Consideration of the Pile-Soil Friction for Earth Pressure of Limited Soil for Double-Row Piles

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Abstract. The soil between the front-row pile and the back-row pile is defined as the limited soil in the structure of the double-row pile. However, the Rankine theory is inappropriate for analyzing the properties of the limited soil. Considering the tangential friction of pile-soil for the double-row pile, this paper develops formula of earth pressure of limited soil when it was in limited equilibrium state. The simplified formula of earth pressure intensity of limited soil is also founded. The analysis shows that the result stemmed from the formula equals to that of numerical simulation (FLAC3D). Therefore, the formula of limited soil is reasonable and valuable. The values of earth pressure intensity of limited soil are the same among different width (2d–5d). They are less than that of positive earth pressure. The value of earth pressure intensity at the lower level of the limited soil is far more less than that of positive earth pressure.

Keywords: Double-row pile · Limited soil earth pressure · Pile-soil friction

1 Introduction

At present, the double-row pile supporting structure has been applied in foundation pit engineering and slope engineering (Liu et al. 2014; Jing and Wei 2010; Nie et al. 2008; Cui et al. 2006). Wang et al. (2015a) analyze the mechanical properties of two types of soils arch for double-row pile, acquiring the ultimate bearing capacity of the soil arches. Numerical simulation is used to investigate the mechanical properties of double-row pile structure during deep foundation pit excavation (Sun 2009; Wang et al. 2015b). Liu and Fu (2011) study computational methodology and stress patterns, making stress behavior of double-row pile is in accordance with the practical project. Involving the pile-soil interaction, a new plane linear finite element model is developed (Zheng et al. 2004). Numerous papers mainly focus on earth pressure calculation of soil retaining wall. However, earth pressure between the double-row pile is merely analyzed. Three gaps exit among current researches. (1) Assumptions proposed by several computational models are unreasonable (Ma et al. 2008; Gao 2000; Li and Guo 2008). For instance, some of them suppose that the soil between the double-row pile is

rigid-plastic material, neglecting soil-pile interaction. (2) Due to the complicated earth pressure the distribution of earth pressure is imprecisely presented based on current numerical and experimental method. (3) Current researches seldom define the soil between the front-row pile and back-row pile as the limited soil. Consequently, This paper develops a computational model of limited soil of double-row pile, considering the soil-pile interaction. The formula of limited soil under the tangential friction of pile-soil for double-row pile is also founded. The results show that the formula is accuracy compared with numerical simulation's outcome.

2   Calculation Model

The Technical Specification of Retaining and Protecting for Building Foundation Excavation (JGJ120-2012) argues that the distance between the front-row pile and back-row pile must be 2d–5d (d is the diameter of pile) (Liu et al. 2003). Therefore the soil between the front-row pile and back-row pile is not defined as the semi-infinite soil but the limited soil (LS). This paper proposes a calculation model of limited soil pressure, considering the tangential friction of pile-soil, based on the previous research results and the Rankine theory. When the limited soil reaches the positive limited state, the calculation model above the slip surface is shown in Fig. 1.
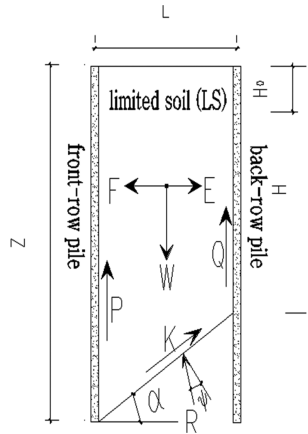

Fig. 1. Computation model of limited soil between double-row pile

The hypothetical condition of the model is that the slip surface is a straight line when the limited soil has reached the positive limited state. The angle between the line and the horizontal direction is α; the space between the front-row pile and the back-row pile is L; the depth of compute point is Z; the gravity of the limited soil W; the supporting force, from the stabled soil block, for the limited soil is R and the friction of the interface is K. Generating from front-row pile and back-row pile, the horizontal forces on the limited soil are E and F as well as the tangential friction are P and Q.

3   Stress Analysis

3.1   Establish the Equation

Assumptions:

(1) The limited soil is the homogeneous;
(2) The limited soil stays in the limit equilibrium state;
(3) The value of stress of limited soil is equal at the same depth.

According to the geometric relationship and the Rankine theory, the gravity W is shown in the Eq. (1) and the relationship between E and F in Eq. (2).

$$W = \gamma L Z - \frac{1}{2}\gamma L^2 \tan \alpha \tag{1}$$

$$\frac{E}{F} = \frac{\frac{1}{2}\gamma(Z - H_0)^2 K_a}{\frac{1}{2}\gamma(H - H_0)^2 K_a} = (\frac{Z - H_0}{Z - L \tan \alpha - H_0})^2 \tag{2}$$

K is shown as formula (3), identifying the friction between the LS and the fixed soil below LS.

$$K = \frac{Lc}{\cos \alpha} \tag{3}$$

The LS can be self-stability and does not interact with surroundings according to the Rankine earth pressure theory that the value of earth pressure can be less than 0 among a certain depth. On the contrary, the LS can interact with surroundings beyond the depth. The formula of depth H_0 is shown in Eq. (4) (Wang et al., 2014), when the value of earth pressure is 0. Consequently, H_0 need to be subtracted when calculating the pile-soil interaction.

$$H_0 = \frac{2c}{\gamma\sqrt{K_a}} \tag{4}$$

The tangential friction P and Q of LS is shown in Eq. (5), when the LS is in the limited equilibrium state, according to the Mohr-Coulomb theory

$$P = E \tan \delta + \bar{c}(Z - H_0) \tag{5}$$

$$Q = N \tan \delta + \bar{c}(H - H_0) \tag{6}$$

In the formula: γ-gravity of the soil (kN/m^3); c, φ-cohesion and internal friction angle; $\bar{c}, \delta$-cohesion and friction angle for interface of the pile-soil.

The stress of the soil between front and back-row pile is in limited equilibrium state, shown in formula (7).

$$\begin{cases} P + Q + K \cdot \sin \alpha + R \cdot \sin(\alpha - \varphi) = W \\ E + K \cdot \cos \alpha = F + R \cdot \sin(\alpha - \varphi) \end{cases} \tag{7}$$

Formula (8) springs from the combination between Eq. (7) and equations from (1) to (6).

$$E = \frac{2\bar{c}H_0 - Lc[\tan \alpha + \cot(\alpha - \varphi)] + (\gamma L - 2\bar{c})(Z - \frac{1}{2}L \tan \alpha)}{\left[1 - (\frac{Z - H_0}{Z - L \tan \alpha - H_0})\right] \cot(\alpha - \varphi) + \left[1 + (\frac{Z - H_0}{Z - L \tan \alpha - H_0})\right] \tan \delta} \tag{8}$$

3.2 Simplified Analysis

The Rankine earth argues that the angle between the slide plane and principle stress σ_1 acting face is $45^0 + \psi/2$. However, the angle between the α and σ_1 will deflect due to the friction of pile-soil. While the action line of σ_1 near the front-row pile turns clockwise, the angle will be less than $45^0 + \psi/2$. On the contrary, when the action line of σ_1 rotates counterclockwise, the angle will be more than $45^0 + \psi/2$. Accordingly, the angle between the α and σ_1 is $45^0 + \psi/2$ whether the interface of pile-soil is smooth or not. Formula (9) can be obtained by substituting $\tan a = \cot(a - \varphi) = \sqrt{K_P}$ into Eq. (8)

$$E = \frac{2\bar{c}H_0 - 2Lc\sqrt{K_p} + (\gamma L - 2\bar{c})(Z - \frac{1}{2}L\sqrt{K_p})}{\left[1 - (\frac{Z - H_0}{Z - L \tan \alpha - H_0})\right]\sqrt{K_p} + \left[1 + (\frac{Z - H_0}{Z - L \tan \alpha - H_0})\right]\tan \delta} \tag{9}$$

$\sqrt{K_p}$- passive earth pressure coefficient

The formula (9) is the earth pressure equation for the limited soil between the front-row pile and back-row pile. The formula (9) shows that value of earth pressure reaches maximum when $\alpha = 45^0 + \psi/2$, considering the friction of the pile-soil interface rough.

3.3 Earth Pressure Intensity

According to current researches, the differential expression of earth pressure intensity is $e = \partial E/\partial Z$. However, the formula (9) is too complex to solve. The research adopts the method of finite difference to solve the intensity of earth pressure approximately. The accuracy of calculation result could be higher and higher as the Z_1 and Z_2 getting closer. In this paper, Z_1 and Z_2 meet the accuracy requirements by the validation.

The formula (10) presents earth pressure intensity.

$$e_{Z=\frac{z_1 + z_2}{2}} = \frac{\Delta E}{\Delta Z} = \frac{E_{Z=Z_2} - E_{Z=Z_1}}{Z_2 - Z_1} \tag{10}$$

4 Numerical Simulation

Numerical simulation is applied to validate formula (9) because it can eliminate irrelevant factors in site monitoring by establishing idealized model. The FLAC3D software best fits limit equilibrium analysis for double-row pile, for simulating the change of earth pressure for double-row pile more accurately than other software in the foundation pit excavation.

4.1 Calculation Parameter Selection

Tables 1, 2 and 3 show the parameter of both the front-row pile and the back-row pile which are in rectangular arrangement.

Table 1. Soil layer of model calculation parameters

Name of soil	Thickness/m	Severe/kN·m^{-3}	Cohesion/kPa	Friction angle $\psi/(°)$	Bulk modulus K/MPa	Shear model G/MPa
Homogeneous soil	40	20	10	18	33.1	11

Table 2. Geometrical parameters of supporting structure

Depth of foundation pit/m	Diameter of pile/m	Distance of front-row pile/m	Distance of double-row pile/m	Embedded depth/m	Length of pile/m	Length and wide for section of beam/m
15.2	0.8	1.6	2.4	11.2	25.6	0.8 × 0.8

Table 3. Calculation parameters support structure

Concrete of pile and beam	E (pile and beam)/ GPa	μ	E Reinforcement/GPa	Diameter of reinforcement/m	Amount of reinforcement for pile and beam
C30	24	0.2	200	0.028	18/24

4.2 Meshing and Simulation of Construction

The size of numerical model is 52 m × 6.4 m × 40 m. The Mohr- Coulomb model is used establish the double-row pile; the pile-unit, beam-unit and shell-unit are respectively applied for the pile, beam and slope. The model does not consider groundwater impact, because it has been precipitated prior to the foundation pit excavation.

The process of excavation is shown in following:

(1) Establishing the model of stratum, calculating the initial stress field under the action of gravity, and setting the displacement to 0.
(2) Establishing double-row pile model, calculating the stress field after finishing the construction of double-row pile, and setting the displacement to 0.
(3) Excavating the depth about 2.4 m, excavating the rest in every depth about 1.6 m till reaching the depth 15.2 m, then calculating the maximum unbalance stress and the distribution for stress field.
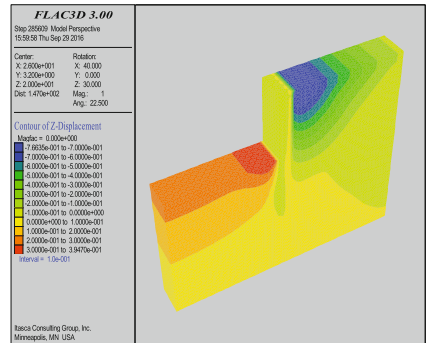(4) Analyzing of calculation and results, as shown in Figs. 2 and 3.
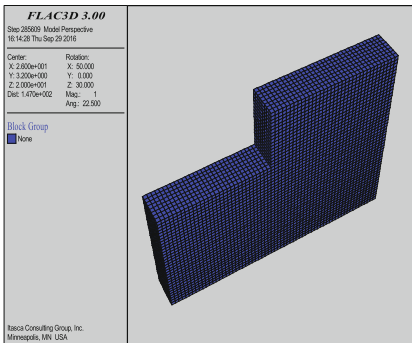
Fig. 2. Model meshing

Fig. 3. Excavation of vertical displacement cloud

5   Comparative Analysis

5.1   Comparison Between Theoretical Calculation and Numerical Simulation

The reference Li and Guo (2008) points out that the value of soil-pile boundary strength reduction factors $\bar{c}, \delta$ can be chosen from 25% to 50% of that of fill. Therefore, this paper opts for one third of medium value when calculating.

The values of earth pressure intensity with depth are obtained and shown in Table 4, while the parameters from Tables 1 and 2 are substituted into the formula (9) and (10).

Table 4. Theoretical calculation value of earth pressure intensity

Depth/m	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Earth pressure/kPa	7.41	10.5	17.03	23.57	28.14	31.73	36.08	38.97	42.58	46.1	48.8	50.7	51.6	52.5

The results of numerical simulation, theoretical calculation and Rankine theory are compared and shown in Fig. 4. The value of Rankine positive earth pressure increased linearly with the depth, more than the value of numerical simulation and theoretical calculation. This result shows that Rankine theory is more suitable for semi-infinite soil rather than limited soil. It will lead to larger calculation results when applied to limited soil and result in using redundant materials in construction.
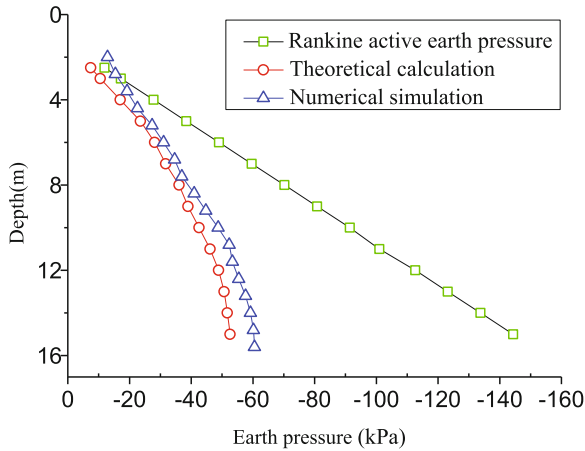

Fig. 4. Calculates the comparison chart

The trend of theoretical calculation for earth pressure is the same as that of numerical simulation. The values of earth pressure intensity both of theoretical calculation and numerical simulation decrease as the depth increases. The earth pressure intensity of the limited soil between front-row pile and back-row pile does not increase linearly with depth, and the slopes of the curves decreases near the bottom of foundation pit.

The value of numerical simulation fits well with that of theoretical calculation, validating the formula (9).

Considering the influence of width of limited soil, this paper takes the width $L = 2d$, $L = 3d$, $L = 4d$, $L = 5d$ (d is diameter of pile) into the formula (9) and (10) separately. The comparison of earth pressure intensity among different width is shown in Fig. 5.

As can be seen from Fig. 5, the trend of earth pressure intensity for limited soil is the same basically in different width. The values of earth pressure intensity in different width are all less than that of Rankine's positive earth pressure. The value of earth pressure intensity for the lower part of the limited soil between front-row pile and back-row pile is far less than that Rankine positive earth pressure.

The intensity of earth pressure at the same depth increases gradually with the increase of width of limited soil, and becomes more close to the value of Rankine active positive earth pressure.
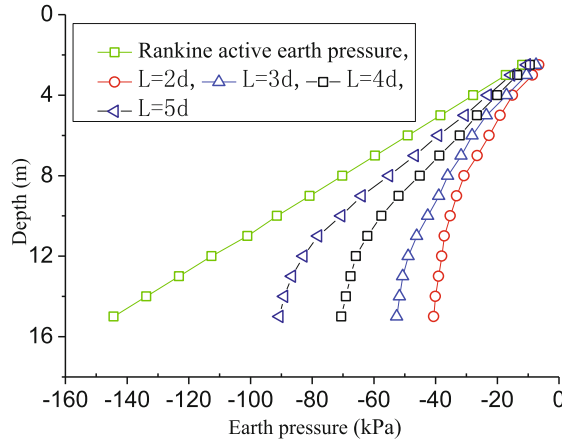
Fig. 5. Comparison of earth pressure intensity in different width

It is use of the FLAC 3D finite difference software for establish the soil of different width to study the influence to the displacement and bending moment. As shown in Figs. 6, 7, 8 and 9.

Knowing from the Figs. 6 and 7, the maximum horizontal displacement of the front and back-row piles are all getting smaller and smaller with increase of width for limited soil. The maximum displacement of the front-row piles are 18.52 mm, 16.77 mm, 15.45 mm and 14.33 mm with increase of width 2d, 3d, 4d and 5d, and the maximum displacement of the back-row piles are 17.6 mm, 17.29 mm, 17.06 mm and 16.93 mm with increase of width 2d, 3d, 4d and 5d. The displacement of the front-row piles is smaller than that of the back-row piles. Specifically the average displacement of front-row piles displacement decreases by 9% and the average displacement of back-row piles decreases by 2%. It is shown that there is a great influence for the displacement of front-row piles with the width of limited soil changes. The front-row piles have play a important role for resisting earth pressure.

The position of maximum displacement for front-row piles is different from that of the back-row piles. the position of maximum displacement for front-row piles is 4 m–4.5 m below the top of pile; the position of maximum displacement for back-row piles is 7 m–8 m below the top of pile. The displacement of front-row piles is larger than that of back-row pile in the same depth.

Knowing from the Figs. 8 and 9, the maximum bending moment of front-row piles is 7 m–8 m below the top of pile, and then the bending moment becomes smaller as the depth increases. The maximum bending moment of front-row pile and back-row piles is increasing gradually with the width of limited soil increases. Specifically the maximum bending moment of the front-row piles are −211.6 kN·m, −242.3 kN·m, −261.9 kN·m and −274.6 kN·m with increase of width 2d, 3d, 4d and 5d, and the maximum bending moment of the back-row piles are −214.1 kN·m, −242.4 kN·m, −253.7 kN·m and −256.5 kN·m with increase of width 2d, 3d, 4d and 5d. The value of maximum
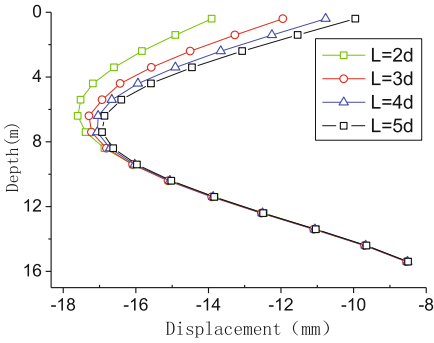
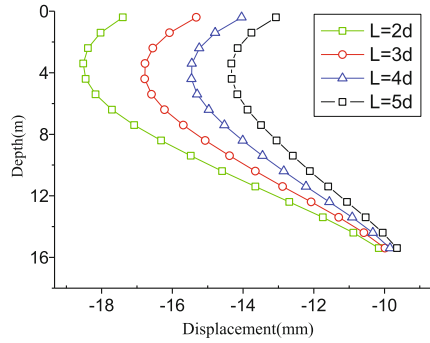**Fig. 6.** Displacement diagram of front-row pile

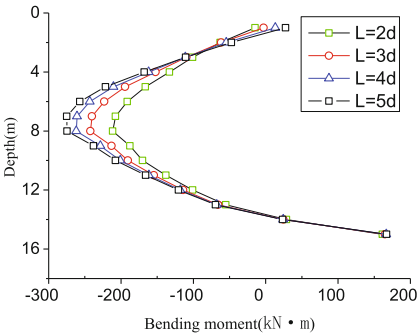**Fig. 7.** Displacement diagram of back-row pile

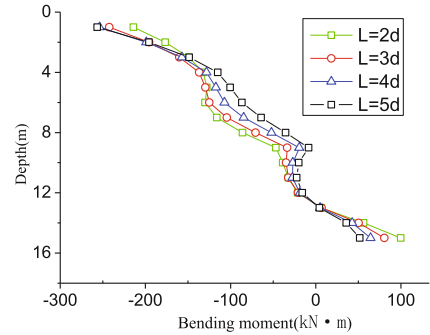**Fig. 8.** Bending moment of front-row pile

Fig. 9. Bending moment of back-row pile

bending moment for front-row pile is the same as the back-row pile basically, and that is to say the double-row piles can work better together.

6   Conclusion

(1) The research shows the calculation formula of limited soil between front-row pile and back-row pile, considering the tangential friction between pile and soil. The calculation results of the formula are best fit those of numerical simulation, and have a better accuracy. The calculation model of limited soil in this paper has a more practical engineering value.

(2) The value of earth pressure of limited soil between front-row pile and back-row pile is different from that of the Rankine theory which increases linearly with the depth. The slope of pressure curve decreases near the bottom of foundation pit.

The value of earth pressure intensity of the lower part of the limited soil is far less than that of Rankine positive earth pressure.

(3) The intensity of earth pressure at the same depth increases gradually with the increase of width of the limited soil when the width changes from 2d to 5d, and become more close to the value of Rankine positive earth pressure.

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