A Three-Dimensional Axisymmetric Arc Sliding Method for Checking Basal Heave Stability of Circular Foundation Pits

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Abstract. Nowadays, a large number of foundation pits in circular plane shape have appeared during urban construction in China. In the design process of the circular foundation pits, it is necessary to check their basal heave stability. Based on the circular arc sliding model of limit equilibrium method, a Three-dimensional Axisymmetric Arc Sliding Method (TAASM), in which the stiffness of the enclosure structure and the spatial effects are considered, was proposed. Furthermore, the TAASM was applied to check a practical engineering and its result was compared with those of other methods. The main conclusions are as follows: (a) The TAASM considers not only the effects of the enclosure structure stiffness and deformation but also the spatial effects of circular foundation pits on the sliding surface; (b) Compared with results of other plane algorithm methods, the result of TAASM is reasonable relatively; And (c) By TAASM, the calculated safety factor of basal heave stability for circular foundation pits will be larger than that by any other existing plane algorithm, so the embedded depth of enclosure structure may be optimized to lower the enclosure structure cost.

Keywords: Circular foundation pits · Basal heave stability · Spatial effects · Arc sliding model · Enclosure structure

1 Introduction

Nowadays, a large number of foundation pits in circular plane have appeared during urban construction in China. For example, the foundation pit of Shanghai Tower located in the Shanghai Lujiazui is in circular plane with diameter 123.4 m and depth 31.2 m (Zhai et al. 2010), the foundation pit of Shanghai Expo 500 kV underground substation is in circular plane with diameter 130 m and depth 34.0 m (Weng et al. 2010), and the south anchorage foundation pit of Wuhan Yangluo Changjiang river bridge is in circular plane with diameter 70 m and depth 45 m (Ruan et al. 2005). The circular foundation pits need to be designed for many cylindrical buildings as well, such as a circular revolving sedimentation tank of hot-rolled steel engineering in circular foundation pit with diameter 33 m and depth of 39.3 m (Li et al. 2006), a circular pool in circular foundation pit with diameter 101 m and depth 16 m (Li et al. 2011),

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and a underground cylindrical 3D garage with diameter 22～36 m and depth 24.5 m (Zhang et al. 2016), and so on.

The process of foundation pit design involves the basal heave stability. There are generally two methods for checking the basal heave stability of foundation pit: the Bearing Capacity Theory Method (BCTM) and the Circular Sliding Method (CSM). With regard to BCTM, Terzaghi had previously given formula for the checking calculations based on the bearing capacity theory, but this method is limited to shallow excavation and clay (Terzaghi 1943). Afterwards, based on the Terzaghi method, Bjerrum had also given formula for the checking calculations, but this method is limited to clay (Bjerrum and Eide 1956). With regard to CSM, many researchers studied how to get the reasonable parameters and analyzed the effects of parameters, such as soil parameters, the width of excavation, and the embedded depth of diaphragm wall, on basal heave stability (Hu et al. 2001; Huang et al. 2008; Zheng et al. Zheng and Cheng 2012). Also, both BCTM and CSM belong to the plane algorithm. However, the circular foundation pits are of self-stability performance because it has the spatial effects during excavation.

In the circular foundation pits, the hoop stress was introduced to modify the earth pressure, for example, hoop stress σ_θ was considered as first main stress. The 3D axisymmetric limit equilibrium equations were established by the slip line method, then the equations were solved to obtain the earth pressures (Ъерезанцев 1952). Afterwards, the solved results were taken to check the basal heave stability of circular foundation pit (Wang 2013). Meanwhile, a new method, in which the definition of the basal heave stability of safety is the soil limit state unload quantity and actual unload quantity ratio, was put forward, but its results are conservative (Wang 2013).

Recently, the Shear Strength Reduction Method (SSRM) has been used to check the basal heave stability of circular foundation pits. In this method, elastoplastic constitutive relationship was adopted for the soil, and the basal heave failure curves of circular foundation pit were gotten (Wang and Xu 2010; Kong et al. 2010). Meanwhile, Finite Element Method (FEM) was applied to analyze enclosure structure of small diameter circular foundation pits for their mechanical properties and the effects of the clay and sand on the basal heave deformation (Huang 2013). However, both the SSRM and FEM highly depend on the parameters and the constitutive model, so they will usually not be recommended for checking the basal heave stability in design process.

Although many researchers used plenty of methods to check the basal heave stability, the spatial effects of foundation pit and enclosure structure are often ignored. Especially for the circular foundation pits, the constraints of the adjacent soil are very strong because the hoop stress affects on the soil. So the circular foundation pits have significant spatial effects and they are of self-stability performance. When the excavation depths are the same, the lateral displacements of enclosure structure in circular foundation pit will be smaller than those in rectangular foundation pit. If a plane algorithm is chosen to calculate the basal heave stability of a circular foundation pit, the results of the embedded depth of enclosure structure will be so large that the enclosure structure will cost much more.

In this paper, based on the circular arc sliding model of limit equilibrium method, a Three-dimensional Axisymmetric Arc Sliding Method (TAASM) was proposed to check the basal heave stability of the circular foundation pits. In TAASM, stiffness and deformation of the enclosure structure were considered by assuming parabolic deformation of the enclosure structure and the hoop stress effects on sliding face were considered by the elastic foundation beam method and the circular ring axisymmetric method.

2 Definitions and Assumptions

Here,the basal heave stability safety factor is also defined as the anti-moment divided by and the sliding moment:

$$k_s = \frac{M_r}{M_s} \tag{1}$$

Where, M_r is the anti-moment; M_s is the sliding moment; and k_s is the safety factor of basal heave stability.

The calculating diagram is shown as Fig. 1.

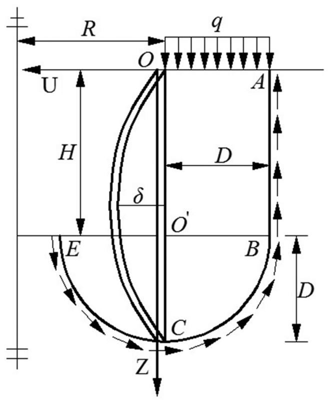

Fig. 1. The calculating diagram

In Fig. 1, H is the depth of excavation; D is the embedded depth of enclosure structure; q is the ground overload; O' is the center of sliding circle; R is the radius of foundation pit; and δ is the maximum lateral displacement of enclosure structure.

The assumptions are made as follows:

(a) Soil slides alone the sliding surface ABCE, on which the shear stress provides the anti-moment;
(b) The constitutive relationship of soil is modeled by Mohr-Coulomb model;
(c) The function $2c\tan(\pi/4 - \varphi/2)$ is ignored in the active earth pressure and the function $2c\tan(\pi/4 + \varphi/2)$ is ignored in the passive earth pressure;
(d) Parabolic deformation is assumed in the enclosure structure;

And (e) The spatial effects on the soil below the bottom are ignored.

According to the assumption (a), the anti-moment is generated by the shear stress on the sliding surface AB, BC, CE and internal support, so

$$M_r = M_1 + M_2 + M_3 + M_h \tag{2}$$

Where, M_1 is anti-moment generated by shear stress on the sliding surface AB;

M_2 is anti-moment generated by shear stress on the sliding surface BC;
M_3 is anti-moment generated by shear stress on the sliding surface CE;
M_h is anti-moment generated by internal support, M_h = 800kN·m for concrete internal support, and M_h = 800kN·m for steel internal support (Wang et al. 2007).

According to the assumption (b), the shear stress can be expressed as:

$$\tau = \sigma \tan \varphi + c \tag{3}$$

Where, σ is the normal stress, τ is the shear stress, c is the cohesive, and φ is the friction angle.

According to the assumption (d), taking the coordinate system in the Fig. 1, the lateral displacement of any point in the enclosure structure can be given by

$$u = -\frac{4\delta}{(D+H)^2} z^2 + \frac{4\delta}{(D+H)} z \tag{4}$$

Where, u and z are the abscissa and the ordinate in UOZ coordinate system, respectively.

3 Formula Derivations

The radial stress and hoop stress generate the shear stress on the sliding surface AB together in the different plane. When the sliding force greater than or equal to the maximum value of the anti-sliding force generated by the radial stress and hoop stress, the sliding body start to slip. And the hoop stress is greater than the radial stress in this condition. So take the anti-moment generated by the hoop stress as the anti-moment generated by sliding surface AB. Furthermore, the hoop stress of hollow cylinder is in Eq. (3). The stress calculation diagram of hollow cylinder is shown as Fig. 2.

Where, R_1 is the internal radius; ρ is the radius of any cross section; R_2 is the external radius; q_1 is the pressure on the internal circular arc; and q_2 is the pressure on the external circular arc.

The stress solution to the hollow cylinder can be obtained through elastic mechanics (Xu 2002) and the hoop stress of hollow cylinder on any cross section is

$$\sigma_\varphi = \frac{\frac{R_2^2}{\rho^2} + 1}{\frac{R_2^2}{R_1^2} - 1} q_1 - \frac{1 + \frac{R_1^2}{\rho^2}}{1 - \frac{R_1^2}{R_2^2}} q_2 \tag{5}$$

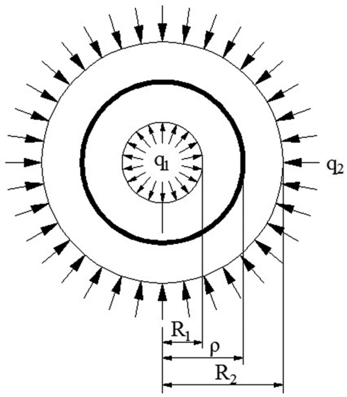

Fig. 2. The stress calculation diagram of hollow cylinder

Where, σ_φ is the hoop stress of hollow cylinder and the tensile stress is stipulated to be positive value.

The effects of circular foundation pit excavation on surrounding soil are limited to some space. If it is assumed that R_2 equals $3R_1$, the pressure on the external circular arc can be considered to be the static earth pressure, i.e.

$$q_2 = k_0 \gamma H \tag{6}$$

Where, k_0 is the coefficient of static lateral earth pressure and here is $k_0 = 1 - \sin\varphi$; γ is the soil unit weight.

Taking the deformation of the enclosure structure into consideration, the pressure q_1 on the internal circular arc can be given by the elastic foundation beam method as follows:

$$q_1 = k_d u \tag{7}$$

$$k_d = \frac{E_d b}{R_0^2} \tag{8}$$

Where, k_d is the equivalent stiffness of the diaphragm wall;

b is the thickness of the diaphragm wall;
R_0 is the radius of the diaphragm wall;
E_d is the circumferential comprehensive compression modulus of the diaphragm wall. Here, $E_d = 0.5 \sim 0.7E$, where E is the elastic modulus of the diaphragm wall and E_d should be taken smaller value when the R_0 is larger or the panel segment of the diaphragm wall is more.

The anti-moment generated on the sliding surface AB can be expressed by the following integral formula:

$$M_1 = \int_0^H \tau D dz \tag{9}$$

Substituting the expressions of Eqs. (3)∼(8) into Eq. (9) and operation leads to

$$\begin{aligned} M_1 = {} & k_1(1-\sin\varphi)(\frac{\gamma H^2}{2}+qH)D\tan\varphi \\ & - k_2[\frac{2\delta}{(D+H)}H^2 - \frac{4\delta}{3(D+H)^2}H^3]Dk_{\mathrm{d}}\tan\varphi + cDH \end{aligned} \tag{10}$$

Where, $k_1 = (\frac{9}{8}+\frac{9R^2}{8(R+D)^2}), k_2 = [\frac{9R^2}{8(R+D)^2}+\frac{1}{8}]$.

The calculating diagram of the anti-moment generated on the sliding surface BC and CE are shown as Fig. 3, respectively.

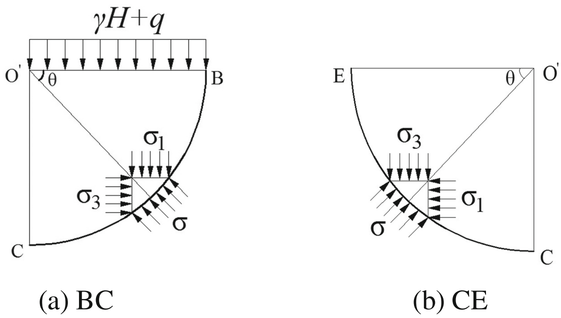

Fig. 3. The calculating diagram of anti-moment on sliding surface

The pressure on the sliding surface BC is considered to be active earth pressure, so the normal stress on the sliding surface BC can be expressed as:

$$\begin{cases} \sigma = \sigma_1\sin\theta + \sigma_3\cos\theta \\ \sigma_1 = D\gamma\sin\theta + q + \gamma H \\ \sigma_3 = k_{\mathrm{a}}\sigma_1 \end{cases} \tag{11}$$

k_{a} is the active earth pressure factor, k_{a} = tan2(π/4-φ/2).

Accordingly, the anti-moment M_2 on sliding surface BC can be derived by the following integral calculus:

$$M_2 = \int_0^{\frac{\pi}{2}} \tau D^2 d\theta = [(\frac{\pi}{4}D\gamma + q + \gamma H) + (\frac{D\gamma}{2} + q + \gamma H)k_a]D^2\tan\varphi + \frac{\pi}{2}cD^2 \tag{12}$$

The pressure on the sliding surface CE is considered to be passive earth pressure, so the normal stress on the sliding surface CE can be expressed as:

$$\begin{cases} \sigma = \sigma_1 \cos\theta + \sigma_3 \sin\ \theta \\ \sigma_1 = k_p \sigma_3 \\ \sigma_3 = D\gamma \sin\theta \end{cases} \tag{13}$$

k_p is the passive earth pressure factor, $k_p = \tan^2(\pi/4 + \varphi/2)$.

Similarly, the anti-moment M_3 on sliding surface CE can be derived by the following integral calculus:

$$M_3 = \int_0^{\frac{\pi}{2}} \tau D^2 d\theta = \frac{k_p D^3 \gamma}{2} \tan\varphi + \frac{\pi}{4} D^3 \gamma\, tan\, \varphi + \frac{\pi}{2} cD^2 \tag{14}$$

Where, M_h = 800kN·m for the concrete internal support, and M_h = 800kN·m for the steel internal support.

The sliding moment M_s is generated by sliding body ABO'O in Fig. 1. And the formula of M_s is expressed:

$$M_s = \frac{(\gamma H + q)D^2}{2} \tag{15}$$

4 An Example for Practical Engineering Checking

The basal heave stability of the foundation pit of Shanghai Expo 500 kV Underground Substation (Wang and Xu et al. 2010) will be checked as an example. The pit is in circular plane with diameter 130 m and depth 34.0 m. Its enclosure structure parameters are: thickness of diaphragm wall 1.2 m, embedded depth of diaphragm wall 23.5 m, and the elastic module of concrete 25 GPa. The field geological compositions and their physical and mechanical parameters are shown in Table 1.

Table 1. The geological compositions and their physical and mechanical parameters

Geological compositions	H/m	γ/kN·m^{-3}	c/kPa	φ/°
Silt plain fill	3.1	18.8	10	20
Mucky silty clay	7.0	17.8	11	26
Mucky silty clay	7.0	17.2	12	18
Clay	9.6	18.2	12	21
Silty clay	4.0	19.6	20	22
Sandy silt	6.5	19.1	2	27
Silty sand	8.3	19.3	2	28
Silty clay	27.8	18.5	10	21
Silty clay with silty sand interbed	4.0	19.4	7	24
Medium sand	4.0	18.6	2	30
Coarse sand	27.8	16.7	1	32

The basal heave stability of the pit was analyzed by the TAASM and its result was compared with those by other method from the reference (Wang and Xu 2010), as shown in Table 2.

Table 2. The safety factors of the basal heave stability for the pit by different methods

Method	BCTM	CSM	TAASM	SSRM
Safety factor	2.50	3.32	3.62	4.90

Because the spatial effects on the sliding surface is considered in the TAASM, the safety factor obtained by this method is larger than that by BCTM or by CSM. However, the mutual effects of soil and enclosure structure and the anti-sliding effects of adjacent soil on the sliding soil are ignored, so the safety factor by TAASM is lower than that by SSRM. The SSRM highly depends on parameters and constitutive model, it is a good method for qualitative and comparative analysis, but it will usually be not recommended for checking basal heave stability in design process.

5 Discussions an Conclusions

The new method (TAASM) is based on the circular arc sliding model, which is limited to the case that the ratio of embedded depth to excavation depth is larger than 0.5. When the ratio of embedded depth to excavation depth is smaller than 0.5, further studies which consider spatial effects should be conduct in the future. The main conclusions are as follows: (a) The TAASM combines the stiffness with the deformation of enclosure structure to check basal heave stability of circular foundation pits and it also takes the spatial effects of circular foundation pit into consideration; (b) The calculation result of TAASM is larger than that of any other plane algorithm method, but it is smaller than that of SSRM. Through the checking of an engineering example, it is proved that the result of TAASM is reasonable; and (c) The calculation result of TAASM is larger than that of the CSM. Under the circumstances of circular foundation pits, if basal heave stability safety factors are the same, the embedded depth of enclosure structure may be optimized to lower the cost of enclosure structure.

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