Target Locating and Tracking Based on the Azimuth and the Change Rate of Doppler Frequency

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1 Introduction

With the development of science and technology and the upgrading of electronic warfare technology, passive location technology has drawn more and more attention. The target location technique, which the observation platform uses the targets' radiation signals or other radiation signals that targets reflect, has developed by leaps and bounds.

In the very beginning, the main positioning method is Direction of Arrival (DOA), but the accuracy is low. With the increasingly improvement of positioning accuracy's requirements, the precise electronic reconnaissance and positioning system theory appeared in the early 1970s, which mainly uses the Time Difference of Arrival (TDOA) and the Difference of Doppler Frequency (DFOA or DD) to measure and position, and the typical application is the PLSS system of the United States. At present, the existing passive positioning systems mainly use the targets' self-radiation signals to locate the targets. In addition to the traditional positioning technology, these systems has introduced a number of new single observer rapid positioning technologies, such as passive ranging, phase changing rate technology, and "out-of-plane" multi-path reflection signal positioning technology, which consists of infrared passive positioning and differential Doppler localization. For the target tracking technique, the scholar Kalman in 1960 and his colleague Bucy proposed the Kalman filtering algorithm [1], and this algorithm is the most basic

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and most widely used. In 1985, T.L. Song and J.L. Speyer proposed the Modified Gain Extended Kalman Filter (MGEKF) [2]. By the end of 20th century, some scholars proposed several nonlinear filtering algorithms such as Bayesian filtering based on nonparametric Monte Carlo simulation and Particle Filtering (PF) [3]. In 2002, Julier proposed the Unscented Kalman Filtering method of non-linear mapping transformation, and this algorithm directly uses the non-linear function to filter.

Since the traditional positioning method is azimuth intersection positioning, and the accuracy is low, so in this paper, we choose to add the change rate of Doppler frequency to the target positioning and tracking process; And in the meantime, the IMM_UKF (Intersecting Multiple Model) will increase the accuracy of tracking the maneuvering targets.

2 Positioning of Maneuvering Target

2.1 Observability Analysis

For the unobservable object, there must be a set of target trajectories corresponding to a same measurement value. In other words, for any trajectory in this set, the observer can exactly get an identical observation value. From another point of view, the target is observable means that a set of observation values can be used to determine the solution and the solution is unique. In this paper, we locate and track the maneuvering targets by the azimuth and the change rate of Doppler frequency. Compared with azimuth tracking, the addition of the change rate of Doppler frequency is supposed to improve the observability of targets to some degree. But, whether it can improve observability of targets in any case should be further studied. According to the principle of particle kinematics, the measurement value can be expressed as follows:

$$\begin{cases} \theta(t_k) = \arctan\left(\frac{x_p(t_k)-x_e(t_k)}{y_p(t_k)-y_e(t_k)}\right) \\ \dot{f}_d(t_k) = -\frac{\ddot{r}(t_k)}{c}f \\ \ddot{r}(t_k) = \frac{v_t^2(t_k)}{r(t_k)} - a_r(t_k) \\ r(t_k) = -\frac{v_t(t_k)}{\dot{\theta}(t_k)} \end{cases} \tag{1}$$

In the above equations, $\theta(t_k)$ represents the measurement value of azimuth angle at time t_k, $\dot{\theta}(t_k)$ represents the change rate of azimuth angle at time t_k, $x_p(t_k)$ represents the abscissa of the observation platform at time t_k, $x_e(t_k)$ represents the abscissa of the target at time t_k, $y_p(t_k)$ represents the ordinate of the observation platform at time t_k, $y_e(t_k)$ represents the ordinate of the target at time t_k, $\dot{f}_d(t_k)$ represents the measurement value of the change rate of Doppler frequency at time t_k, $r(t_k)$ represents the radial distance between the target and the platform at time t_k,

$\ddot{r}(t_k)$ represents the radial acceleration of the target relative to the platform at time t_k, $v_t(t_k)$ represents the tangential velocity of the target relative to the platform at time t_k, $a_r(t_k)$ represents the acceleration component of the target caused by the force to which it's subjected in the direction between the target and the platform at time t_k, f represents the coming frequency of the target, c represents the velocity of light in vacuum. So we can get from above equations:

$$\begin{aligned} x_e(t_k) &= \frac{-c \cdot \dot{f}_d(t_k) + a_r(t_k) \cdot f}{f \cdot \dot{\theta}^2(t_k)} \cos(\theta(t_k)) + x_p(t_k) \\ y_e(t_k) &= \frac{-c \cdot \dot{f}_d(t_k) + a_r(t_k) \cdot f}{f \cdot \dot{\theta}^2(t_k)} \sin(\theta(t_k)) + y_p(t_k) \end{aligned} \tag{2}$$

Obviously we can see, the positioning solution is dependent on the value of $\dot{\theta}^2(t_k)$ and $\dot{f}_d(t_k)$. Considering the existing measurement error in practice, a larger measurement error of azimuth angle will decrease the observability of the target because $\dot{\theta}^2(t_k)$ will be affected by the measurement error of azimuth angle, either.

2.2 Target Initial Positioning

Under the circumstance of passive location of maneuvering targets, although there is the radial distance between the target and the platform that we can get from the change rate of Doppler frequency, and we can locate the target just by the combination of the azimuth angle and the change rate of Doppler frequency, but the existing measurement error may cause a huge error on the positioning of targets, and the filtering performance won't be assured unless we optimize the results of target positioning.

Figure 1 indicates the geometric relation between platform trajectory and radiation source. At time t_k, the azimuth angle $\theta(x_e(t_k), y_e(t_k), t_k)$ and the change rate of Doppler frequency $\dot{f}_d(x_e(t_k), y_e(t_k), t_k)$ that the radar measures can be expressed respectively as follows:

$$\begin{cases} \theta(t_k) = \arctan\left(\frac{y_e(t_k) - y_p(t_k)}{x_e(t_k) - x_p(t_k)}\right) + \sigma_\theta \\ \dot{f}_d(t_k) = -\frac{\dot{\theta}^2(t_k) \cdot r(t_k) + a_r(t_k)}{\lambda} + \sigma_{\dot{f}_d} \end{cases} \tag{3}$$

In which, σ_θ and $\sigma_{\dot{f}_d}$ represent the measurement error of the azimuth of target and the change rate of Doppler frequency respectively, and they are often considered as priori information; $r(t_k)$ represents the radial distance between the target and the observation platform at time t_k, $a_r(t_k)$ represents the radial acceleration of the target, λ represents the wavelength of target emitter signal. Obviously, we can see that the

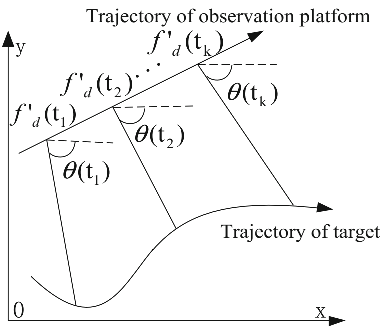


Fig. 1 Geometric relation between platform trajectory and radiation source

accuracy of $\theta(t_k)$ and $\dot{f}_d(t_k)$ depends on the measurement error and the geometric relation between the platform trajectory and the radiation source.

The main idea of initial positioning is as below: As to the target concerned, it's assumed that the target is not maneuvered at the initial time, then the positioning result in a period of time will be calculated, and each individual positioning point will be smoothed to the latest time, and the mean values of smoothed results will be the initial positioning results.

In the above process, we calculate the wavelength λ by the detected target's radiation source signal frequency f instead of signal carrier frequency f_c, but due to the Doppler frequency, f and the real frequency f_c are different. So in this case, we define:

$$f = f_c + f_d = \left(1 - \frac{\dot{r}}{c}\right) f_c \tag{4}$$

The relative frequency error caused by it is: $\frac{\Delta f}{f_c} = \frac{f_d}{f_c} = -\frac{\dot{r}}{c}$, due to $|\dot{r}/c| \ll 1$, the bias can be neglected.

2.3 Tracking Maneuvering Targets

Because tracking targets is a nonlinear filtering process, so in this paper, we choose the Unscented Kalman Filter, and there is no need to linearize the nonlinear system. And we choose the intersecting multiple model (IMM) algorithm to track targets due to the changing motion form of maneuvering targets. From time k − 1 to k, The recursive process of the intersecting multiple model (IMM) algorithm which contains N models is as follows:

1. The intersection of state estimation
   Let the transition probability from model i to model j be $P_{t_{ij}}$

$$P_{t_{ij}} = \begin{bmatrix} P_{t_{11}} & P_{t_{12}} & \cdots & P_{t_{1N}} \\ P_{t_{21}} & P_{t_{22}} & \cdots & P_{t_{2N}} \\ \vdots & \vdots & \ddots & \vdots \\ P_{t_{N1}} & P_{t_{N2}} & \cdots & P_{t_{NN}} \end{bmatrix} \tag{5}$$

   Let $\hat{X}^j(k-1|k-1)$ be the state estimation of filter j at time k − 1, and $P^j(k-1|k-1)$ be the corresponding state covariance, and $u_{k-1}(j)$ be the probability of model j at time k − 1, and $i,j = 1,2...,N$. So after intersection, the input of each filter at time k is:

$$\hat{X}^{oj}(k-1|k-1) = \sum_{i=1}^{N} \hat{X}^i(k-1|k-1)u_{k-1|k-1}(i|j) \tag{6}$$

   In which, $\begin{cases} u_{k-1|k-1}(i|j) = \frac{1}{\bar{C}_j} P_{t_{ij}} u_{k-1}(i) \\ \bar{C}_j = \sum\limits_{i=1}^{N} P_{t_{ij}} u_{k-1}(i) \end{cases}$

$$P^{oj}(k-1|k-1) = \sum_{i=1}^{N} \left\{ P^i(k-1|k-1) + \left[\hat{X}^i(k-1|k-1) - \hat{X}^{oj}(k-1|k-1)\right] \times \left[\hat{X}^i(k-1|k-1) - \hat{X}^{oj}(k-1|k-1)\right]^T \right\} u_{k-1|k-1}(i|j)$$

2. Model updating
   Let the state vector $\hat{X}^{oj}(k-1|k-1)$ and its variance $P^{oj}(k-1|k-1)$ and the observed value $Z(k)$ as the input of model j at time k; by means of the selected filters, we can calculate the output of each model, which are $\hat{X}^j(k|k)$, $P^j(k|k)$, $j = 1,2,...,N$.
3. Model possibility calculation
   If the filtering residual of model j is v_k^j, and the corresponding covariance is S_k^j, and assumed to obey the Gauss distribution in the meantime, then the possibility of model j is:

$$\Lambda_k^j = \frac{1}{\sqrt{|2\pi S_k^j|}} \exp\left[-\frac{1}{2}(v_k^j)'(S_k^j)^{-1} v_k^j\right] \tag{7}$$

   In which, $\begin{cases} v_k^j = Z(k) - H^j(k)\hat{X}^j(k|k-1) \\ S_k^j = H^j(k)P^j(k|k-1)H^j(k)' + R(k) \end{cases}$

4. Model probability updating

$$u_k(j) = \frac{1}{C}\Lambda_k^j \bar{C}_j \tag{8}$$

In which, $C = \sum_{i=1}^{N} \Lambda_k^i \bar{C}_i$

5. Model output

Let $\hat{X}(k|k)$ and $\hat{X}(k|k)$ be the output of the intersecting model respectively at time k, and we can get:

$$\hat{X}(k|k) = \sum_{i=1}^{N} \hat{X}^i(k|k) u_k(i) \tag{9}$$

$$\begin{aligned} P(k|k) = \sum_{i=1}^{N} u_k(i) \Big\{ P^i(k|k) + \big[\hat{X}^i(k|k) - \hat{X}(k|k)\big] \\ \times \big[\hat{X}^i(k|k) - \hat{X}(k|k)\big]^T \Big\} \end{aligned} \tag{10}$$

As we can see from the algorithm above, since the measurement information is used in the filtering estimation and the model probability calculation, the adaptive adjustment model can be used to track the maneuvering target, and tracking maneuvering targets without maneuver detection, tracking without time lag.

3 Simulation Test and Experiment

3.1 Observability Analysis of Target

Experiment 1: There are 3 targets to be observed in order to verify the observability. The observation platform is stationary at the coordinate (10 km, 20 km), and all three targets' initial point is at the coordinate (50 km, 50 km) and their initial location is all the same. One of their motion forms is uniform rectilinear, and the trajectory line and the line between the target and the platform is at a 45° angle; One is the radial motion along the path between target and platform; The other is uniform circular motion around the platform. The azimuth angle error is 0.1°, the change rate error of Doppler frequency is 1 Hz/s. Three targets' motion forms are corresponding to three states as follows: Both of the change rate of Doppler frequency and the azimuth angle are observable; The change rate of Doppler frequency is observable and the azimuth angle is not; Only the azimuth angle is observable. And the results of simulation are demonstrated in Fig. 2.

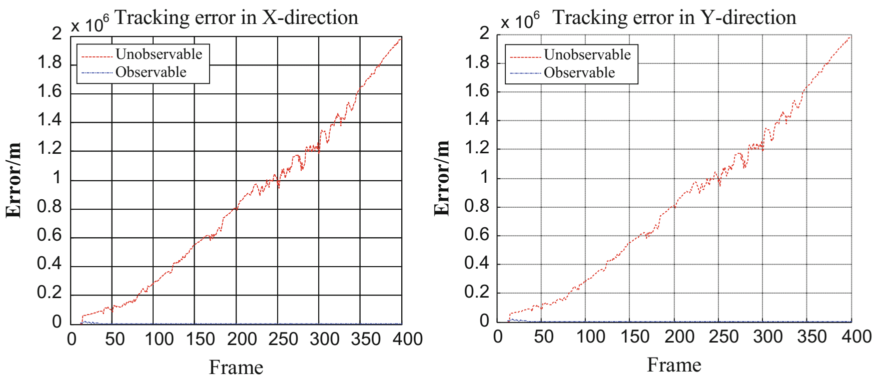


Fig. 2 Observability positioning error

The results shows that when the change rate of Doppler frequency is observable and the azimuth angle is not, the location error is far greater than that when only the azimuth angle is observable. As for that, increasing one-dimensional information of Doppler frequency change rate does not necessarily improve the observability of the target. If and only if the both observability of them is good, we can improve the tracking effects through improving observability.

3.2 Initial Analysis of Target

Experiment 2: The simulation environment is the same as that in Experiment 1, and the purpose is to find the influence of target's observability on initial positioning; 300 times Monte Carlo simulation will be carried out, and the results are shown in Table 1.

Experiment 3: The measurement error of azimuth angle is 0.05°, 0.1°, 0.2° respectively; The measurement error of change rate of Doppler frequency is 0.5, 1, 3 Hz/s respectively; and the results are shown in Table 2.

The above results shows, with the worse observability of the target, the initial positioning accuracy will decrease. In addition, Experiment 3 shows, with larger

Table 1 Influence of target's observability on initial positioning

Target's motion form	Observability	RMS error of positioning	
		X direction	Y direction
45°	Good	8.42×10^3	8.89×10^3
Along between target and platform	Not	2.73×10^8	2.73×10^8
Circular around the platform	Weak	1.006×10^5	1.001×10^5

Table 2 Influence of measurement error on initial positioning

Measurement error of azimuth (°)	Measurement error of change rate of Doppler frequency (Hz/s)	RMS error of positioning	
		X direction	Y direction
0.05	0.5	3.30×10^3	3.64×10^3
	1	3.41×10^3	3.76×10^3
	3	5.99×10^3	6.57×10^3
0.1	0.5	7.15×10^3	7.51×10^3
	1	7.75×10^3	8.17×10^3
	3	7.86×10^3	8.34×10^3
0.2	0.5	2.97×10^4	3.04×10^4
	1	2.98×10^4	2.97×10^4
	3	3.44×10^4	3.62×10^4

measurement error of the azimuth angle and the change rate of Doppler frequency, the positioning accuracy decreases greatly.

3.3 Tracking Analysis of Target

Experiment 4: the observation platform is at the coordinate (0 km, 0 km), and in uniform rectilinear motion at 550 km/h; the azimuth angle is −60°, and its measurement error is 0.1°, and the change rate of Doppler frequency is 1 Hz/s, and the data updating rate is 1 Hz. The starting point of target is at the coordinate (100 km, 100 km), and the target is set as uniform rectilinear motion at a speed of 300 m/s and the azimuth angle is 135°; UKF tracking purely by azimuth angle and UKF

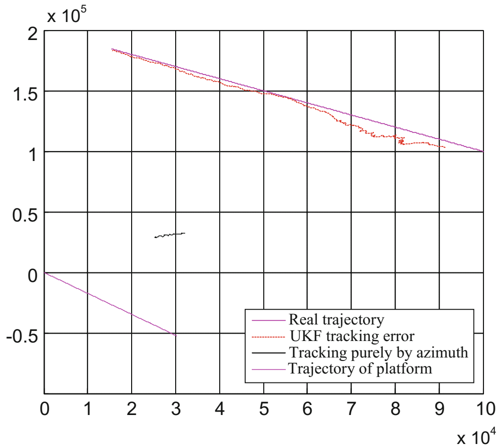


Fig. 3 Tracking results

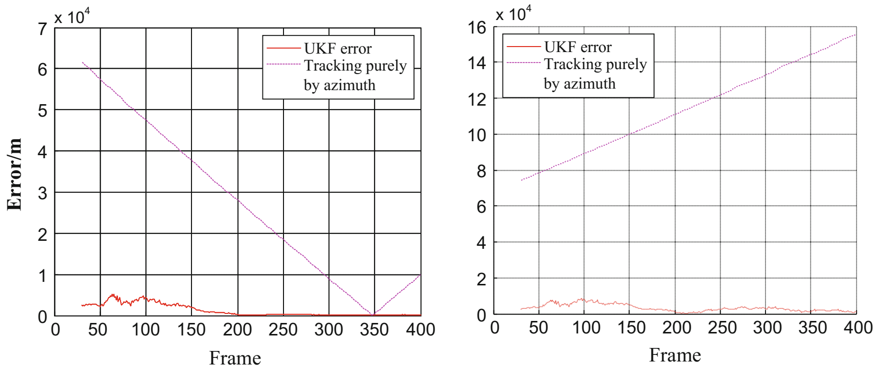


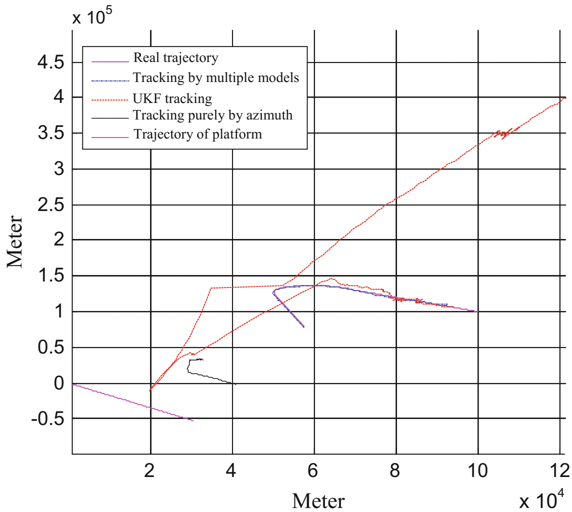
Fig. 4 Tracking error in X-direction and Y-direction respectively

tracking by both of azimuth angle and change rate of Doppler frequency will be carried out respectively (Figs. 3 and 4).

As is shown above, compared with tracking purely by azimuth angle, the effect of tracking by the addition of the change rate of Doppler frequency is more good and satisfactory.

Experiment 5: the environment is set as same as that in Experiment 4. The starting point of target is at the coordinate (100 km, 100 km). In the first, the target is set as uniform rectilinear motion for a period of time at a speed of 300 m/s and the azimuth angle is 135°, then the target turns and move as uniform rectilinear motion again. UKF tracking purely by azimuth angle, UKF tracking by both of azimuth angle and change rate of Doppler frequency and IMM_UKF tracking by

Fig. 5 Tracking results



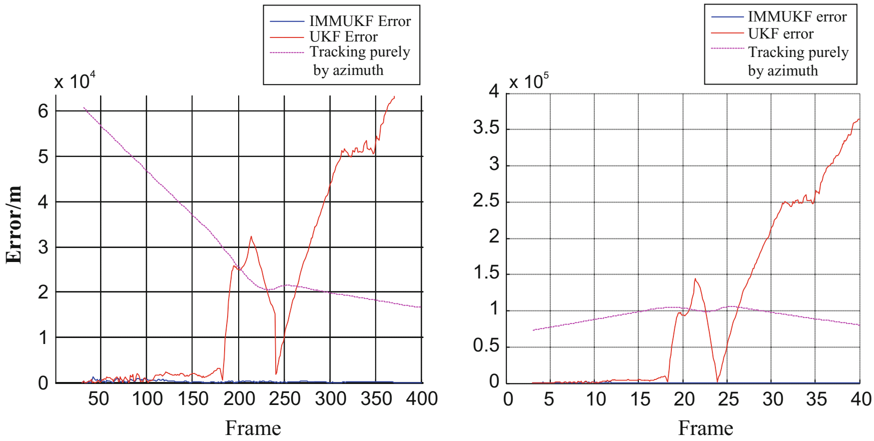


Fig. 6 Tracking error in X-direction and Y-direction respectively

both of azimuth angle and change rate of Doppler frequency will be carried out respectively (Figs. 5 and 6).

The Experiment 5 shows that, in the initial phase of uniform rectilinear motion, the tracking effects of UKF and IMM_UKF with the addition of the change rate of Doppler frequency are good and satisfactory, but when the target maneuvers, the IMM_UKF can quickly converges, but the algorithms which don't use intersecting multiple model (IMM) distinctly diverge.

4 Conclusions

This paper mainly studied the positioning and tracking maneuvering targets in observability, initial positioning and tracking; Compared with tracking purely by azimuth, the addition of the change rate of Doppler frequency, if both of them are well observable in the meantime, would increase the accuracy of initial positioning greatly, then improve the tracking performance. The experiments also shows that the intersecting multiple model (IMM) would achieve real-time tracking for maneuvering targets.

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