

Evaluation of Performance of Walls in Deep Excavations Using Wall Deflection Paths

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Abstract. This paper discusses the concept of wall deflection path and reference envelope and illustrates its applications on evaluating the performance of diaphragm walls in deep excavations carried out in thick soft deposits. Numerical analyses were performed for a cut-and-cover section of the Taipei Metro to verify this concept and to quantify the influences of four factors, i.e., the preloading of struts, wall thickness, wall length, and width of excavation on wall deflections. It has been found that the width of excavation is the most influential factor on maximum wall deflections while the thickness and the length of walls have dominating influence on toe stability.

Keywords: Taipei Metro · Underground construction · Deep excavation · Wall deflection path · Reference envelope

1 Introduction

Structures adjacent to deep excavations were frequently damaged to a certain degree as ground moved, or even collapsed in disastrous events. Serious failures of ground may cause traffic congestion or disrupt the function of lifelines, leading to chaos and panics of people, and may lead to injunction of the constructions. Therefore, it is vital to take precautions whenever a deep excavation is carried out in the neighbourhood of sensitive structures, vital utilities, and trunk roads. This is particularly true for metro constructions which are normally carried out in densely populated city areas with heavy traffic. On the other hand, the performance of the retaining system of a deep excavation may be affected by the structures in the vicinity. For example, the basements of highrise buildings together with retaining structures left in-place after the completion of construction are very likely to reduce wall deflections. For metro constructions, there are always entrances, ventilation shafts, etc., structurally connected to underground stations and the rigidity of the diaphragm walls is much increased due to the irregularity of the walls, and the wall deflections are much reduced as a result.

Ground settlement, which is one of the primary factors affecting the structures adjacent to excavations, is closely related to wall deflections. Wall deflections thus become the most important subject in evaluating the performance of diaphragm walls in deep excavations and assessing the potential risks of damaging the structures in vicinity. Conventionally numerical analyses are conducted by using two-dimensional models to evaluate the performance of retaining structures, i.e., walls and struts. Since the existing underground structures adjacent to excavations are normally omitted in

these analyses, it is unrealistic to compare the results obtained with the observed performance of walls. The conclusions made in such a way may be misleading. For these reasons, it is desirable to have a means to quantify the influence of adjacent structures and many other factors which may affect wall deflections, so the performance of walls can be faithfully evaluated.

Discussed herein is the concept of wall deflection path and reference envelope firstly suggested in Moh and Hwang (2005) and Hwang et al. (2006). Applications of this concept in evaluating the performance of diaphragm walls in the Taipei Basin were later illustrated in Hwang and Moh (2007, 2008) and Hwang et al. (2007a, 2016). Numerical analyses have been subsequently conducted by Hsiung and Hwang (2009); Chao et al. (2010a, b); Hwang et al. (2012) and Hwang and Moh (2017) to substantiate the concept and to study the sensitivity of various factors on wall deflections.

This study is an extension of the above-mentioned studies aiming at generalization of the conclusions reached therein by providing data obtained from additional numerical analyses.

2 Case Studied

Wall deflections are routinely monitored by using inclinometers. The readings obtained are inevitably affected by the movements at the tips which are assumed to be fixed and wall deflections at other depths are calculated accordingly. Moh and Hwang (2005) and Hwang et al. (2007b) recommended to calibrate inclinometer readings to account for the movements at the inclinometer tips by assuming that the joints between the struts at the first level and the diaphragm walls would not move once these struts were preloaded. This

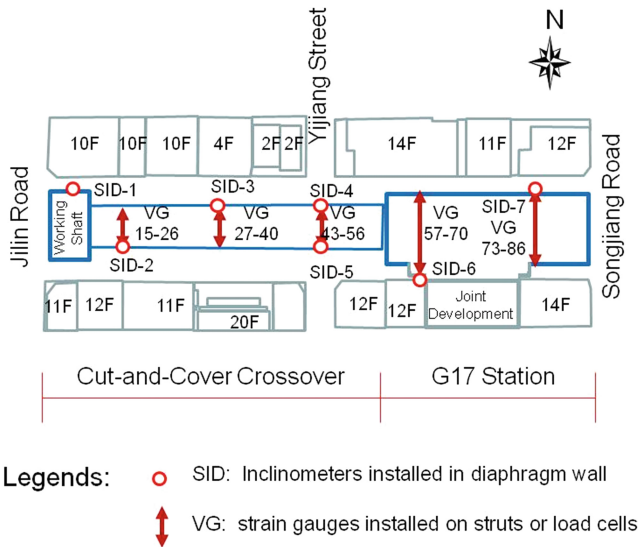


Fig. 1. Layout of G17 Station of the Green Line of Taipei Metro and the adjoining cut-and-cover crossover

recommendation was based on the finding that the changes in the lengths of these struts would be minimal as the load increments and/or decrements in the struts at the 1st level would be very small. To illustrate this point, the readings obtained in the cut-and-cover construction for the crossover, refer to Fig. 1, next to G17 Station (Songjiang Nanjing Station) of Line 3 (the Green Line) of Taipei Metro are discussed herein. The excavation was carried out to depths varying from 17.7 m to 20.2 m below the ground level in 7 stages and the pit was retained by diaphragm walls of 1 m in thickness installed to a depth of 35 m and braced by steel struts at 6 levels as depicted in Fig. 2.

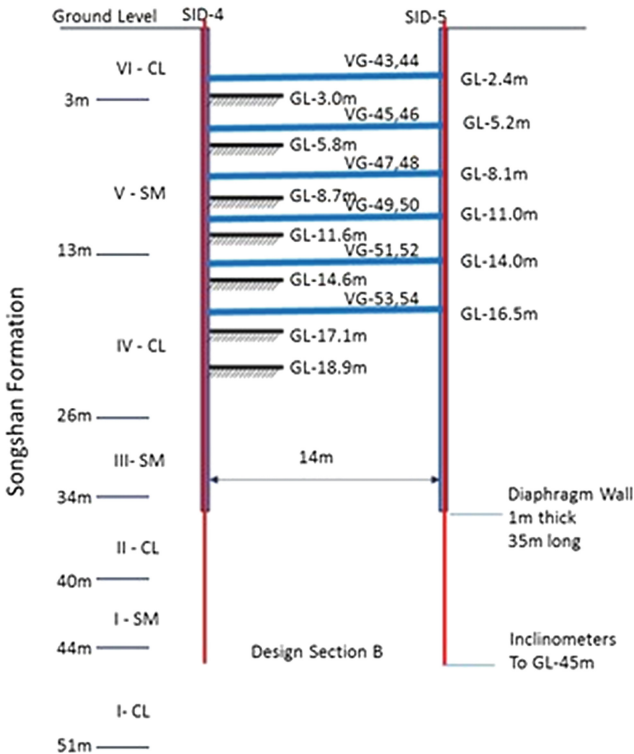


Fig. 2. Excavation scheme for the crossover next to G17 Station of Taipei Metro

3 Correction of Inclinometer Readings

Figure 3 shows the strut loads recorded by two strain gauges, namely, VG-43 and VG-44, installed on the strut at the 1st level between Inclinometers SID-4 and SID-5 which were installed in the diaphragm walls and extended by 10 m below the toes of walls. As can be noted, the strut was preloaded to 71 tones, which is the average of the readings of the two gauges, at the beginning. The load in the strut increased to an average of 87 tons in the second stage of excavation and dropped to a minimum of -3 tons in the subsequent stages. For a strut length of 14 m and a Young's Modulus,

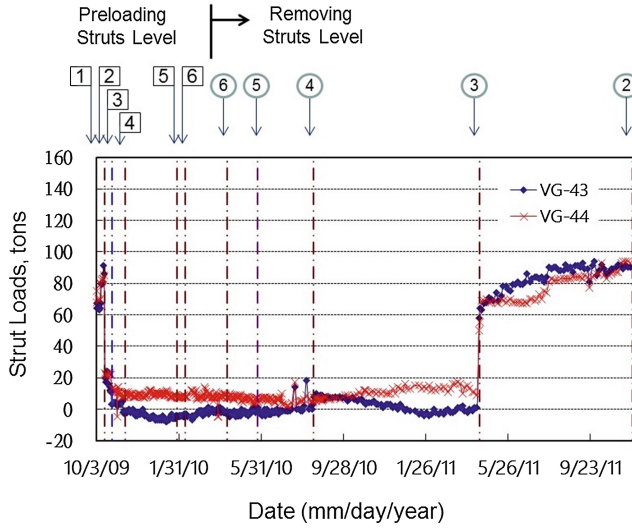


Fig. 3. Loads in the 1st level strut at the locations of Inclinometers SID-4 and SID-5

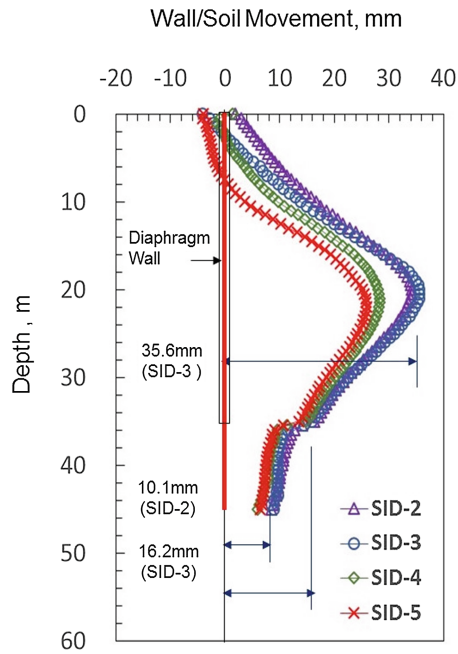


Fig. 4. Inclinometer readings, duly corrected to accounted for the movements at the tips

i.e. the E-value, of $200,000 \text{ N/mm}^2$ for steel, the increment of 16 tons, i.e., from 71 tons to 87 tons, corresponds to a shortening of 0.6 mm of the strut, or an inward movement of 0.3 mm at each end; and the decrement of 74 tons, i.e., from 71 tons to -3 tons, corresponds to a lengthening of 3 mm of the strut, or an outward movement of 1.5 mm at each end. Movements of such magnitudes are negligible for practical purposes and the joints between the struts and the diaphragm walls can indeed be assumed fixed for calibrating the inclinometer readings at other depths.

The readings, duly calibrated accordingly, obtained at the final excavation stage by the 4 inclinometers installed in the diaphragm walls on the two sides of the crossover are shown in Fig. 4. A maximum wall deflection of 35.6 mm was obtained at the location of SID-3. It is interesting to note that the diaphragm wall toes moved by 16.2 mm at the location of SID-3. It has become quite common nowadays to install inclinometers in diaphragm walls and stop at the toe levels of the walls to reduce construction costs. The toe movements of such magnitudes were, nearly, 50% of the maximum wall deflections for the case studied and back analyses would certainly lead to misleading conclusions if inclinometer readings were not corrected. It can also be noted from Fig. 4, the tips of the inclinometers still moved by, as much as, 10.1 mm, or 28% of the maximum wall deflections, even with a 10 m extension below the toes of the diaphragm walls.

4 Numerical Analyses

Numerical analyses were conducted by using the two-dimensional finite element computer program PLAXIS (PLAXIS BV 2011) to compare the performance of the walls with the theoretical predictions. Figure 5 shows the finite element model adopted. The strength parameters of soils suggested by the Detailed Design Consultant of the project are given in Table 1 (MAA 2005). The Mohr-Coulomb Model was adopted to

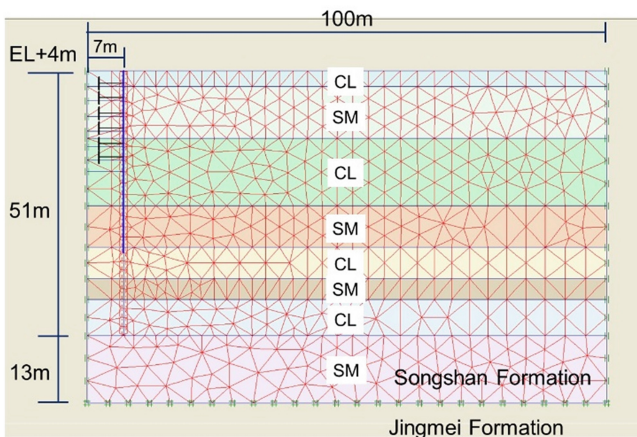


Fig. 5. Finite element model for the crossover next to G17 Station

Table 1. Soil properties and soil parameters adopted in the PLAXIS analyses (MAA 2005)

Depth (m)	Soil type	γ_t (kN/m ³)	N (blows)	S_u (kPa)	c' (kPa)	Φ' (deg)	E' (MN/m ²)	Poisson's ratio, ν'
0–3	CL	18.8	4	25			12.5	0.35
3–13	SM	19.2	8	–	0	32	20.0	0.30
13–26	CL	18.6	6	65			32.5	0.35
26–34	SM	19.4	13	–	0	32	32.5	0.30
34–40	CL	18.9	13	110			55.0	0.35
40–44	SM	19.7	21	–	0	32	52.5	0.30
44–51	CL	19.9	20	145			72.5	0.35

simulate the stress-strain behaviour of soils. The Young's moduli, E' , were correlated to soil strengths by using the following empirical relationships:

$$E' = 500 S_u \text{ (for clayey soils)} \tag{1}$$

$$E' = 2.5 N \text{ (in MPa for sandy soils)} \tag{2}$$

in which S_u = undrained shearing strength, and N = blow counts in standard penetration tests (SPT). Young's moduli are not uniquely related to soil strengths. The coefficients relating these two depend on many things and can only be established empirically by back analyses. The values shown above are applicable at this site in conjunction with the soil strengths given in Table 1.

The deepest borehole reached a maximum depth of 51 m and a remark is noted in the borehole log indicating that the soils below this depth are Type SM (silty sand). A N value (blow counts in standard penetration tests) of 25 was thus assumed for the soils below the depth of 51 m. Based on the local geology, it was estimated that the gravelly Jingmei Formation lies at a depth of, roughly, 64 m. The Jingmei Formation is

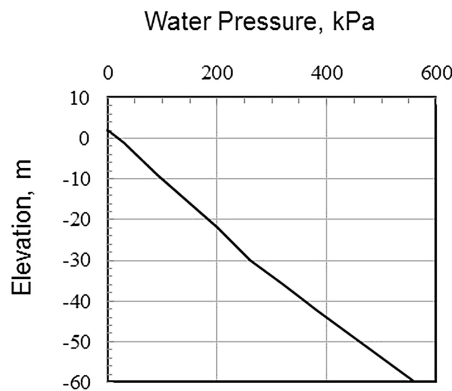


Fig. 6. Groundwater pressures on the outer face of diaphragm wall adopted in the numerical analyses

a hard gravelly strata and can be deemed as a hardpan for engineering purposes, therefore the bottom of the finite element model was assumed at this depth. The water pressures acting on the outer face of the diaphragm wall are shown in Fig. 6. Inside the pit, the water level was assumed to be at a depth of 1 m below the bottom of the excavation in each stage.

The diaphragm walls were simulated by plate elements and an E value of 25,000 MPa was adopted for concrete with an unconfined compressive strength, i.e. f'_c value, of 28 MPa. The EI (I = moment of inertia) and EA (A = sectional area) values of the diaphragm walls were reduced by 30%, giving a value of 1,464 MN*m for the former and 17,570 MN/m for the later, following the normal practice to account for the influence of cracking, creep and relaxation during excavation. The structural properties of the struts and the magnitude of preloading adopted in the analyses are shown in Table 2.

Table 2. Stiffness of struts adopted in numerical analyses

Level	Members	Sectional area A (cm ²)	Stiffness, AE/S (MN/m)	Preload kN/m
1	1H350x350x12x19	1 × 173.9	891	178
2, 3	1H400x400x13x21	1 × 218.7	1121	339, 494
4	1H428x407x20x35	1 × 360.7	1849	428
5, 6	1H414x405x18x28	1 × 295.4	1514	458, 538

Note: Spacing between struts S = 4 m

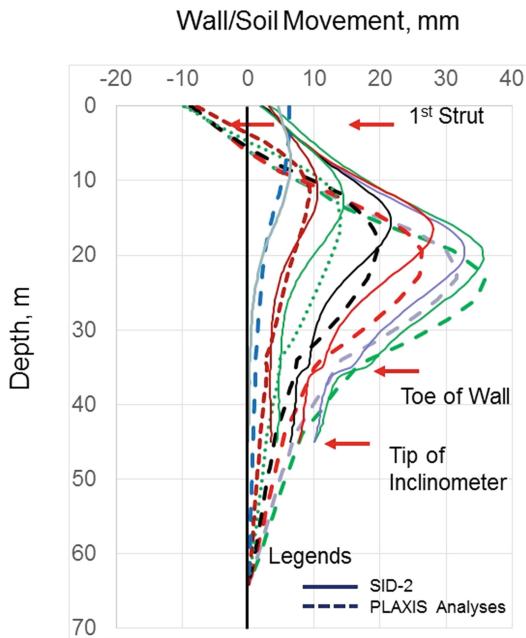


Fig. 7. Comparison of inclinometer readings with computed wall deflections for the case with 100% preloads at all levels

Of the 4 inclinometers installed on the two sides of the crossover, SID-2 and SID-3 gave the largest wall deflections, refer to Fig. 4. The first set of readings of SID-3 was taken after the struts at the first level had been preloaded, therefore, the readings are inappropriate for discussion. The wall deflections obtained from the analyses are compared with the readings obtained by Inclinometer SID-2 in Fig. 7, and as can be noted, the maximum wall deflections in all the stages of excavation are in a fair agreement with the readings of Inclinometer SID-2.

4.1 Effects of Preloading of Struts

The profile of wall deflections obtained from the numerical analyses for the 1st stage of excavation is in a very good agreement with the inclinometer readings. On the other hand, a large outward movement was computed as a result of the preloading of the struts at the first level as depicted in Fig. 8(a). These struts were preloaded to 72.5 tons (the average of 2 readings) on 1 January 2009 and, as depicted in Fig. 8(b), the inclinometer readings taken on 5 January indicated that the wall had hardly moved as compared to those taken on 27 December 2008.

The large outward movements obtained from numerical analyses were presumably due to the use of Mohr-Coulomb Model which much under-estimates the soil moduli in the early stages of excavation. Secondly, the preloads are line load applied to all the struts at the same level simultaneously in two-dimensional numerical analyses; while in reality, struts were preloaded individually, one by one. Each time a strut was preloaded, the load was essentially a point load resisted by the entire wall and the wall movement would be smaller than what would be if all the struts at the same level were preloaded simultaneously. Wall movements due to subsequent preloading of neighbouring struts

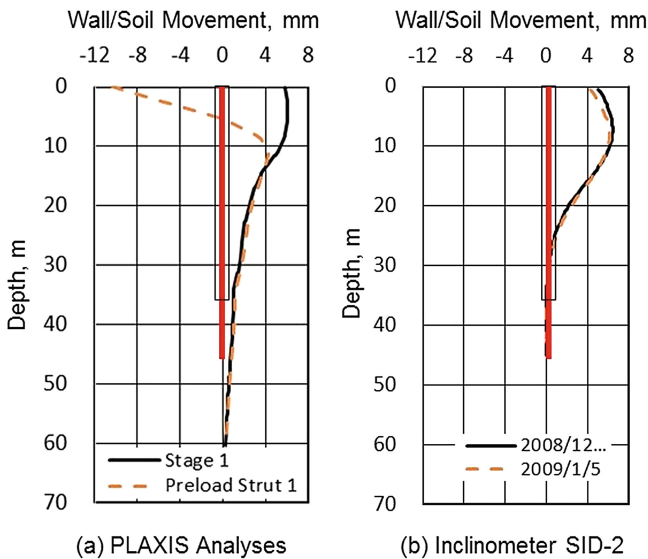


Fig. 8. Influences of preloading the strut at the first level on wall deflections

would be small because the loads in the struts which had already been preloaded were not sustaining.

Figure 9 shows the fact that the strut load dropped to nearly a half in a few days subsequent to preloading presumably due to the preloading of neighbouring struts. This residual load can be considered as an effective load in all the struts at the first level and should be the load to be adopted in numerical analyses. However, this reduction appears to be applicable to the first level only. As depicted in Fig. 10, strut loads at the second level did not drop much after preloading.

The computed wall deflections with the preloads at the first level reduced by 50% are compared with the inclinometer readings in Fig. 11. As can be noted, the outward movements shown in Fig. 7 are much reduced and the agreement between the computed wall deflections and the inclinometer readings is much improved. However, the improvement is limited to shallow depths and the maximum wall deflection in each stage of excavation is unaffected.

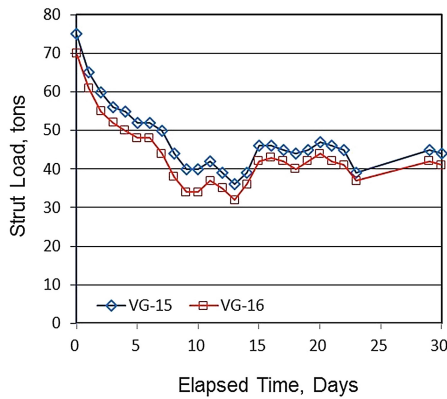


Fig. 9. Strut loads at the first level subsequent to preloading

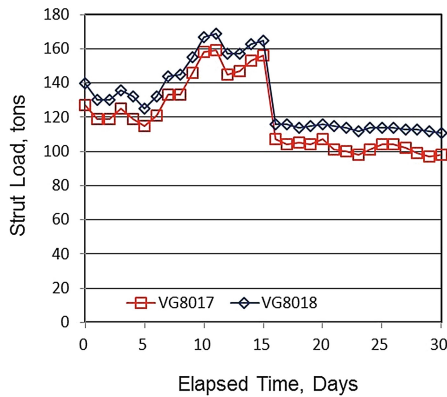


Fig. 10. Strut loads at the second level subsequent to preloading

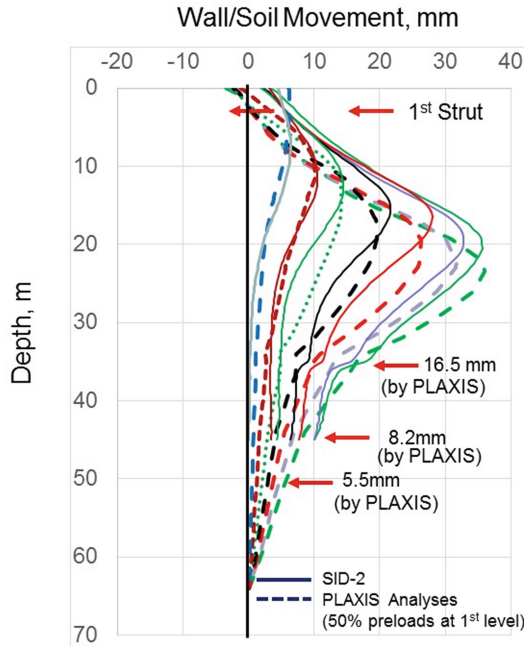


Fig. 11. Comparison of inclinometer readings with the computed wall deflections for the benchmark case with the preloads at the first level reduced by 50%

The case shown in Fig. 11 is considered as the “benchmark case” for parametric studies. For academic interest, analyses have also been carried out for the case with preloads in all stages of excavation ignored and the results are shown in Fig. 12. As can be noted that the shapes of the deflection profiles computed agree very well with those obtained based on the inclinometer readings. However, the maximum wall deflection increases from 36 mm to 44.5 mm at the end of the final excavation.

4.2 Movements at Diaphragm Wall Toes

What is of great significance is the fact that the large movements at the toes of the diaphragm walls and at the tips of inclinometers shown in Fig. 4 were confirmed by the analyses. The computed movements at the first strut level, the toe of diaphragm walls and at the tips of inclinometers are listed in Table 3. As can be noted, the movement at the 1st strut level increased from 1.53 mm after preloading of the strut (inward) to 2.59 mm at the end of Stage 2 excavation, and then dropped to 0.17 mm at the end of the final excavation. These increments and decrements agree well with those deduced from strut loads in Sect. 3. Movements of such magnitudes are indeed negligible in comparison with the large movements at the diaphragm wall toes and at the tips of inclinometers. The procedure of calibrating inclinometer readings by assuming that the diaphragm wall at the first strut level would not move once the struts were preloaded is thus proved to be valid.

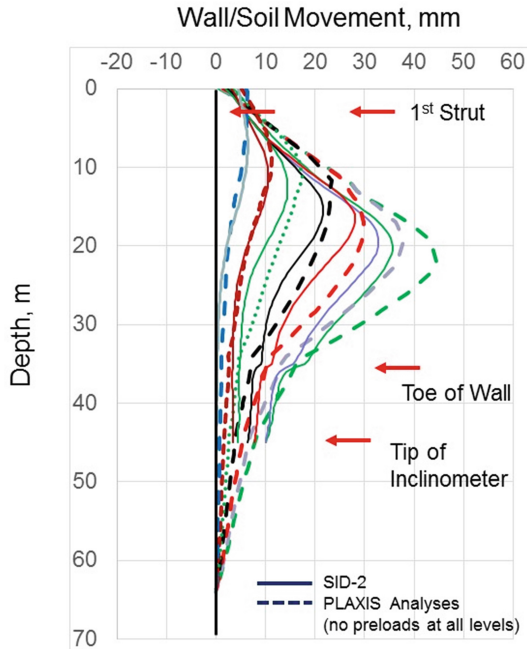


Fig. 12. Comparison of inclinometer readings with the computed wall deflections for the case without preloads at all levels

Table 3. Comparison of inclinometer readings with movements computed for the benchmark case

Location	Lateral movement, mm					
	1 st Strut level		Diaphragm wall toe		Inclinometer tip	
Excavation stages and struts preloading	PLAXIS	SID2	PLAXIS	SID2	PLAXIS	SID2
Stage 1	6.26	5.90	1.09	0.21	0.76	0
Preload 1 st Strut	1.53	5.26	1.20		0.82	
Stage 2	2.59	5.26	2.80	3.41	1.63	3.49
Preload 2 nd Strut	0.94		2.92		1.70	
Stage 3	0.47	5.26	4.54	4.87	2.79	4.35
Preload 3 rd Strut	0.62		4.89		2.65	
Stage 4	0.27	5.26	7.39	9.03	3.85	6.34
Preload 4 th Strut	0.43		7.10		4.06	
Stage 5	0.25	5.26	10.31	11.16	5.05	7.74
Preload 5 th Strut	0.33		10.42		5.09	
Stage 6	0.23	5.26	13.06	15.63	6.89	9.99
Preload 6 th Strut	0.23		13.33		6.36	
Stage 7	0.17	5.26	16.50	16.35	8.15	10.11

Notes: Positive values for inward movements and negative values for outward Movements
 Readings of Inclinometer SID-2 have been corrected to account for the movements at the tip

5 Concept of Wall Deflection Path and Reference Envelope

The drastic differences between the wall deflection profiles for different inclinometers shown in Fig. 4 fully demonstrate the ambiguity faced in back analyses. Different results will be obtained if a different set of inclinometer readings is selected to compare with the results of numerical analyses. To establish a consistent methodology for back analyses, Moh and Hwang (2005) suggested to plot wall deflections versus depth of excavation, in a log-log scale, and designate such plots as “wall deflection paths”. It was further suggested to take the upper envelope of the wall deflection paths obtained at a specific site, or for a specific set of parameters, as “reference envelope” to compare with the results of conventional two dimensional numerical analyses which are usually conducted for excavations in green field without underground structures or utilities in the vicinity. This suggestion was based on the belief that wall deflections are likely to be reduced by the presence of underground structures and facilities as well as many factors and the upper envelope of wall deflection paths will be closer to what will be obtained in numerical analyses carried out for excavations in green field.

For convenience, reference envelopes are defined by the wall deflections for a depth of excavation of 4 m, i.e., Δ_4 and the wall deflection projected to a depth of excavation of 100 m, i.e., Δ_{100} . Accordingly, the reference envelope for the case of interest can be expressed as $\Delta_4 = 6 \text{ mm}$ and $\Delta_{100} = 250 \text{ mm}$ as depicted in Fig. 13. The projection of reference envelope to a depth of excavation of 100 m, however, is merely for convenience in defining reference envelopes and by no means implies the validity of the relationship between wall deflection and depth of excavation beyond the final depth of excavation.

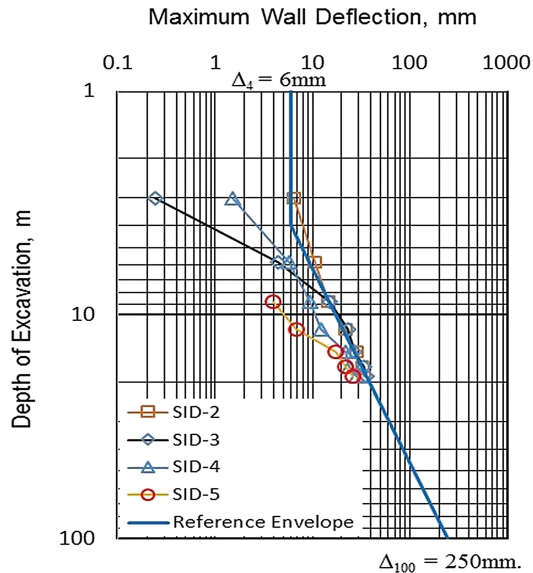


Fig. 13. Observed wall deflection paths and reference envelopes for the crossover

Excavation and preloading of struts at excavation sites are never carried out in the ways specified in designs. They are carried out in a rather unpredictable sequence as site constraints and project progress always prevail. Coordination among subcontractors is often difficult, and as such, over-excavation occurs rather frequently and delay in strutting is quite common. For these reasons, the data for shallow excavations are erratic and should be ignored, and only the data for excavations exceeding 10 m in depth should be considered in establishing reference envelopes as suggested in previous studies (Moh and Hwang 2005; Hwang et al. 2006, 2007a, b). After all, deep excavations are the ones of primary concern.

For numerical analyses, as discussed in Sect. 4.1, wall deflections at shallow depths are likely to be affected by how the preloads at the first and the second levels are applied at the site, therefore the data points for shallow excavations should be ignored. The wall deflection path obtained by PLAXIS analyses for the benchmark case, i.e., with the preload at the first strut level reduced by 50%, is given in Fig. 14 and can be defined by $\Delta_4 = 6 \text{ mm}$ and $\Delta_{100} = 250 \text{ mm}$. This wall deflection path is identical to the reference envelope given in Fig. 13.

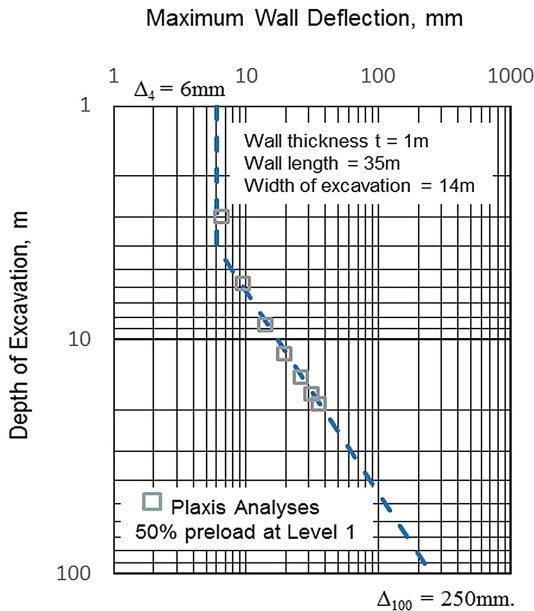


Fig. 14. Wall deflection path obtained by PLAXIS analyses for the benchmark case

6 Factors Affecting Wall Deflections

The concept of wall deflection path is a quite useful tool in evaluating the influences of various factors on wall deflections for excavations in deep soft deposits. For illustration, the influences of four of the primary factors, namely,

- (1) Preloading of struts,
- (2) Wall thickness,
- (3) Wall length,
- (4) Width of excavation,

which affect the magnitudes of wall deflections are studied and discussed as follows.

6.1 Influence of Preloading of Struts

As depicted in Fig. 8, the preloads applied at the first strut level affect the wall deflections in the early stages of excavation. However, as can be noted by comparing Fig. 11 with Fig. 7, the reduction of the preloads at the first level affect the maximum wall deflection at each stage of excavation only slightly. On the other hand, Fig. 12 shows the fact that the maximum wall deflection would be much increased if preloads at all the strut levels are omitted. The influences of preloads on wall deflection paths for these three scenarios can easily be visualized from Fig. 15. The reduction of preloads at the first strut level does not affect the Δ_{100} value while the omission of preloads at all the strut levels increases the Δ_{100} value from 250 mm to 400 mm. The Δ_4 values remain the same as they, in theory, correspond to the first stage excavation before preloading.

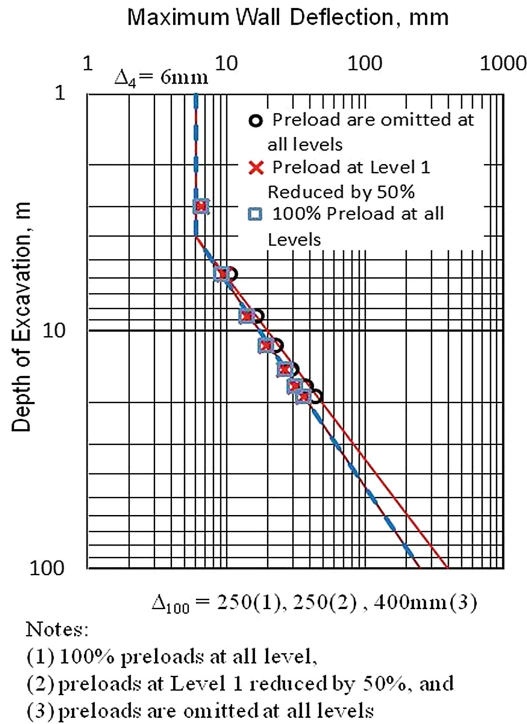


Fig. 15. Influences of preloading of struts on wall deflections obtained by PLAXIS analyses

6.2 Influence of Wall Thickness

The wall deflection paths for walls of 0.6 m and 1.5 m in thickness are depicted in Fig. 16. This figure together with Fig. 14 show that the Δ_{100} value increases from 180 mm to 250 mm as the thickness of wall decreases from 1.5 m to 1 m and further increases to 320 mm as the thickness of the wall decreases to 0.6 m. The Δ_4 values, which correspond to the deflection in Stage 1 excavation before preloading of the struts, are unaffected by the thickness of diaphragm wall. For a depth of excavation of 18.9 m, the maximum wall deflections will be 31 mm, 36 mm and 41 mm for walls with thicknesses of 1.5 m, 1 m and 0.6 m, respectively. It is interesting to note by comparing Fig. 15 with Fig. 16 that the preloads have similar effects as thickening the diaphragm wall, i.e., increasing the rigidity of the retaining system.

For academic interest, analyses were carried out assuming that the excavation were proceeded further below the design depth of 18.9 m using the same excavation scheme just to see how the walls would perform. Excavation from the depths of 18.9 m to 28.0 m are supported with the level 7 to level 10 struts. The stiffness for these struts follows that of the level 6 strut (Table 3). As can be noted from Fig. 16, the deflection paths remain linear till a depth of excavation of 23.8 m is reached. Beyond this depth, the rapid increases in wall deflections indicate instability of the diaphragm wall toes. It should be noted that only the force equilibrium was considered in this exercise with all other factors, such as seepage and blow-in, excluded in the analyses. Furthermore, the wall was assumed to be linear elastic with unlimited structural capacity. Therefore readers are warned not to take this finding for granted in designs.

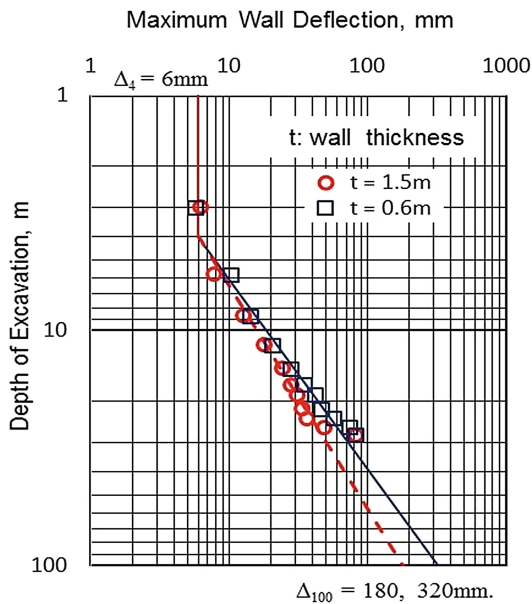


Fig. 16. Influences of thickness of diaphragm wall on wall deflections obtained by PLAXIS analyses

6.3 Influences of Wall Length

Similarly, analyses were carried out to a depth of 28 m to see how wall deflections would be affected by the lengths of walls. Figure 17 shows the wall deflections for walls of 30 m, 35 m and 40 m in length. As can be noted, for excavations to the depth of 18.9 m, the maximum wall deflections are essentially the same in these 3 cases. The wall deflections start to deviate noticeably from a depth of excavation of 18.9 m. For walls of 40 m in length, deflections were slightly below the reference envelope, suggesting that lengthening walls does help in maintaining toe stability.

6.4 Influences of Width of Excavation

Analyses were performed for different widths of excavation and the results are given in Fig. 18. As can be noted that the Δ_4 values are more or less proportional to the width of excavation. This is one of the reasons that only the inclinometers for excavations of similar widths, i.e., SID-2 to SID-5 are included in the comparison and other inclinometers are excluded. It is emphasized again that only the data between depths of excavation in the range of 10 m to 20 m should be considered in constructing wall deflection paths.

What is even more interesting is the fact that the 4 wall deflection paths converge to a same Δ_{100} value. This finding is consistent with what was given in Hwang et al. (2012), (Chao et al. 2010a, b) and Hsiung et al. (2016).

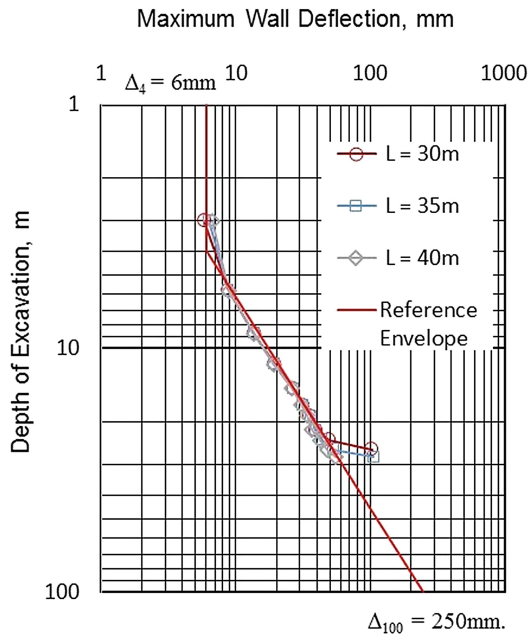


Fig. 17. Influences of wall length on wall deflections obtained by PLAXIS analyses

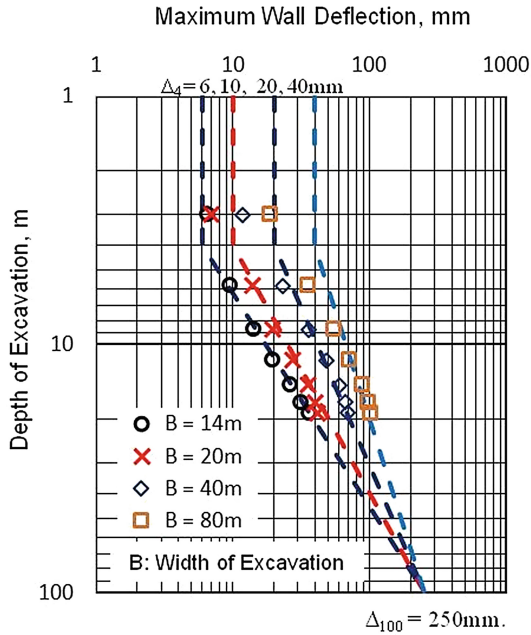


Fig. 18. Influences of width of excavation on wall deflections obtained by PLAXIS analyses

7 Conclusions

Numerical analyses have been conducted for a cut-and-cover section of the Taipei Metro. The results of analyses verify the various input parameters such as the Young’s moduli for clay and sand deposits, the stiffness for the struts and the magnitude for strut preloading. The calibrated numerical model validates the wall deflection path concept, which plots wall deflections versus depth of excavation, in a log-log scale. The influences of four factors, i.e., the preloading of struts, wall thickness, wall length, and the width of excavation on wall deflections were quantified by using this concept.

The foregoing discussions lead to the following conclusions:

1. The concept of wall deflection path is a very useful tool to evaluate the performance of diaphragm walls in deep excavations in thick soft sand and clay deposits with high groundwater tables.
2. The influences of various factors can be readily quantified by comparing the wall deflection paths obtained with the reference envelope which is the wall deflection path representative of a specific site or for a specific combination of influencing factors.
3. The width of excavation, the thickness of wall and the magnitude of preloads have pronounced influence on the maximum wall deflections.
4. The preloads have similar effects as thickening the diaphragm wall and, hence, reduce the maximum wall deflections.

5. The influence of the length of wall to wall deflections will be insignificant until a critical depth of excavation is reached. Such critical depth will depend on the bending stiffness of the wall or base heave modes of failure.

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