Fuzzy Linear Elastic Dynamic Analysis of 2-Dimensional Semi-rigid Steel Frame with Fuzzy Fixity Factors

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Abstract. This paper proposes a fuzzy finite element procedure for dynamic analysis of planar steel frame structures with fuzzy input parameters. The fixity factors of beam – column and column – base connections, loads, mass per unit volume and damping ratio are modeled as triangular fuzzy numbers. The Newmark-β numerical integration method is applied to determine the displacement of the linear dynamic equilibrium equation system. The α-level optimization using the Differential Evolution (DE) involving integrated finite element modeling is proposed to apply in the fuzzy structural dynamic analysis. The efficiency of proposed methodology is demonstrated through example problem relating to for the twenty-story, four-bay portal steel frame.

Keywords: Steel frame · Fuzzy connection · Fuzzy structural dynamic · Differential evolution algorithm

1 Introduction

In the dynamic analysis of steel frame structures with semi-rigid connections, rigidity of the connection (or fixity factor of the connection), loads, mass per unit volume, and damping ratio have a significant influence on the time-history response of steel frame structure [3, 8, 10, 11]. In practice, however, many parameters like worker skill, quality of welds, properties of material and type of the connecting elements affect the behavior of a connection, and this fixity factor is difficult to determine exactly. Therefore, in a practical analysis of structures, a systematic approach is needed to include the uncertainty in the joints behavior and the fixity factor of a connection modeled as the fuzzy number is reasonable [7]. In addition, the uncertainty of input parameters such as the external forces, mass per unit volume and damping ratio are also described in form of the fuzzy numbers.

In recent years, the fuzzy static analysis, and the fuzzy stability analysis for planar steel frame structures with the fuzzy connections have been reported [5–7, 14]. However, the fuzzy dynamic analysis for determining the fuzzy time-history response

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by using exact approach has been limited. For the rigid frame, Tuan *et al.* (2015) presented an approach by using Response Surface Method (RSM) for fuzzy free vibration analysis of linear elastic structure in which response surfaces (surrogate functions) in terms of complete quadratic polynomials are presented for model quantities and all fuzzy variables are standardized [13]. The usage of the RSM shows that this approach has effectiveness for the complex structural problems with a large number of fuzzy variables. However, the RSM is only suitable for problems which all fuzzy variables are modeled as symmetric triangular fuzzy numbers. For the problems with non-symmetric triangular fuzzy numbers, the fuzzy structural analysis must use another approach. Anh *et al.* presented an optimization algorithm for fuzzy analysis by combining the Differential Evolution (DE) with the α-level optimization [1]. DE is a global optimization technique, which combines the evolution strategy and the Monte Carlo simulation, and is simple and easy to use [4, 12].

In this paper, the fuzzy displacement - time dependency of planar steel frame structure is determined in which the fixity factor, loads, and mass per unit volume are described in the form of triangular fuzzy numbers. A procedure is based on finite element model by combining the α-level optimization with the Differential Evolution algorithm (DEa). The Newmark-β average acceleration numerical integration method is applied to determine the displacements from the linear dynamic equilibrium equation system of the finite element model. A twenty-floor, four-bay portal steel frame structure is considered. The deterministic results of the proposed algorithms are also compared with ones of the SAP2000 software.

2 Finite Element with Linear Semi-rigid Connection

The linear dynamic equilibrium equation system is given as following

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{P(t)\} \tag{1}$$

where $\{\ddot{u}\}$, $\{\dot{u}\}$, and $\{u\}$ are the vectors of acceleration, velocity, and displacement respectively; $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices respectively; $\{P(t)\}$ is the external load vector. The viscous damping matrix $[C]$ can be defined as

$$[C] = \alpha_M[M] + \beta_K[K] \tag{2}$$

where α_M and β_K are the proportional damping factors which defined as

$$\alpha_M = \xi\frac{2\omega_1\omega_2}{\omega_1+\omega_2};\, \beta_K = \xi\frac{2}{\omega_1+\omega_2} \tag{3}$$

where ξ is the damping ratio; ω_1 and ω_2 are the natural radian frequencies of the first and second modes of the considered frame, respectively.

In this study, a the frame element with linear semi-rigid connection is shown in Fig. 1, with E - the elastic modulus, A - the section area, I - the inertia moment, m - the mass per unit volume, k_1 and k_2 - rotation resistance stiffness at connections.

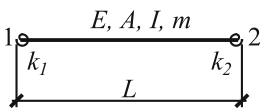

Fig. 1. Frame element with linear semi-rigid connection

The element stiffness matrix - $[K^{el}]$ and the mass matrix - $[M^{el}]$ of the frame are given as following [2]:

$$\left[K^{el}\right] = \begin{bmatrix} \frac{EA}{L} & & & & & \\ 0 & k_{22} & & & symmetric & \\ 0 & k_{32} & k_{33} & & & \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & & \\ 0 & k_{52} & k_{53} & 0 & k_{55} & \\ 0 & k_{62} & k_{63} & 0 & k_{65} & k_{66} \end{bmatrix} \tag{4}$$

where

$$k_{22} = k_{55} = -k_{52} = \frac{12EI}{L^3}\frac{(s_1 + s_2 + s_1 s_2)}{(4 - s_1 s_2)}; k_{32} = -k_{53} = \frac{6EI}{L^2}\frac{s_1(s_2 + 2)}{(4 - s_1 s_2)} \tag{5a}$$

$$k_{33} = 2k_{63} = \frac{12EI}{L}\frac{s_1}{(4 - s_1 s_2)}; k_{62} = -k_{65} = \frac{6EI}{L^2}\frac{s_2(s_1 + 2)}{(4 - s_1 s_2)}, k_{66} = \frac{12EI}{L}\frac{s_2}{(4 - s_1 s_2)} \tag{5b}$$

and

$$\left[M^{el}\right] = \frac{mAL}{420(4 - s_1 s_2)^2}\begin{bmatrix} 140(4 - s_1 s_2)^2 & & & & & \\ 0 & m_{22} & & & symmetric & \\ 0 & m_{32} & m_{33} & & & \\ 70(4 - s_1 s_2)^2 & 0 & 0 & 140(4 - s_1 s_2)^2 & & \\ 0 & m_{52} & m_{53} & 0 & m_{55} & \\ 0 & m_{62} & m_{63} & 0 & m_{65} & m_{66} \end{bmatrix} \tag{6}$$

where

$$\begin{aligned} m_{22} &= 16\left(140 - 49s_2 + 8s_2^2\right) + 4s_1^2\left(32 - 55s_2 + 32s_2^2\right) + 4s_1\left(224 - 328s_2 + 50s_2^2\right); \\ m_{33} &= 4L^2 s_1^2\left(32 - 31s_2 + 8s_2^2\right) \end{aligned} \tag{6a}$$

$$\begin{aligned} m_{55} &= 64\left(35 + 14s_2 + 2s_2^2\right) + 4s_1^2\left(32 - 50s_2 + 32s_2^2\right) - 4s_1\left(196 - 328s_2 + 55s_2^2\right); \\ m_{66} &= 4L^2 s_2^2\left(32 - 31s_2 + 8s_1^2\right) \end{aligned} \tag{6b}$$

$$m_{32} = 2Ls_1\left(32\left(7 - 5s_2 + s_2^2\right) + s_1\left(64 - 86s_2 + 25s_2^2\right)\right) \tag{6c}$$

$$m_{52} = 1120 - 56s_2 - 128s_2^2 + 2s_1\left(-28 - 184s_2 + 5s_2^2\right) + 2s_1^2\left(-64 + 5s_2 + 41s_2^2\right) \tag{6d}$$

$$m_{53} = Ls_1\left(4\left(98 - 25s_2 - 16s_2^2\right) - s_1\left(128 + 38s_2 - 55s_2^2\right)\right);$$
$$m_{63} = L^2 s_1 s_2 (64s_2 - 124 + s_1(64 - 31s_2)) \tag{6e}$$

$$m_{65} = -2Ls_2\left(224 + 64s_2 + s_1^2(32 + 25s_2) - 2s_1(80 + 43s_2)\right) \tag{6f}$$

where $s_i = Lk_i/(3EI + Lk_i)$ denote the fixity factor of semi-rigid connection at the boundaries (i = 1, 2).

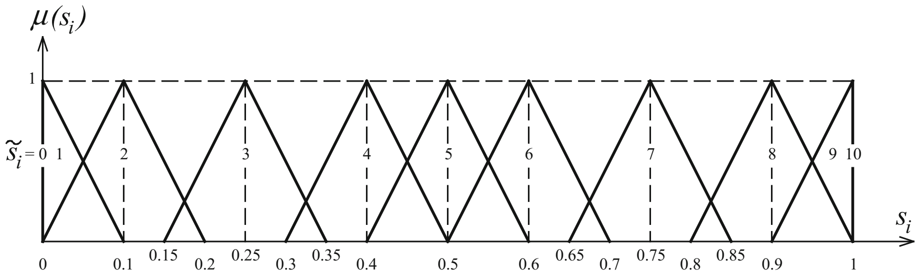

Fig. 2. Membership functions of fuzzy fixity factors.

In Eq. (1), when fixity factors of connections, external loads, mass per unit volume and damping ratio are given by fuzzy numbers, the displacements of joints are also the fuzzy numbers. In steel structures, the common fuzzy connections can be defined by linguistic terms as shown in Fig. 2. Eleven linguistic terms are assigned numbers from 0 to 10 ($\tilde{s}_i = 0, 1, \ldots 10$) [7].

In the classical finite element method (FEM), in Eq. (1), the displacement-time dependency of the joints is determined by solving the linear dynamic equilibrium equation system. The Newmark-β method has been chosen for the numerical integration of this equation system because of its simplicity [9]. The fuzzy displacement is determined by the fuzzy finite element method (FFEM) using the a-cut strategy with the optimization approaches. FFEM is an extension of FEM in the case that the input quantities in the FEM are modeled as fuzzy numbers. In this study, an optimization approach is presented in the next sections: the differential evolution algorithm (DEa).

3 Procedure for Fuzzy Structural Analysis

3.1 Linear Elastic Dynamic Analysis Algorithm

The Newmark-β average acceleration method is based on the solution of an incremental form of the equations of motion. For the equations of motion (1), the incremental equilibrium equation is:

$$[M]\{\Delta\ddot{u}\} + [C]\{\Delta\dot{u}\} + [K]\{\Delta u\} = \{\Delta P\} \tag{7}$$

where $\{\Delta\ddot{u}\}$, $\{\Delta\dot{u}\}$, and $\{\Delta u\}$ are the vectors of incremental acceleration, velocity, and displacement respectively; $\{\Delta P\}$ is the external load increment vector. The displacement of the joint at each time step is determined by this algorithm of linear elastic dynamic analysis.

3.2 α-Level Optimization Using Differential Evolution Algorithm (DEa)

For fuzzy structural analysis, the α-level optimization is known as a general approach in which all the fuzzy inputs are discretized by the intervals that are equal α-levels. The output intervals are then searched by the optimization algorithms. The optimization process is implemented directly by the finite element model and the goal function is evaluated many times in order to reach to an acceptable value. In this study, the output intervals are the displacement intervals at each time step, and the solution procedure is proposed by combining the Differential Evolution algorithm (DEa) with the α-level optimization. DEa which is a population-based optimizer, which is suggested by Storn and Price [12]. The DEa has shown better than the genetic algorithm (GA) and is simple and easy to use.

4 Numerical Illustration

The example is considered by fuzzy elastic dynamic analysis a twenty-story, four-bay linear semi-rigid portal steel frame subjected to fuzzy loads $\tilde{P}(t)$ concentrated at joints as shown in Fig. 3. The elastic modulus is $E = 210 \times 10^6$ kN/m^2, damping ratio $\xi = 0.05$. Fuzzy terms were considered to be triangular fuzzy numbers with 20% absolute spread [15]. The fuzzy mass per unit volume of the columns and the beams are $\tilde{m}_1 = (7.85, 0.785, 0.785)$ and $\tilde{m}_2 = (50, 5, 5)$ (included load dead from slab), respectively. The fuzzy fixity factor at column base is $\tilde{s}_1 = 9$. The fuzzy fixity factor at the ends of beams from story 1 to story 4 is $\tilde{s}_2 = 8$, from story 5 to story 8 is $\tilde{s}_3 = 7$, from story 9 to story 14 is $\tilde{s}_4 = 6$, and from story 9 to story 14 is $\tilde{s}_5 = 5$. The fuzzy loads are: $\tilde{P}(t) = \tilde{P}\sin(\pi t)$ $(0 \le t \le 2$ s), and $\tilde{P}(t) = 0$ ($t > 2$ s), in which $\tilde{P} = (40, 4, 4)$. The section properties used for analysis of the frame are shown in Table 1.

A time step Δt of 0.05 s is chosen in the dynamic analysis. Since the fuzzy fixity factor at column base is the non-symmetric triangular fuzzy number, the fuzzy displacement is determined by using the α-level optimization in combination with the DEa

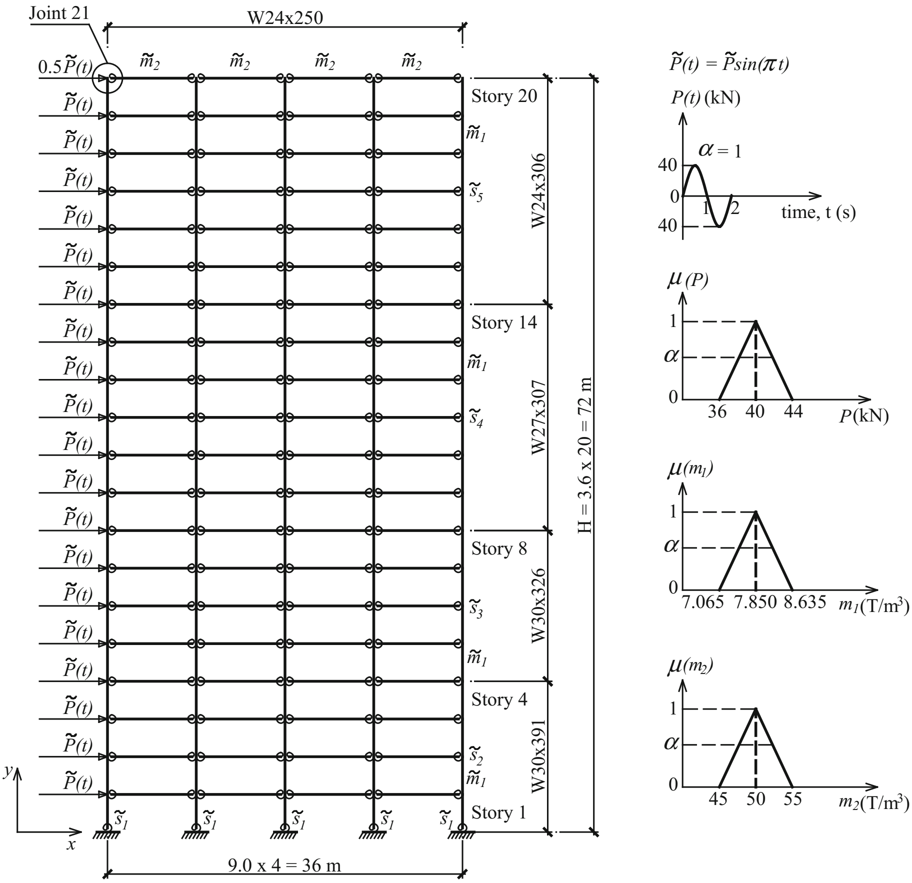

Fig. 3. Portal steel frame with fuzzy input parameters.

Table 1. Section properties used for analysis of the portal steel frame

Member	Section	Cross-section area, A (m^2)	Moment of inertia, I (m^4)
Column (1st to 4th story)	W30x391	7.35E−02	8.616E−03
Column (5th to 8th story)	W30x326	6.17E−02	6.993E−03
Column (9th to 14th story)	W27x307	5.82E−02	5.453E−03
Column (15th to 20th story)	W24x306	5.79E−02	4.454E−03
Beam (1st to 20th story)	W24x250	4.74E−02	3.534E−03

which is programmed in MATLAB. The parameters for DE are: the NP = 50, F = 0.5, C_r = 0.9. The optimization process is stopped after 40 iterations.

The time dependency of the displacement (at the central input values) in x direction at joint 21 of this frame up to t = 20 s is plotted in Fig. 4, and match well with that of the SAP2000 software. With the fuzzy result for the displacement-time dependency up

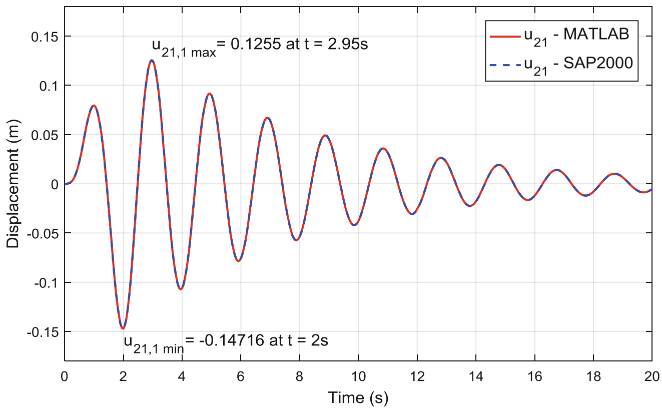

Fig. 4. Time-displacement response at joint 21 in x direction of the twenty-story frame in MATLAB and SAP2000.

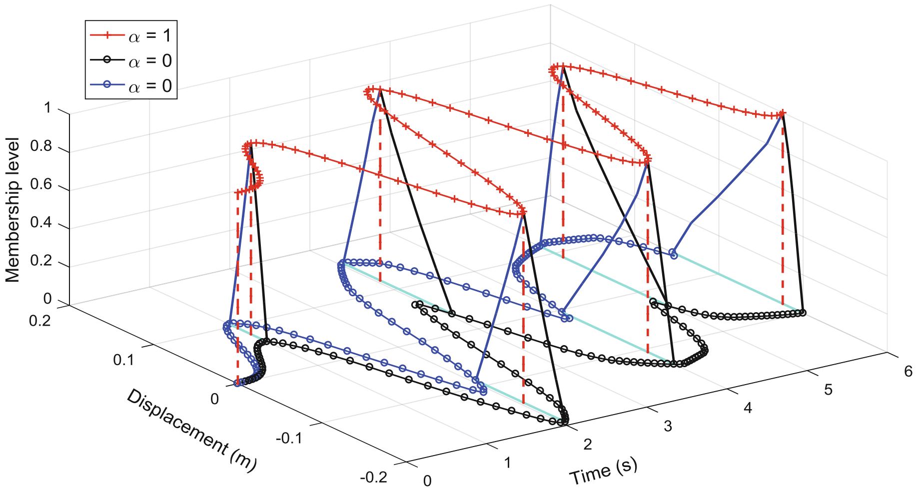

Fig. 5. Fuzzy displacement-time response at joint 21 in x direction of the twenty-story frame.

to t = 6.0 s of the joint 21 in x direction of that frame, Fig. 5 shows the fuzzy displacement-time response and the membership function of fuzzy displacement at different times in 3D-axis; Fig. 6 shows the envelope of fuzzy displacement and the values: $\inf(u_{21,0\ \min}) = -0.1938$ m at $t = 2.05$ s and $\sup(u_{21,0\ \max}) = 0.1651$ m at $t = 3.05$ s; and Fig. 7 shows the membership function of fuzzy displacement and the displacement (at central value) from the SAP2000 software, with t = 1.00, 2.05, 3.05, 4.00, 5.00, and 6.00 s. This Fig. 7 shows a significant difference between the shape of the fuzzy displacement membership functions at the different times.

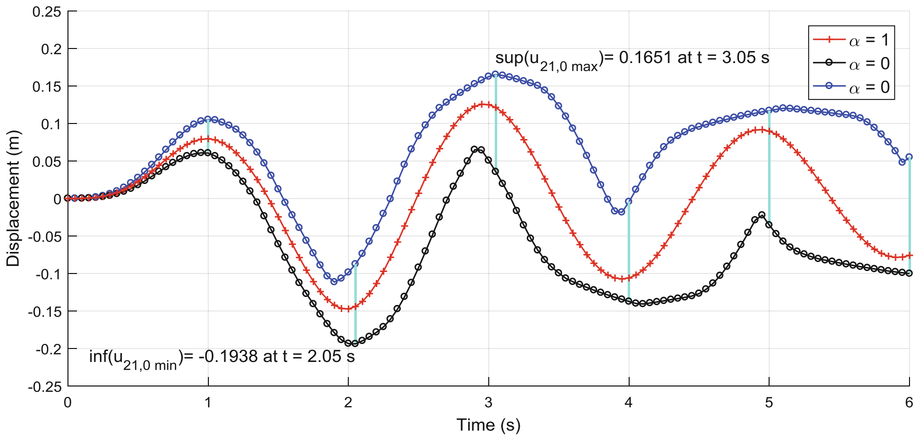

Fig. 6. Envelope of Fuzzy displacement at joint 21 in *x* direction of the twenty-story frame.

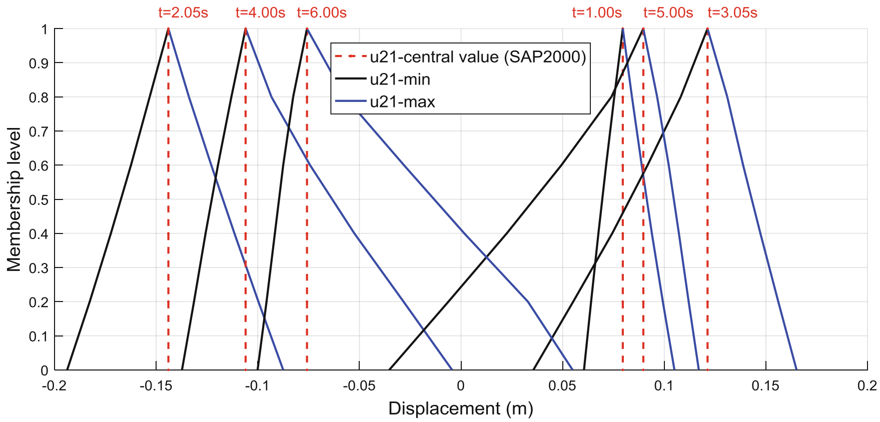

Fig. 7. The membership function of Fuzzy displacement at joint 21 in *x* direction at different times of the twenty-story frame.

5 Conclusions

A fuzzy dynamic analysis procedure is proposed for the linear elastic semi-rigid steel frame dynamic analysis with the input triangular fuzzy numbers. The fuzzy finite element analysis based on the Differential Evolution (DE) in combination with the α-level optimization, in which the Newmark-β average acceleration method is applied to determine the deterministic displacement. The fuzzy input parameters such as fixity factors of connections, external forces, mass per unit volume, and damping ratio have significant influence on the time dependency of the fuzzy displacement. The numerical examples illustrated that this procedure is applied efficiently. The deterministic results are also compared with ones of the SAP2000 software. However, using simple linear elastic semi-rigid connection model is suitable for the structural system assumed that its displacement is small. As the displacement is large, it is necessary to include geometric nonlinearity and fuzzy initial geometric imperfections into advanced fuzzy dynamic analysis of steel frames, this may be subject of studies in the future.

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