Quantitative Analysis of Shear Mark Based on Maximum Lyapunov Exponent Algorithm

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1 Introduction

The surface topography has unsteady stochastic characteristics. This fact has been discovered by Sayles et al. in the 1970s. The problem of surface topography recognition is studied by time series analysis [1] on the basis of stochastic process theory. The analysis of the two-dimensional profile curve and the three-dimensional surface are treated as one-dimensional signals and three-dimensional signals [2]. In the research of past 20 years, some progress is made in this field. Wang and other researchers wrote the paper [3], etc. results. The achievements show that the time series have been applied in the study of surface topography.

The surface profile curve of shearing marks is treated as one-dimensional signals. The maximum Lyapunov exponent is applied to the study of surface characteristics analysis of shearing marks. The analysis method of surface characteristics of shearing marks based on maximum Lyapunov exponent is proposed. The basis of the reasonable calculation of the maximum Lyapunov exponent is the determination of delay time τ and the embedded dimension d in the phase space reconstruction of time series. Therefore in this paper the determination of these two quantities is analyzed emphatically. The delay time of different time series is determined by mutual information function method. The minimum embedding dimension d is determined according to the improved false neighbor method which is proposed by Cao [4]. The correlation dimension is calculated for the surface profile curve of shearing marks [5, 6] on the basis. The research provides a method of analysis for surface characteristics recognition of shearing marks.

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2 Experimental Conditions and Data Acquisition

Scissors, wire cutters and wire clippers are used as shearing tools to shear the lead wires. Several samples of shearing marks are made. The samples are observed under the stereo microscope. The samples which can reflect the stability characteristics of the shearing marks are chosen to be collected digitally.

In the experiment, the shearing marks are collected digitally by using Austria Infinite Focus auto-zoom three-dimensional surface topography measurement device. A three-dimensional shearing mark is stored in computer. In this experiment, the objective magnification is 10 times and the sampling resolution is 1.1 μm. The stable part of the mark characteristic is marked by the application software. The profile curve which is perpendicular to the marks surface is obtained. Profile curve which is perpendicular to the mark surface formed by scissors, wire cutters, wire clippers are shown in Fig. 1.

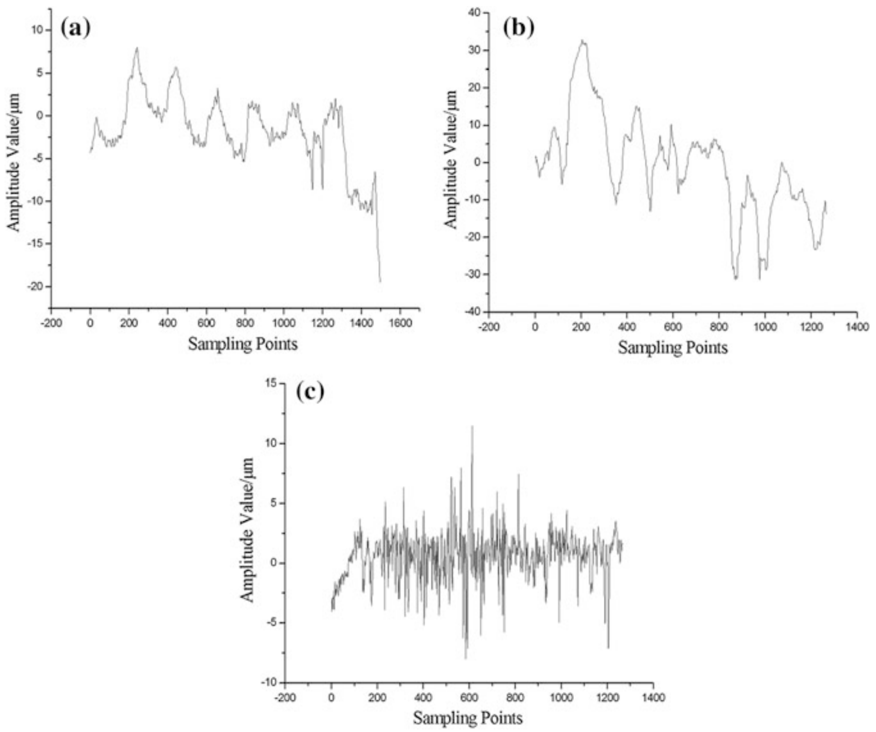

Fig. 1 Three kinds of profile curves

3 Two Parameter Determination in Restructuring of Phase Space

The delay time τ is one of the two important parameters of phase space reconstruction. In order to make the reconstructed phase space fully reflect the system dynamic characteristics, two parameters must be properly selected. When the selection of delay time τ is too big or too small, the dynamic characteristic of system is unable to reflect and the calculated chaotic characteristics may be inaccurate or even incorrect. In this paper, the delay time τ is determined by using mutual information function method.

Supposes the observation time series is $\{x_n\}$, and then observes the mutual information of variables between n and $n+1$ time, the definition is:

$$I(\tau) = \sum_{n=1}^{N} p(x_n, x_{n+\tau}) \ln[p(x_n, x_{n+\tau})/p(x_n)p(x_{n+\tau})] \tag{1}$$

In the formula, $p(x_n)$, $p(x_{n+\tau})$ and $p(x_n, x_{n+\tau})$ are the probability. The function value of the time series $\{x_n\}$ is obtained. Draw the curve under the coordinate system of $I(\tau) - \tau$. The value of τ, which corresponding to the first minimum point, is the time delay.

The delay time τ of phase space reconstruction of three kinds of profile curves is calculated by formula (1) according to determine principles of delay time τ—the mutual information function method.

It can be seen from Fig. 2 the delay time τ of the phase space reconstruction is different with different profile curves.

Embedding dimension d is one of the important parameters of the phase space reconstruction. The improved false neighbour method which is proposed by Cao has no subjective parameters in determining the embedding dimension d. It is suitable the high-dimensional dynamic system and the small data quantity. The random signals can be distinguished and the signals can be determined. The principle is:

Suppose the time series is $\{x_n\} n = 1, 2, \ldots, N$, the delay time τ is determined by formula (1), and then a time vector is constructed, such as $X_i(d) = \{x_i, x_{i+\tau}, \ldots, x_{i+(d-1)\tau}\}\, i = 1, 2, \ldots, N - (d-1)\tau$. The d is embedding dimension. A quantity $\beta(i, d)$ is introduced refers to the false neighbour method, and to order:

$$\beta(i, d) = \frac{\left\| X_i(d+1) - X_{n(i,d)}(d+1) \right\|}{\left\| X_i(d) - X_{n(i,d)}(d) \right\|} \quad i = 1, 2, \ldots, N - d\tau \tag{2}$$

In formula (2) $\|X_k(d) - X_l(d)\| = \max_{0 \le j \le m-1} \left\| x_{k+j\tau} - x_{l+j\tau} \right\|$, $n(i, d)$ is an integer, the size is $1 \le n(i, d) \le N - d\tau$, $X_{n(i,d)}(d)$, is the nearest point of $X_i(d)$ in the reconstruction d space.

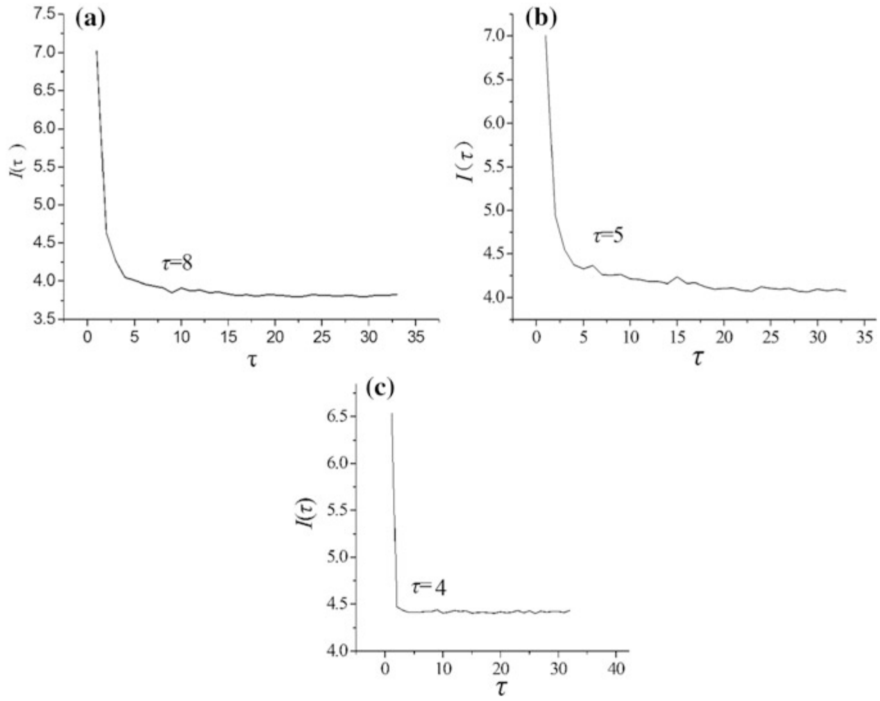

Fig. 2 Delay time τ of different profile curves

In order to avoid that different threshold of distance of two neighbor points have to select the different threshold of distance of two neighbor points, an average value of all parameters $n(i, d)$ is defined, it is:

$$E(d) = \frac{1}{N - d\tau} \sum_{i=1}^{N-d\tau} \beta(i.d) \tag{3}$$

It can be seen by formula (3), the function value $E(d)$ is only related to the embedded dimension d and the delay time τ, obviously, when the delay time τ is given, the function changes is changed only with the embedding dimension d, therefore $E_1(d)$ is defined:

$$E_1(d) = E(d + 1)/E(d) \tag{4}$$

Draw the curve under $E_1(d) - d$ coordinate system; the $E_1(d)$ value does not change with the increase of d, the d corresponding to is the minimum embedding dimension.

The minimum embedding dimension d is determined according to the improved false neighbor method which is proposed by Cao. The minimum embedding

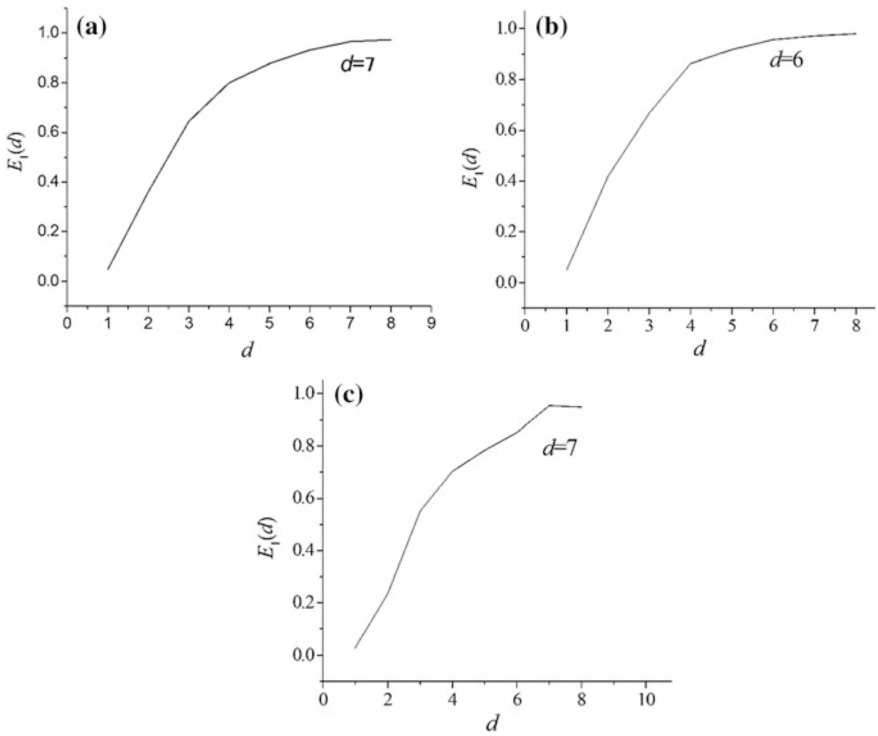

Fig. 3 The minimum embedding dimension of three profile curves

dimension of phase space reconstruction of three kinds of profile curves are calculated by formula (4). Figure 3 shows the minimum embedding dimension of three kinds of profile curves using the method of Cao.

It can be seen from Fig. 3, the embedding dimension of phase space reconstruction is different with different profile curves.

4 Calculating of Maximum Lyapunov Exponent of Profile Curves

A well-known and widely used method of computing the largest Lyapunov exponent, also known as direct method, is proposed by Wolf et al. [7]. Wolf algorithm is often used for experimental data analysis. And another direct method is proposed by Sato et al. [5], which is very similar to the Wolf algorithm; this algorithm is very simple in the calculation of a few parameters. At the logarithmic scale, the average doubling increases of adjacent track distance is studied by the prediction error, it is defined as follows:

$$p(k) = \frac{1}{Nt_s} \sum_{n=1}^{N} \log_2\left(\frac{\left\|X^{n+k} - X^{nn+k}\right\|}{\left\|X^n - X^{nn}\right\|}\right) \tag{5}$$

The curve is plotted under the $p(k) - k$ coordinates, and the slope of the straight part is the largest Lyapunov exponent.

Figure 4 shows the maximum Lyapunov exponent estimated by formula (5) for different profile curves.

Figure 4 shows the maximum Lyapunov exponent for different profile curves. It can be seen from Fig. 4, the maximum Lyapunov exponent is different with different profile curves.

The reciprocal of the largest Lyapunov exponent is defined as the quantitative index of the mark surface profile curve, the expression is:

$$f = 1/\lambda \tag{6}$$

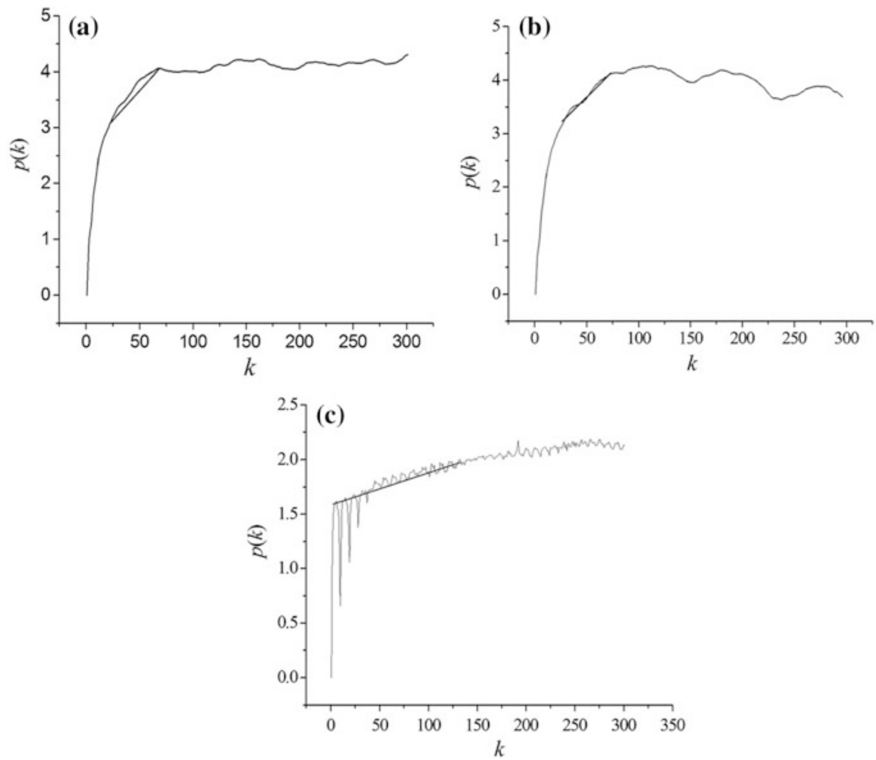

Fig. 4 the maximum Lyapunov exponent of different profile curves

Table 1 Calculating of largest Lyapunov exponent of different profile curves

	Sample a	Sample b	Sample c
Delay time	8	5	4
Embedding dimension	7	6	7
Maximum Lyapunov exponent	0.0280	0.0192	0.0031
Quantitative indexes	35.72	52.08	322.58

Formula (6) can be used to calculate the quantitative index of the trace surface, as shown in Table 1.

The maximum Lyapunov exponent and the quantitative index of different mark surface profile curves are shown in Table 1. It can be seen from the calculation results, the maximum Lyapunov exponents and the quantitative indexes are different with different profile curves.

5 Conclusions

The embedded dimension d and the delay time τ are two important parameters of phase space reconstruction. In order to make the reconstructed phase space fully reflect the chaotic characteristics of the system, these two parameters must be properly selected. It is also the prerequisite and basis for the accurate calculation of chaotic characteristic quantities such as such as maximum Lyapunov exponent. It can be seen that the embedding dimension and delay time are different with different profile curves according to the analysis of three kinds of profile curves.

The maximum Lyapunov exponent of three profile curves is calculated based on the appropriate selection of embedded dimension and delay time. The parameter can reflect the dynamic characteristics of the nonlinear system, and it can be used to analyze and identify the surface characteristics of shearing marks. The maximum Lyapunov exponent and the quantitative indexes are different with different surface of marks. It can be quantitatively marked surface topography characteristics.

Therefore, the maximum Lyapunov exponent and the quantitative indexes can be used to describe the characteristics of nonlinear system. It can be used to analyze and identify the surface characteristics of shearing marks. It provides a method to analyze the surface features of shearing marks.

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