

One Dimensional Consolidation Theory Considering the Strain Rate Effect

Quan Liu, Yue-bao Deng^(✉), Fei Chen, and Sen-jie Tong

Institute of Geotechnical Engineering, Ningbo University,
Ningbo, Zhejiang, China
dengyuebao@nbu.edu.cn

Abstract. The traditional Terzaghi's one dimensional consolidation theory assumes a linear stress-strain relationship for soil. But in reality, the stress-strain relationship together with the permeability behavior of soft soil is usually nonlinear. Following Davis's infiltration hypothesis, an assumption about the permeability index and the compression index is made. The effect of the strain rate on the compression behavior soft soil is also taken into considering. Based on these assumptions, a new one dimensional consolidation theory considering the strain rate is established. The effective stress-strain-strain rate model is thus set up. The effect of the strain rate on the consolidation process is analyzed. The conclusion shows that the consolidation rate considering the strain rate is faster than it by traditional one-dimensional consolidation method. Also, it found that the higher the strain rate is, the faster the consolidation rate will be.

Keywords: Soft clay · Consolidation · Settlement · Strain rate effect · Analytical solution

1 Introduction

It has been generally accepted that the stress-strain relationship of saturated clay is strongly influenced by the strain rate. As early as 1936, Buisman found the soil deformation would develop slowly over time. In the early 1960s, Crawford (1964) considering the effect of strain rate during consolidation and found the strain rate had a significant effect on the soil deformation. By summing up a large group of one-dimensional compression tests of different types of clay, Leroueil [5, 6] further explored the effect of strain rate and found the pre-consolidation stress σ'_p increased with the strain rate, especially in the double logarithmic coordinates the pre-consolidation stress and strain rate showed a linear relationship. In addition, many triaxial test results show that the strain rate effect can be extended to the whole initial yield surface, the larger the initial yield surface as the strain rate increased [4, 8]. At the same time Leroueil [7] pointed out that in the last 20 years, people had done lot of consolidation tests on natural clay with different strain rates (CRS) (for eastern clay in Canada; Crawford 1963, Vailetal 1979; Leroueil et al. 1983, etc.) All the experiments showed a similar result that the strain rate affects the behaviour of clay [1, 12, 13]. Jarrett (1967) found that for a given strain the higher the strain rate, the higher the effective stress would be. After that, Burghignoli (1979) and Larsson (1981) also

verified the validity of this view according to different types of consolidation tests. Duncan [3] also pointed out that both indoor and field tests indicated that the compression of the clay depended on the change of the strain rate. And the excessive strain rate of indoor tests will prevent the compression process while the relatively low strain rate of site tests will accelerate the compression process.

Many scholars have improved the one-dimensional consolidation theory of Terzaghi, and developed into a new one-dimensional consolidation theory considering the nonlinearity of compression and permeability. For example, based on the hypothesis that the permeability coefficient and the compression coefficient are changed synchronously during the consolidation, Davis et al. [2] (1965) first got the analytical solution of saturated clay under the self-stress was uniform. Barden et al. (1965), Mesri et al. (1974) used the currently accepted $e\text{-log}\sigma'$ and $e\text{-log}k_v$ linear models, obtained the corresponding compression curve by using finite difference method. In domestic, Xie Kanghe et al. [9–11] (1996, 2002) based on Davis's assumption, obtained the one-dimensional nonlinear consolidation analysis under the condition of single and double layers foundation.

In summary, many scholars have studied the impact of the strain rate, but quantitative study of the impact of strain rate is still blank, and how to introduce strain rate factor into consolidation theory is still important. Based on available cases this paper proposed a new consolidation model with considering of stress rate. Finally numerical analysis is used to verify the correctness of the model.

2 Constitutive Model of Soil

Leroueil (2006) summed up the behavior of soft soil. Professor Šuklje (1957) presented a test result about soil samples, which describes the relation between the pore ratio to the effective stress curve; as Fig. 1 shows.

The abscissa is the effective stress, and the ordinate is the pore ratio. The strain rates in the test are between 10^{-4} 1/s to 10^{-13} 1/s. It can be seen from the figure that the compression behaviour of soil is corresponding to the strain rate. A new constitutive model of soft soil considering the strain rate can be gained according to fit these measured points. Figure 2 shows the part of measured value and fitting results using the new expressions, i.e., Eq. (1).

The relationship between the fitting curve and the measured point is shown in Fig. 2. The different curves are the stress-strain relation at different strain rates. When the abscissa is 0, the intersection with the ordinate is 0.98. It thus can be gained that the initial porosity ratio $e_0 = 0.98$.

The strain rate in Fig. 1 is 10^{-4} 1/s. It can be expressed as the following equation:

$$e - 0.98 = -0.1223 \lg \sigma' \quad (1)$$

In Eq. 1, the initial pore ratio is 0.98. It can be referred from Eq. (1) that the compression index a is corresponding to the strain rate. Figure 3 shows the fitting results between compression index a and strain rate.

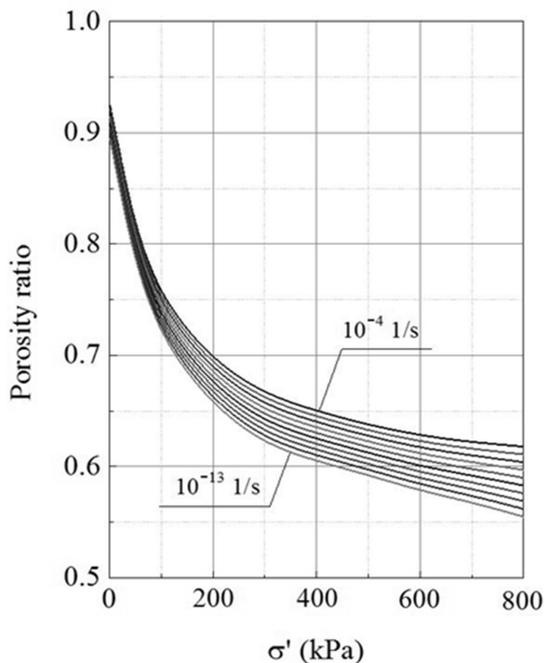


Fig. 1. Compression test result (from Šuklje 1957)

According to Fig. 3, it can be gained that

$$a = A + \alpha \lg \dot{\epsilon} \quad (2)$$

Parameters A and α are the values relate the type of soil. According to the test result of Šuklje (1957), there are $A = 0.114$, and $\alpha = -0.002$.

Substituting Eq. (2) to Eq. (1), then

$$e - 0.98 = -(A + \alpha \lg \dot{\epsilon}) \cdot \lg \sigma' \quad (3)$$

Equation (3) is gained from test result of Šuklje (1957). Extending this equation to the general soft soil, it can be obtained that

$$e = e_0 - (A + \alpha \lg \dot{\epsilon}) \cdot \lg (\sigma' / \sigma'_0) \quad (4)$$

where e_0 is initial porosity ratio, σ'_0 is the initial effective stress. Equation (4) is the proposed constitute model for soft soil considering the strain rate effect in the present paper.

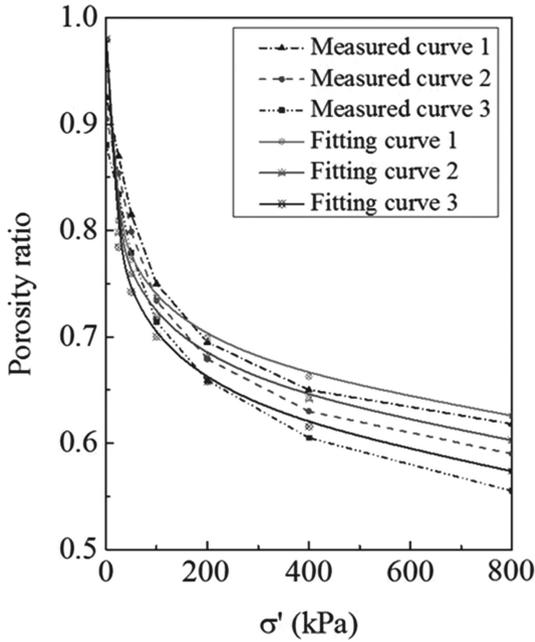


Fig. 2. Measured and fitting result for the compression test result from Šuklje (1957)

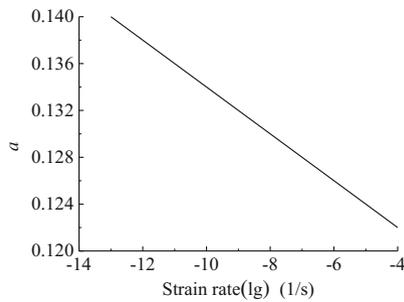


Fig. 3. The relationship between strain rate and slope

3 Consolidation Theory Considering Strain Rate

3.1 Basic Assumption

In order to solve the deformation of saturated soil during infiltration consolidation, it is usually calculated by the one-dimensional consolidation theory proposed by Terzaghi. Here, the basic assumptions are that: (1) the load area is much larger than the thickness of the compressible soil; (2) the pore water in the foundation mainly flows along the vertical direction.

On the other hand, it has been found that the compressive modulus and the permeability coefficient change with time. It means the soil has a non-linear characteristic during the consolidation process. On the basis of the existing research, a new one-dimensional nonlinear consolidation equation is established by using the newly accepted $e \sim \log \sigma'$ value and $e \sim \log k_v$ models:

$$e - e_0 = C_c \log(\sigma'_0/\sigma') \quad (5a)$$

$$e - e_0 = C_k \log(k_v/k_{v0}) \quad (5b)$$

Where C_k is permeability index, k_{v0} is initial permeability coefficient. According to the Eqs. (5a) and (5b), there is

$$k_v = k_{v0} (\sigma'_0/\sigma')^{C_c/C_k} \quad (6a)$$

It can be deduced if $C_c/C_k = 1$ that

$$k_v = k_{v0} \frac{\sigma'_0}{\sigma'} \quad (6b)$$

Therefore, the basic assumption of the one-dimensional consolidation theory considering the strain rate is as follows: (1) the soil is homogeneous, isotropic and fully saturated; (2) soil particles and pore water are incompressible; (3) the additional stress in the soil along the horizontal plane is infinite distribution, so the soil deformation and water flow are vertical; (4) the seepage of water in the soil is subject to Darcy's law; (5) The consolidation deformation is small, that is, the deformation is completely caused by the dissipation of excess pore water pressure in the soil; (6) the permeability coefficient k_v of the soil is changed during the consolidation process; (7) the compression index C_c is change during the consolidation process.

For the $e \sim \log \sigma'$ value model, the slope of the line in Fig. 3 is the compression index C_c . The compression index is varied with strain rate. However, it can be seen from Table 1 that the variation range is small. Thus, in the following deducing, value C_c is fixed as a constant.

3.2 Determination of the Varied Consolidation Coefficient

The consolidation coefficient is calculated as

$$c_v = \frac{k_v}{m_v \gamma_w} \quad (7a)$$

where m_v is volume compression coefficient. Here it should be noted that m_v is a changed value; it can be deduced from the following equation.

$$m_v = -\frac{1}{1+e_1} \cdot \frac{\partial e}{\partial \sigma'} = \frac{0.434C_c}{(1+e_1)\sigma'} \quad (7b)$$

Where e_1 is porosity ratio under natural condition. Combining Eqs. (7a) and (7b), there is

$$c_v = \frac{k_v}{m_v \gamma_w} = \frac{k_{v0} \sigma'_0 (1+e_1)}{0.434 \gamma_w} \cdot \frac{1}{C_c} \quad (7c)$$

It should be noted that c_v in Eq. (7c) is varied with permeability coefficient, porosity ratio and strain rate. Equation (7c) can be reformed as

$$c_v = \frac{C}{C_c} \quad (7d)$$

Where

$$C = \frac{k_{v0} \sigma'_0 (1+e_1)}{0.434 \gamma_w} \quad (7e)$$

3.3 Controlling Equation

According the Darcy's law, the average seepage velocity of any section is

$$v = k_v i = -\frac{k_v}{\gamma_w} \cdot \frac{\partial u}{\partial z} \quad (8)$$

then

$$\frac{\partial v}{\partial z} dz = \frac{\partial}{\partial z} \left(-\frac{k_v}{\gamma_w} \cdot \frac{\partial u}{\partial z} \right) dz \quad (9)$$

Substituting Eqs. (5b) and (7c) to the above equation, there is

$$\frac{\partial v}{\partial z} dz = -B c_v C_c \left[\frac{1}{\sigma'} \cdot \frac{\partial^2 u}{\partial z^2} - \frac{1}{\sigma'^2} \frac{\partial u}{\partial z} \cdot \frac{\partial \sigma'}{\partial z} \right] dz \quad (10)$$

where $B = 0.434/(1+e_1)$.

According to the basic assumption (1), there is $V_v = V_w$, then

$$\frac{\partial V_w}{\partial t} = -\frac{\partial}{\partial t} \left(\frac{e}{1+e_1} dx dy dz \right) \quad (11)$$

Further, based on the relationship between strain and stress, there is

$$\frac{\partial e}{\partial t} = -\frac{0.434C_c}{\sigma'} \cdot \frac{\partial \sigma'}{\partial t} \quad (12)$$

Substituting Eq. (12) to Eq. (11), there is

$$\frac{\partial V_w}{\partial t} = \frac{0.434C_c}{(1+e_1)\sigma'} \cdot \frac{\partial \sigma'}{\partial t} dx dy dz = \frac{BC_c}{\sigma'} \cdot \frac{\partial \sigma'}{\partial t} dx dy dz \quad (13)$$

According to continuous conditions, that the reduction in pore volume in the soil unit should be equal to the amount of water flowing out of the unit in the same period, then

$$\frac{\partial V_w}{\partial t} dt = \frac{\partial q}{\partial z} dz dt \quad (14a)$$

Because of $q = v dx dy$, it can be further deduced that

$$\frac{\partial V_w}{\partial t} dt = \frac{\partial v}{\partial z} dz dx dy dt \quad (14b)$$

Combining Eqs. (13) and (14b), then

$$\frac{BC_c}{\sigma'} \cdot \frac{\partial \sigma'}{\partial t} = \frac{\partial v}{\partial z} \quad (15)$$

Substituting Eq. (15) to Eq. (10), there is

$$\frac{1}{\sigma'} \cdot \frac{\partial \sigma'}{\partial t} = -c_v \left[\frac{1}{\sigma'} \cdot \frac{\partial^2 u}{\partial z^2} - \frac{1}{\sigma'^2} \frac{\partial u}{\partial z} \cdot \frac{\partial \sigma'}{\partial z} \right] \quad (16)$$

According to the principle of effective stress, $\sigma' = \sigma - u$, then

$$\frac{1}{\sigma'} \cdot \frac{\partial \sigma'}{\partial t} = -\frac{C}{C_c} \left[\frac{1}{\sigma'} \cdot \frac{\partial^2 u}{\partial z^2} + \frac{1}{\sigma'^2} \left(\frac{\partial u}{\partial z} \right)^2 \right] \quad (17)$$

The above equation is the one-dimensional consolidation differential equation considering the strain rate for the saturated soil.

4 Analytical Solutions

It can be seen from Eq. (17) that the newly present consolidation equation is a non-linear equation. Here, we make the following assumption for further deducing.

$$\omega = \ln \frac{\sigma'}{\sigma'_f} \quad (18a)$$

where σ'_f is the final effective stress. Then, there are

$$\frac{\partial \omega}{\partial t} = \frac{1}{\sigma'} \frac{\partial \sigma'}{\partial t} \quad (18b)$$

$$\frac{\partial \omega}{\partial z} = \frac{1}{\sigma'} \frac{\partial \sigma'}{\partial z} \quad (18c)$$

$$\frac{\partial^2 \omega}{\partial z^2} = -\frac{1}{\sigma'^2} \cdot \left(\frac{\partial \sigma'}{\partial z} \right)^2 + \frac{1}{\sigma'} \cdot \frac{\partial^2 \sigma'}{\partial z^2} \quad (18d)$$

Substituting the above equations into Eq. (17), and combining the principle of effective stress, there is

$$\frac{C}{C_c} \cdot \frac{\partial^2 \omega}{\partial z^2} = \frac{\partial \omega}{\partial t} \quad (19)$$

To solve the above equation, the initial conditions and boundary conditions are needed. According to the classical Terzaghi's one dimensional consolidation theory, there are:

The initial condition: $t = 0$, $u = p_0$; thus $\omega = \ln(1-p_0/\sigma'_f) = \ln(\sigma'_0/\sigma'_f)$;

The boundary conditions:

- (1) permeable boundary for the up boundary, that is $z = 0$, $u = 0$, thus $\omega = 0$;
- (2) impermeable boundary for the base boundary, that $z = H$, $\partial u/\partial z = 0$, and $\partial \omega/\partial z = 0$;

Additionally, when t tends to infinity, there is $u = 0$ and $\omega = 0$.

Combining the above equation and conditions, there is

$$\begin{cases} \frac{\partial \omega}{\partial t} = \frac{C}{C_c} \cdot \frac{\partial^2 \omega}{\partial z^2} \\ \omega(0, t) = 0, \frac{\partial \omega}{\partial z} \Big|_{z=H} = 0 & (0 \leq z \leq H) \\ \omega(z, 0) = \ln \frac{\sigma'_f - p_0}{\sigma'_f} \end{cases} \quad (20)$$

Equation (20) can be solved by the separation variable method. Here, it is assumed that:

$$\omega(z, t) = X(z)T(t) \quad (21)$$

Then, Eq. (20) can be reformed to the following two equations:

$$\begin{aligned} X''(z) + \lambda X(z) &= 0 \\ T'(t) + \lambda \frac{C}{C_c} T(t) &= 0 \end{aligned} \quad (22)$$

Firstly, function $X(z)$ can be solved as:

$$\begin{cases} X''(z) + \lambda X(z) = 0 \\ X(0) = X_z(H) = 0 \end{cases} \quad (23)$$

The following three cases λ about will be analyzed:

- (1) if $\lambda < 0$, nontrivial solution cannot be gained for Eq. (23). In fact, it is known from the ordinary differential equation that the general solution of the equation is

$$X(z) = Ae^{\sqrt{-\lambda}z} + Be^{-\sqrt{-\lambda}z} \quad (24a)$$

It can be gained by the boundary condition ($A = B = 0$) that $X(z)$ will be fixed as zero.

- (2) if $\lambda = 0$, it also cannot be gained a nontrivial solution. The general solution for this condition is:

$$X(z) = Az + B \quad (24b)$$

According to the boundary conditions (i.e., $A = B = 0$), it can be deduced that $X(z)$ will be fixed as zero.

- (3) if $\lambda > 0$, the general solution is

$$X(z) = A \cos \sqrt{\lambda}z + B \sin \sqrt{\lambda}z \quad (24c)$$

According to the boundary conditions, there is

$$X(0) = A = 0, X_z(H) = B\sqrt{\lambda} \cos \sqrt{\lambda}H = 0 \quad (24d)$$

It can thus be gained that

$$\lambda = \lambda_n = \left(\frac{n\pi}{2H} \right)^2 \quad (25)$$

where $n = 1, 3, 5, \dots$. Thus, a family of non-zero solution can be gained as

$$X_n(z) = \sin \frac{n\pi z}{2H} \quad (n = 1, 3, 5, \dots) \tag{26}$$

Substituting the above equations to the Eq. (22) about $T(t)$, then

$$T' = -\left(\frac{n\pi}{2H}\right)^2 \frac{C}{C_c} T \tag{27a}$$

The general solution for this equation is

$$T_n(t) = C_n e^{-\left(\frac{n^2\pi^2 C t}{4H^2 C_c}\right)} \quad (n = 1, 3, 5, \dots) \tag{27b}$$

then

$$\omega(z, t) = \sum_{n=1}^{\infty} X_n(z) T_n(t) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi z}{2H} \cdot e^{-\left(\frac{n^2\pi^2 C t}{4H^2 C_c}\right)} \quad (n = 1, 3, 5, \dots) \tag{28a}$$

Where

$$C_n = \frac{2}{H} \int_0^H \phi(t) \sin \frac{n\pi z}{2H} dz = \frac{2}{H} \cdot \ln \frac{\sigma'_0}{\sigma'_f} \cdot \int_0^H \sin \frac{n\pi z}{2H} dz = \frac{4}{n\pi} \cdot \ln \frac{\sigma'_0}{\sigma'_f} \tag{28b}$$

Suming up the above equations and substituting them into Eq. (21), there is

$$\omega = \ln \frac{\sigma'_0}{\sigma'_f} \cdot \sum_{m=0}^{\infty} \left(\frac{2}{M} \sin \frac{Mz}{H}\right) \exp\left(-\frac{M^2 C}{H^2 C_c} t\right) \quad (m = 0, 1, 2, \dots) \tag{28c}$$

Where $m = (n-1)/2$, $M = \pi(2m + 1)/2$.

Substituting Eq. (18a) into Eq. (28c), there is

$$\frac{\sigma'_f - u}{\sigma'_f} = \left(\frac{\sigma'_0}{\sigma'_f}\right) \sum_{m=0}^{\infty} \left(\frac{2}{M} \sin \frac{Mz}{H}\right) \exp\left(-\frac{M^2 C t}{H^2 C_c}\right) \tag{29}$$

$$u = \sigma'_f \left(1 - \left(\frac{\sigma'_0}{\sigma'_f}\right) \sum_{m=0}^{\infty} \left(\frac{2}{M} \sin \frac{Mz}{H}\right) \exp\left(-\frac{M^2 C t}{H^2 C_c}\right)\right) \tag{30a}$$

Lastly, it can be reformed as

$$u = \sigma'_f \left(1 - \left(\frac{\sigma'_0}{\sigma'_f} \right) \sum_{m=0}^{\infty} \left(\frac{2}{M} \sin \frac{Mz}{H} \right) \exp \left(- \frac{M^2 C}{H^2 (A + \alpha \lg \dot{\epsilon})} t \right) \right) \quad (30b)$$

This is the one-dimensional consolidation solution considering the strain rate about the excess pore water pressure.

Additionally, the average degree of consolidation can be calculated as:

$$U = \frac{\int_0^H (e_0 - e) dz}{\int_0^H (e_0 - e_f) dz} \quad (31a)$$

Where e_0 is the initial Porosity ratio; e_f is the final porosity ratio. According to the relationship of porosity ratio and effective stress, the following equation can be gained from Eq. (31b).

$$U = \frac{\int_0^H \left(\lg \frac{\sigma'_f - u}{\sigma'_0} \right) dz}{\int_0^H \left(\lg \frac{\sigma'_f}{\sigma'_0} \right) dz} \quad (31b)$$

Substituting Eq. (30b) into Eq. (31b), then

$$\begin{aligned} U &= \frac{\int_0^H \left(\lg \frac{\sigma'_f}{\sigma'_0} \cdot \left(\frac{\sigma'_0}{\sigma'_f} \right) \sum_{m=0}^{\infty} \left(\frac{2}{M} \sin \frac{Mz}{H} \right) \exp \left(- \frac{M^2 C}{H^2 (A + \alpha \lg \dot{\epsilon})} t \right) \right) dz}{\int_0^H \left(\lg \frac{\sigma'_f}{\sigma'_0} \right) dz} \\ &= 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp \left(- \frac{M^2 C}{H^2 (A + \alpha \lg \dot{\epsilon})} t \right) \end{aligned} \quad (31c)$$

Finally, the average degree of consolidation considering strain rate effect is gained as

$$U = 1 - \sum_{m=0}^{\infty} \frac{2}{M^2} \exp \left(- \frac{M^2 C}{H^2 (A + \alpha \lg \dot{\epsilon})} t \right) \quad (31d)$$

5 Computation and Analysis

Comparison between the classical Terzaghi's consolidation theory and the newly present theory is made in the following. In the computation, the constitute model of soil is gained from the test result by Šuklje (1957). Other conditions are listed here:

$H = 1 \text{ m}$; $k_{v0} = 10^{-8} \text{ m/s}$; $\sigma'_{0} = 1 \text{ kPa}$; $\sigma'_{f} = 101 \text{ kPa}$; $e_{1} = 1.2$. The strain rate is assumed changed from 10^{-4} 1/s to 10^{-13} 1/s .

Figure 4 shows the pore water pressure at depth $z = 0.8 \text{ m}$. Figure 4(a) is the result by Terzaghi's consolidation theory, while Fig. 4 (b) is the result by the present method. It can be seen from the two figures that (1) under the same total stress, initial permeability coefficient and natural porosity ratio, the pore water pressure dissipation rate considering the strain rate is faster than the dissipation rate by the classical one-dimensional consolidation theory; (2) the dissipation for the pore water pressure at depth $z = 0.8 \text{ m}$ is about 2.5 years when considering the strain rate, while it needs 4.4 years for the same condition by the classical method; (3) the higher the strain rate is, the faster the pore water pressure dissipates; (4) a strain rate of 10^{-4} is about 0.3 years faster than it under strain rate 10^{-13} for 90% dissipation of pore water pressure.

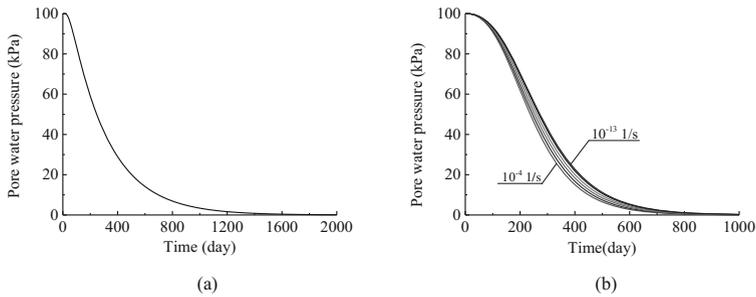


Fig. 4. The relationship between pore water pressure and time at $z = 0.8 \text{ m}$ (a) by Terzaghi's classical consolidation theory; (b) by the present method

Figure 5 shows the average degree of consolidation with time. Figure 5(a) is the result by Terzaghi's consolidation theory; Fig. 5 (b) is the result by the present method. It can be seen from the two figures that (1) under the same permeability coefficient and natural porosity ratio, the consolidation time considering the strain rate is shorter than the consolidation time by the classical one-dimensional consolidation theory; (2) the

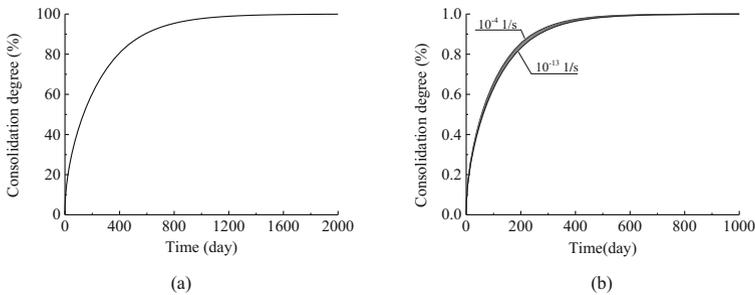


Fig. 5. The relationship between average degree of consolidation and time (a) by Terzaghi's classical consolidation theory; (b) by the present method

time to 99% of degree of consolidation is 1.9 years for the present method, while it is 4.1 years for the same condition by Terzaghi's method; (3) the higher the strain rate is, the faster the consolidation rate will be; (4) comparing to the consolidation under 10^{-3} 1/s, it is 0.16 years faster to reach 90% of degree of consolidation for 10^{-4} 1/s strain rate condition.

6 Conclusions

- (1) Under the same conditions, the dissipation rate of pore water pressure considering the strain rate effect is faster than that with an average constant strain rate by classical consolidation theory.
- (2) The greater the strain rate is, the faster the dissipation of pore water pressure will be.
- (3) Under the same conditions, the consolidation rate considering the strain rate by the newly proposed method is faster than that by the classical one-dimensional consolidation theory.
- (4) The greater the strain rate is, the faster the consolidation rate will be.

References

1. Cheng, C.M., Yin, J.H.: Strain-rate dependent stress-strain behavior of undisturbed Hong Kong Marine deposits under oedometric and triaxial stress states. *Mar. Georesources Geotechnol.* **23**, 61–92 (2005)
2. Davis, E.H., Raymond, G.P.: A nonlinear theory of consolidation. *Geotechnique* **15**(2), 161–173 (1965)
3. Duncan, J.M.: Limitations of conventional analysis of consolidation settlement. *J. Geotech. Eng.* **119**, 1333–1359 (1993)
4. Gao, Y.B., Wang, Z.W.: Effect of strain rate on undrained shear strength of clays. *Chin. J. Rock Mech. Eng.* **24**(2), 5779–5783 (2005). (In Chinese)
5. Graham, J., Crooks, J.H.A., Bell, A.L.: Time effects on the stress - strain behaviour of natural soft clays. *Geotechnique* **33**(3), 327–340 (1983)
6. Leroueil, S., Kabbaj, M., Tavenas, F., Bouchard, R.: Stress-strain-strain rate relation for the compressibility of sensitive natural clays. *Geotechnique* **35**(2), 159–180 (1985)
7. Leroueil, S.: Recent developments in consolidation of natural clays. *Can. Geotech. J.* **25**, 85–107 (1988)
8. Leroueil, S., Marques, M.E.S.: Importance of strain rate and temperature effects in geotechnical engineering. In: Sheaharl, T.C. (ed.) *Measuring and Modeling Time Dependent Soil Behavior*, No. 61, pp. 1–60. Geotechnical Special Publication, New York (1996)
9. Xie, K.H., Pan, Q.Y.: One-dimensional consolidation theory of arbitrary layers under time-dependent loading. *Chin. J. Geotech. Eng.* **17**(5), 80–85 (1995). (In Chinese)
10. Xie, K.H., Li, B.H., Li, Q.L.: A nonlinear theory of consolidation under time-dependent loading. In: *Proceedings of the 2nd International Conference on Soft Soil Engineering*, [s.l.]: [s. n.], pp. 193–198 (1996)

11. Xie, K.H., Zheng, H., Li, B.H., et al.: Analysis of one-dimensional nonlinear consolidation of layered soils under time-dependent loading. *J. Zhejiang Univ.* **37**(4), 426–431 (2003). (In Chinese)
12. Yin, J.H., Cheng, C.M.: Comparison of strain-rate dependent stress-strain behavior from K_0 -consolidated compression and extension tests on natural Hong Kong marine deposits. *Mar. Georesources Geotech.* **24**, 119–147 (2006)
13. Zhu, J.G., Yin, J.H.: Strain-rate dependent stress-strain behaviour of over consolidated Hong Kong marine clay. *Can. Geotech. J.* **37**(6), 1272–1282 (2000)