-month, 18-month, and 3-year surveys of the Taiwan Birth Cohort Study (TBCS). The mean age of the mothers when giving birth was 28.88 years (SD = 4.85), and the majority had received either 10–12 years (40%) or more than 13 years (46%) of formal education	Family transition type (married stable, cohabiting stable, single stable, married unstable, single unstable); exposure to income poverty; family APGAR score; parenting quality (cognitive stimulation, emotional support	Not measured	Not measured

randomly selected from six kindergarten schools (serving children aged 5–6), which were, in turn, selected with a stratified randomized approach in order to represent a range of demographics. The children were native speakers of Greek, Caucasian, and 57% had attended prekindergarten (serving children aged 4–5). They measured formal home numeracy environment by adapting into Greek, the LeFevre et al. (2009) scale that measured parent self-report on frequency of teaching child to identify numbers, count objects, sort objects, count, and simple calculations. The results from their path analyses indicated parents' teaching of numeracy skills predicted mathematics fluency through the effects of verbal counting and that their overall model accounted for 26–27% of the variance (Manolitsis et al., 2013).

12.9.1.2 Supportive Home–School Relationship

Carmichael et al., (2014), in their exploratory study, measured how supportive home–school relationship, one of the factors in their exploratory ecological theory, would predict children's performance on a standardized numeracy assessment. They conducted their study on a subsample of 2,450 Australian kindergarten children (aged between 8.25 and 10; 53% male; 2% indigenous status) from the third wave of the Longitudinal Study of Australian Children (LSAC) who completed their numeracy component of the Australian National Assessment Program—Literacy and Numeracy (NAPLAN) test for the first time. They measured supportive home–school relationship through these measures: parent provision of academic support based on parent reports on two survey items, parent/school interaction based on teacher response to one survey item, and parent/teacher communication based on parents response to six survey items adapted from a measure in the Early Childhood Study of Kindergarteners (ECLS-K)—Base Year. They found parental involvement as measured by teachers' perception of parental involvement, parent help with homework, and parental communication explained 11% of the variation.

12.9.2 Propensity Indicators Examined by International Colleagues

We examined the literature to identify longitudinal studies of early mathematics achievement that were conducted by researchers outside of the USA and which focused on variables that have not been examined by O-P theorists based in the USA to date. Within disciplines such as educational psychology, mathematics education, and education policy, researchers adopt common definitions of theoretical constructs (e.g., "self-regulation," "inquiry learning," and "educational opportunities") and major journals in these disciplines publish the work of scholars from around the world. As such, the studies described below refer to these shared constructs. That said, the relative weight of the shared constructs could differ by country (e.g., family income could be a stronger predictor of performance in countries that have wider income disparities—as has been found in PISA); these constructs could be manifested, understood, and measured in distinct ways in these countries; there may be factors in each category that are unique to particular countries that await to be identified. Our brief review is intended to initiate the dialogue and welcome collaborations that help reveal these national differences to help extend and improve the model.

12.9.2.1 Working Memory

Alloway and Passolunghi (2011) extended current understanding of the relationship between working memory and mathematical skills in their investigation of the contribution of working memory and verbal ability to mathematical skills on a sample of 206 typically developing Italian children aged 7 and 8 (109 boys) recruited from four mainstream schools located in the northwest of Italy. Working memory was assessed by 12 tests from the Automated Working Memory Assessment (AWMA, Alloway, 2007), a computer-based standardized battery that provides three assessments of verbal short-term memory, three assessments of visuospatial memory, three assessments of verbal working memory, and three assessment of visuospatial working memory. They found that working memory accounted for four to nine percent of the variance and that the general model accounted for 17–35% of the variance (Alloway & Passolunghi, 2011).

Fitzpatrick and Pagani (2012) tested their hypothesis that better working memory skills at 35 months will predict better kindergarten classroom engagement and academic performance on a subsample of 1824 children (approx. 50% females with average age of 29 months) from the Québec Longitudinal Study of Child Development (QLSCD). Working memory was measured by the Imitation Sorting Task (Alp, 1994) and was assessed at 29 and 41 months by trained examiners. Using stepwise regression, they found their model explained 17–35% of the variance of mathematics skills with general ability explaining around 13–26% of the variance and working memory explaining around 4–9% of the variance. Specifically, they found a positive association between early working memory scores and later classroom engagement, number knowledge, and receptive vocabulary.

12.9.2.2 Quality of Parent–Child Relationship

Wu and Chiang (2015) examined the relationships and potential pathways between family structure transitions and early childhood development on the 19,499 children (53% were boys, 8% were preterm births) who completed the 6-month, 18-month, and 3-year surveys of the Taiwan Birth Cohort Study (TBCS). The mean age of the mothers when giving birth was 28.88 years (SD = 4.85), and the majority had received either 10–12 years (40%) or more than 13 years (46%) of formal education. They measured family transition type (married stable, cohabiting stable, single stable, married unstable, and single unstable) and parenting quality (degree of cognitive stimulation and emotional support). They found caregiver's psychosocial functioning and/or parenting quality can partly buffer against the risks of poorer cognitive or socioemotional development that accompany certain family transition types.

12.10 Conclusion and Implications for Theory Building, Policy and Practice

Earlier we noted that an important first step in knowing how to increase the mathematics skills of young children and thereby equip them for later success in school is to build a theory using the results of large-scale, longitudinal multivariate studies. The process taken so far is to add variables to the model that explain additional variance once other consistent predictors have been included and controlled, and delete variables that drop out when other more powerful and authentic predictors have been included (e.g., race after income is controlled). At present, the set of approximately 20 predictors in the three categories that remain significant account for about 50–60% of the variance in mathematics achievement (e.g., Byrnes & Miller-Cotto, 2016; Wang et al., 2013). Hence, much is known about why individuals and groups differ in achievement, but additional variables wait to be discovered and their relative importance in each country needs to be established. Once these additional variables are discovered and the amount of explained variance moves closer to 100%, scholars can then propose and test a theoretical model that effectively integrates the variables and describes how they relate to each other. This simple yet versatile model can then be tested in experimental investigations to verify the causal role of each of the variables. Once scholars and practitioners know why certain children show higher levels of mathematics achievement than others by the end of the preschool period, effective forms of intervention can be created to target the causal factors identified in the theory.

In prior studies of the O-P framework, we identified and verified the predictive role of antecedent factors such as family socioeconomic status (family income and maternal education), parent educational expectations for their children, age, birth weight, gender, and ethnicity. With respect to opportunity factors, we identified and confirmed the predictive role of several aspects of instruction (the content that is presented, the style of presentation, and the dosage per week of either a full curriculum or supplemental activities). Our international colleagues have identified several others as well as the ones we describe in this chapter: home numeracy environment and supportive home–school relationships. With respect to propensity factors, we have identified and confirmed the role of prior knowledge, motivation, and self-regulation in our studies.

We encourage other researchers to build on this work in order to create the most accurate, predictive, and generalizable model of early children mathematics achievement. This way, we can collaborate in guiding early mathematics policy-makers, practitioners, and professional and advocacy organizations by providing them with a simple yet flexible framework on early mathematics achievement to scaffold understanding, generate and test hypotheses, and facilitate and adapt targeted interventions to address context- and cultural-specific problems.

References

Alexander, K. L., Entwisle, D. R., & Dauber, S. L. (1993). First-grade classroom behavior: Its short- and long-term consequences for school performance. *Child Development, 64*(3), 801–814.

Alloway, T. P. (2007). *Automated working memory assessment*. London: Harcourt Assessment.

Alloway, T. P., & Passolunghi, M. C. (2011). The relationship between working memory, IQ, and mathematical skills in children. *Learning and Individual Differences, 21,* 133–137. doi:10.1016/j.lindif.2010.09.013.

Alp, I. E. (1994). Measuring the size of working memory in very young children: The imitation sorting task. *International Journal of Behavioral Development, 17,* 125–141.

Aunio, P., Heiskari, P., Van Luit, J. E. H., & Vuorio, J. M. (2015). The development of early numeracy skills in kindergarten in low-, average- and high-performance groups. *Journal of Early Childhood Research, 13*(1), 3–16. doi:10.1177/1476718X14538722.

Bollen, K. A. (2011). Evaluating effect, composite, and causal indicators in structural equation models. *MIS Quarterly, 35*(2), 359–372.

Bowen, J., Gibson, F., & Hand, P. (2002). Educational outcome at 8 years for children who were born extremely prematurely: A controlled study. *Journal of Paediatrics and Child Health, 38* (5), 438–444.

Byrne, B. M. (1998). *Structural equation modeling with LISREL, PRELIS, and SIMPLIS: Basic concepts, applications, and programming*. New York, NY: Lawrence Erlbaum Associates, Taylor & Francis Group.

Byrnes, J. P. (2003). Factors predictive of mathematics achievement in White, Black, and Hispanic 12th graders. *Journal of Educational Psychology, 95,* 316–326.

Byrnes, J. P. (2011). Academic achievement. In B. Brown & M. Prinstein (Eds. in chief), *Encyclopedia of Adolescence* (pp. 1–9). Amsterdam: Elsevier.

Byrnes, J. P., & Miller, D. C. (2007). The relative importance of predictors of mathematics and science achievement: An opportunity-propensity analysis. *Contemporary Educational Psychology, 32,* 599–629. doi:10.1016/j.cedpsych.2006.09.002.

Byrnes, J. P., & Miller-Cotto, D. (2016). The growth of mathematics and reading skills in segregated and diverse schools: An opportunity-propensity analysis of national database. *Contemporary Educational Psychology*, *46*, 34–51. doi:10.1016/j.cedpsych.2016.04.002

Byrnes, J. P., & Wasik, B. A. (2009). Factors predictive of mathematics achievement in kindergarten, first and third grades: An opportunity-propensity analysis. *Contemporary Educational Psychology, 34,* 167–183. doi:10.1016/j.cedpsych.2009.01.002.

Carmichael, C., MacDonald, A., & McFarland-Piazza, L. (2014). Predictors of numeracy performance in national testing programs: Insights from the longitudinal study of Australian children. *British Educational Research Journal, 40*(4), 637–659. doi:10.1002/berj.3104.

Catsambis, S. (2001). Expanding knowledge of parental involvement in children's secondary education: Connections with high school seniors' academic success. *Social Psychology of Education, 5*(2), 149–177.

Chard, D. J., Baker, S. K., Clarke, B., Jungjohann, K., Davis, K., & Smolkowski, K. (2008). Preventing early mathematics difficulties: The feasibility of a rigorous kindergarten mathematics curriculum. *Learning Disability Quarterly, 31*(1), 11–20 (Mathematics Instruction and Learning Disabilities).

Clarke, B., Smolkowski, K., Baker, S., Fien, H., Doabler, C., & Chard, D. (2011). The impact of a comprehensive tier I core kindergarten program on the achievement of students at risk in mathematics. *The Elementary School Journal, 111*(4), 561–584.

Clements, D. H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks Project. *Journal for Research in Mathematics Education, 38* (2), 136–163.

Clements, D. H., & Sarama, J. (2008). Experimental evaluation of the effects of a research-based preschool mathematics curriculum. *American Educational Research Journal, 45*(2), 443–494.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. New York: Routledge.

Clements, D. H., Sarama, J., Spitler, M. E., Lange, A. A., & Wolfe, C. B. (2011). Mathematics learned by young children in an intervention based on learning trajectories: A large-scale cluster randomized trial. *Journal for Research in Mathematics Education, 42*(2), 127–166.

Coleman, J. S. (1987). Families and schools. *Educational Researcher, 16,* 32–38.

Collins, J., & David, R. (1990). The differential effect of traditional risk factors on infant birth weight among Blacks and Whites in Chicago. *American Journal of Public Health, 80,* 679–681.

Davis-Kean, P. E. (2005). The influence of parent education and family income on child achievement: The indirect role of parental expectations and the home environment. *Journal of Family Psychology, 19*(2), 294–304.

DiPerna, J. C., Volpe, R. J., & Elliott, S. N. (2005). A model of academic enablers and mathematics achievement in the elementary grades. *Journal of School Psychology, 43,* 379–392.

Dowker, A. (2008). Individual differences in numerical abilities in preschoolers. *Developmental Science, 11*(5), 650–654.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., …, Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428–1446.

Duncan, G. J., Morris, P. A., & Rodrigues, C. (2011). Does money really matter? Estimating impacts of family income on young children's achievement with data from random-assignment experiments. *Developmental Psychology, 47*(5), 1263–1279. doi:10.1037/a0023875.

Early, D., Barbarin, O., Bryant, D., Burchinal, M., Chang, F., Clifford, R., … Barnett, W. S. (2005). *Pre-kindergarten in seven states: NCEDL's multi-state study of pre-kindergarten & study of state-wide early education programs (SWEEP)*. Chapel Hill: NC: University of North Carolina.

Eccles, J. S. (2005). Influences of parents' education on their children's educational attainments: The role of parent and child perceptions. *London Review of Education, 3*(3), 191–204. doi:10.1080/14748460500372309.

Farran, D. C., Lipsey, M. W., Watson, B., & Hurley, S. (2007). *Balance of content emphasis and child content engagement in an early reading first program*. Paper presented at the American Educational Research Association.

Fehrmann, P. G., Keith, T. Z., & Reimers, T. M. (1987). Home influence on school learning: Direct and indirect effects of parental involvement on high school grades. *Journal of Educational Research, 80*(6), 330–337.

Fitzpatrick, C., & Pagani, L. S. (2012). Toddler working memory skills predict kindergarten school readiness. *Intelligence, 40,* 205–212.

Furstenberg, F. F., Cook, T. D., Eccles, J., Elder, G. H., & Sameroff, A. (1999). *Managing to make it: Urban families and adolescent success*. Chicago, IL: University of Chicago.

Geary, D. C. (2007). An evolutionary perspective on learning disability in mathematics. *Developmental Neuropsychology, 32*(1), 471–519.

Georges, A. (2009). Relation of instruction and poverty to mathematics achievement gains during kindergarten. *Teachers College Record, 111*(9), 1–12.

Graham, T. A., Nash, C., & Paul, K. (1997). Young children's exposure to mathematics: The child care context. *Early Childhood Education Journal, 25,* 31–38.

Hailikari, T., Nevgi, A., & WKomulainen, E. (2008). Academic self-beliefs and prior knowledge as predictors of student achievement in mathematics: A structural model. *Educational Psychology*, *28*(1), 59–71.

Hillemeier, M., Morgan, P., Farkas, G., & Maczuga, S. (2010). Perinatal and socioeconomic risk factors for variable and persistent cognitive delay at 24 and 48 months of age in a national sample. *Maternal and Child Health Journal,* 1–10.

Hindman, A. H., Skibbe, L. E., Miller, A., & Zimmerman, M. (2010). Ecological contexts and early learning: Contributions of child, family, and classroom factors during Head Start, to literacy and mathematics growth through first grade. *Early Childhood Research Quarterly, 25,* 235–250.

Jones, K. K., & Byrnes, J. P. (2006). Characteristics of students who benefit from high-quality mathematics instruction. *Contemporary Educational Psychology, 31*(3), 328–343.

Jordan, N. C., Kaplan, D., Olah, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, 77*(1), 153–175. doi:10.1111/j.1467-8624.2006.00862.x.

Kaestle, C. F. (1993). The awful reputation of education research. *Educational Researcher, 22*(1), 26–31.

Kline, R. B. (2011). Principles and practice of structural equation modeling (3rd ed.). New York, NY: The Guilford Press.

Kort, W., Schittekatte, M., & Compaan, E. (2008). CELF-4-NL Test voor diagnose en evaluatie van taalproblemen. Handleiding [Test for the evaluation of language problems. Manual]. Amsterdam: Pearson.

Kurdek, L. A., & Sinclair, R. J. (1988). Relation of eighth graders' family structure, gender, and family environment with academic performance and school behavior. *Journal of Educational Psychology, 80*(1), 90–94.

Lamy, C. E., Frede, E., Seplocha, H., Strasser, J., Jambunathan, S., Juncker, J. A., et al. (2004). *Inch by inch, row by row, gonna make this garden grow: Classroom quality and language skills in the Abbott Preschool Program*. Trenton, NJ: New Jersey Department of Education, Office of Early Childhood Education.

Lee, V. E., & Burkam, D. T. (2002). *Inequality at the starting gate: Social background differences in achievement as children begin school*. Washington, DC: Economic Policy Institute.

LeFevre, J., Skwarchuk, S., Smith-Chant, B., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science, 41,* 55–66.

Lung, F-W., Shu, B-C, Chiang, T-L., Y Lin, S-J. (2011). Maternal mental health and childrearing context in the development of children at 6, 18 and 36 months: A Taiwan birth cohort pilot study. *Child: Care, Health and Development, 37*(2), 211–223. doi:10.1111/j.1365-2214.2010.01163.

Manolitsis, G., Georgiou, G. K., & Tziraki, N. (2013). Examining the effects of home literacy and numeracy environment on early reading and math acquisition. *Early Childhood Research Quarterly, 28,* 692–703. doi:10.1016/j.ecresq.2013.05.004.

McClelland, M. M., Cameron, C. E., Connor, C. M., Farris, C. L., Jewkes, A. M., & Morrison, F. J. (2007). Links between behavioral regulation and preschoolers' literacy, vocabulary, and mathematics skills. *Developmental Psychology, 43,* 947–959.

Murphy, P. K., Alexander, P. A. (2002). What counts? The predictive powers of subject-matter knowledge, strategic processing, and interest in domain-specific performance. *Journal of Experimental Education*, *70*(3), 197–214.

National Association for the Education of Young Children. (2002). *Early childhood mathematics: Promoting good beginnings*. Washington, D. C.: Author.

Perry, M. A., & Fantuzzo, J. W. (2010). A multivariate investigation of maternal risks and their relationship to low-income, preschool children's competencies. *Applied Developmental Science, 14*(1), 1–17.

Praet, M., & Desoete, A. (2014). Number line estimation from kindergarten to grade 2: A longitudinal study. *Learning and Instruction, 33,* 19–28. doi:10.1016/j.learninstruc.2014.02.003.

Ramani, G. B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children's numerical knowledge through playing number board games. *Child Development, 79*(2), 375–394.

Ramani, G. B., & Siegler, R. S. (2011). Reducing the gap in numerical knowledge between low- and middle-income preschoolers. *Journal of Applied Developmental Psychology, 32*(3), 146–159.

Ransdell, S., & Hecht, S. (2003). Time and resource limits on working memory: Cross-age consistency in counting span performance. *Journal of Experimental Child Psychology, 86*(4), 303–314. doi:10.1016/j.jecp.2003.08.002.

Rubin, D. B. (1987). *Multiple imputation for nonresponse in surveys*. New York: Wiley.

Rutchick, A. M., Smyth, J. M., Lopoo, L. M., & Dusek, J. B. (2009). Great expectations: The biasing effects of reported child behavior problems on educational expectancies and subsequent academic achievement. *Journal of Social and Clinical Psychology, 28*(3), 392–413.

Sajaniemi, N., Hakamies-Blomqvist, L., Mäkelä, J., Avellan, A., Rita, H., & von Wendt, L. (2001). Cognitive development, temperament and behavior at 2 years as indicative of language development at 4 years in pre-term infants. *Child Psychiatry and Human Development, 31*(4), 329–346.

Schunk, D. H., & Zimmerman, B. J. (2013). Self-regulation and learning. In W. M. Reynolds, G. E. Miller, & I. B. Weiner (Eds.), *Handbook of psychology, Vol. 7: Educational psychology* (2nd ed.) (pp. 45–68). New York, NY: Wiley.

Semel, E., Wiig, E. H., & Secord, W. A. (2008). CELF 4 Nl clinical evaluation of language fundamentals. Amsterdam: Pearson.

Shavelson, R. J., & Towne, L. (2002). *Scientific research in education*. Washington, D. C.: National Academy Press.

Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology, 101,* 545–560.

Soares, D. L., Lemos, G. C., Primi, R., & Almeida, L. S. (2015). The relationship between intelligence and academic achievement throughout middle school: The role of students' prior academic performance. *Learning and Individual Differences, 41,* 73–78.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly, 19*(1), 99–120.

Sternberg, R. J., Grigorenko, E., & Bundy, D. A. (2001). The predictive value of IQ. *Merrill-Palmer Quarterly,* 1–41.

Thurstone, N. L., & Thurstone, T. G. (1968). PMA Primary Mental ability (trad. it. Batteria Primaria di Abilità). Firenze: Organizzazioni Speciali.

Tudge, J. R. H., & Doucet, F. (2004). Early mathematical experiences: Observing young Black and White children's everyday activities. *Early Childhood Research Quarterly, 19,* 21–39.

Walters, P. B., & Lareau, A. (2009). Introduction. In P. B. Walters, A. Lareau, & S. H. Ranis (2009), *Educational research on trial: policy reform and the call for scientific rigor* (pp. 1–13). New York, NY: Routledge.

Wang, A. H. (2010). Optimizing early mathematics experiences for children from low-income families: A study on opportunity to learn. *Early Childhood Education Journal, 37*(4), 295–302. doi:10.1007/s10643-009-0353-9.

Wang, A. H, Firmender, J. M., Power, J. R., & Byrnes, J. P. (2016). Understanding the characteristics and effects of school-based early mathematics learning opportunities in early childhood settings: A meta-analytic review. *Early Education and Development*, doi:10.1080/10409289.2016.1116343.

Wang, A. H., Shen, F., & Byrnes, J. (2013). Does the opportunity-propensity framework predict early mathematics skills for low-income pre-kindergarten children? *Contemporary Educational Psychology, 38,* 259–270. doi:10.1016/j.cedpsych.2013.04.004.

Wechsler, D. (2003). Wechsler intelligence scale for children: fourth edition (San Antonio, TX: Psycho-logical Corporation).

Wigfield, A., & Cambria, J. (2010). Students' achievement values, goal orientations, and interest: Definitions, development, and relations to achievement outcomes. *Developmental Review, 30* (1), 1–35.

Wood, D., Kurtz-Costes, B., & Copping, K. E. (2011). Gender differences in motivational pathways to college for middle class African American youths. *Developmental Psychology, 47* (4), 961–968.

Wu, J. C., & Chiang, T. L. (2015). Family structure transitions and early childhood development in Taiwan: Evidence from a population-based birth cohort study. *International Journal of Behavioral Development, 39*(3), 275–284. doi:10.1177/0165025414544.

Authors Biography

Dr. Aubrey H. Wang is Associate Professor and Director of Doctor of Education Program at Saint Joseph's University. Aubrey has worked in educational policy and research, and educational leadership for 20 years. Aubrey has worked as a research team member at the Consortium for Research in Education and the Educational Testing Service. Aubrey's research interests include how to use research and policy to improve early mathematics learning, school readiness, and achievement gap. Currently, she is leading an international team to investigate causes of school success and discrepancies for young children in the USA, Canada, and Taiwan using multivariate analyses.

Dr. James P. Byrnes is Professor of Educational Psychology and Applied Developmental Science at Temple University in Philadelphia, PA, USA. His research focus has been on developing and testing the Opportunity-Propensity Model of achievement in a variety of small-scale and large-scale multivariate studies. Other areas of interest include the importance of conceptual knowledge in acquiring mathematical skills, the information value of neuroscientific findings for psychological theories, and developing a model to explain adolescent and adult risk-taking. He is the proud spouse of Barbara Wasik, an expert in early childhood development, and the proud parent of Tom and Julia.