

The Comparative Study of Mars Entry Phase's Guidance Methods

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1 Introduction

As one of the closest planets to earth, Mars is similar to earth in many ways [1]. Therefore, it has become one of the most important planets for deep space exploration [2]. Considering that the Mars atmosphere's density is only 1% of the earth's atmosphere, so compared with the spacecraft return to earth, weak control capability and parameter uncertainty should be considered, and the adaptive ability of guidance method has become a hot spot in the field of Mars Exploration [3]. So far, the world has conducted 39 Mars Exploration, but only 7 landing missions are successful [4, 5].

The guidance methods of entry phase are generally divided into the method of tracking nominal trajectory and predictor-corrector method. The former one has the advantages of simple control law, easy realization, but it is sensitive to the initial entry conditions. In order to improve the accuracy, the first one is to study the trajectory tracking method with robust performance and adaptive ability, and the second one is to study the trajectory planning algorithm online. Such as, the sliding mode variable structure control (SMC) is adopted, which can obtain better guidance accuracy [6]. However, this method is difficult to be used in engineering practice because of the buffet. Benito and Mease use nonlinear model predictive control which can achieve high guidance accuracy. The predictor-corrector guidance method uses numerical or analytical methods for predicting the placement of entry phase's final point and the final conditions. The correctional step is to get the instructions of bank angle to nullify the final errors in real-time. Now, with the rapid development of the aerospace computer, it provides the basis for the application of numerical prediction guidance method.

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Foreign scholars commonly use the predictor corrector guidance method based on iterative numerical correction to get the instructions of bank angle. The all-coefficient adaptive predictor-corrector guidance method which is proposed by Hu Jun is different from the traditional predictor-corrector guidance method [7]. In this method, the characteristic model which describes the relationship between guidance increment and the error of generalized predictive range is established. The guidance law calculates the characteristic increment based on the range error from the predictor step, and then the new range error is predicted with the new guidance instructions. Compared with the traditional predictor-corrector guidance method, this method is a non-iterative method which relieves the calculation burden of computer, so it can be used in engineering practice problems.

In this paper, the all-coefficient intelligent adaptive predictor-corrector guidance method is used. The first-order characteristic model is used which is different from the traditional second order characteristic model in [7]. What's more, the method is compared with the traditional predictor-corrector guidance method based on iteration and the robust guidance method of tracking nominal trajectory, so the method which is more suitable for engineering application can be found.

2 The Formulation of Mars Entry Guidance Problem

The entry guidance problem for the Mars detector is to determine bank angle commands such that the Mars detector can meet the final constraints. In this paper, the entry guidance model is given as follows:

$$\dot{r} = v \sin \gamma \quad (1)$$

$$\dot{\theta} = \frac{v \cos \gamma \sin \psi}{r \cos \phi} \quad (2)$$

$$\dot{\phi} = \frac{v \cos \gamma \cos \psi}{r} \quad (3)$$

$$\dot{v} = -D - g \sin \gamma \quad (4)$$

$$\dot{\gamma} = \frac{1}{v} \left[L \cos \sigma + \left(\frac{v^2}{r} - g \right) \cos \gamma \right] \quad (5)$$

$$\dot{\psi} = \frac{1}{v} \left[\frac{L \sin \sigma}{\cos \gamma} + \frac{v^2}{r} \cos \gamma \sin \psi \tan \phi \right] \quad (6)$$

where r is radial distance from the center of the Mars, θ and ϕ are the longitude and latitude, v is the velocity, γ is the flight path angle, ψ is the velocity azimuth angle

and σ is the bank angle. L and D are the acceleration of aerodynamic lift and drag. The aerodynamic lift and drag forces are given by

$$L = \frac{0.5\rho v^2 C_L S}{m} \quad (7)$$

$$D = \frac{0.5\rho v^2 C_D S}{m} \quad (8)$$

3 The Guidance Methods of Mars Entry Guidance

In this section, the all-coefficient intelligent adaptive predictor-corrector guidance method based on the first-order characteristic model is introduced. The traditional predictor-corrector guidance method based on iteration is also introduced simply.

3.1 *The All-Coefficient Intelligent Adaptive Predictor-Corrector Guidance Method*

During every period of guidance, the actual landing point is predicted with the Eqs. (1)–(6) using the fixed step 4th order Runge-Kutta method, and then the predicted range error can be obtained. With the range error, the guidance method calculates the modification value of bank angle. In the all-coefficient intelligent adaptive predictor-corrector guidance method, the guidance method of integral type is used. In other words, the modification value is added on the guidance instructions of last guidance period, so the integrator must be used in the guidance method. In this method, the elimination of the error is evenly distributed from the current point to the terminal point of the whole interval.

In the correction step, the relation between the variation of lift characteristic quantity and range change must be known. Measured from the nominal terminal time, if the bank angle changes earlier, the amount of range change is larger. A special situation is researched that the detector is always in a nominal flight state before the adding of the increment of lift characteristic variable. Although it is a special case, it can be seen that the relationship between the increment and the range error. From the Fig. 1, we can see that the range change $D_1(t)$ when the increment is 0.1 and -0.1 . The definition is given that $D(t) = D_1(t)/0.1$ is the time-varying dynamic amplification factor between the increment of the lift characteristic variable and the range change [7].

Although the above dynamic amplification factor is the nominal situation, it has great significance for the guidance, because the time-varying dynamic factor for the non-nominal situation can be divided into two parts artificially. The two parts are

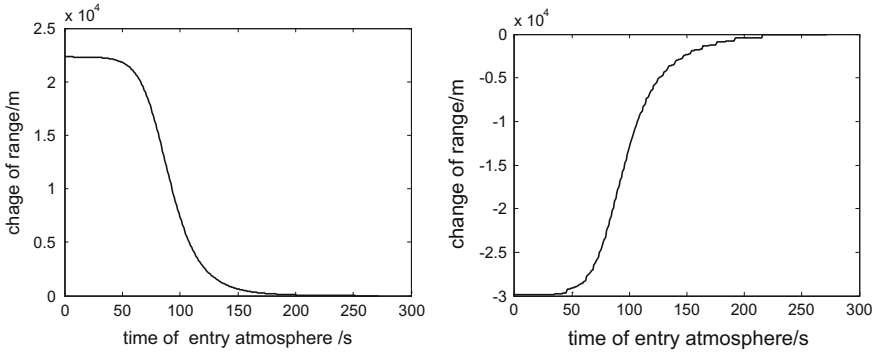


Fig. 1 The change value of range when the increment is 0.1 and -0.1

the known $\bar{D}(t)$ and the unknown $\Delta D(t)$ [8]. The unknown $\Delta D(t)$ is calculated by the following intelligent adaptive algorithm.

In the guidance method, the relationship between the range error and the increment is represented by the time-varying dynamic system based on first-order characteristic model, and the coefficients of the first-order characteristic model are estimated online. The all-coefficient intelligent adaptive theory is built on the sum of all coefficients is 1. In order to use the all-coefficient adaptive theory, input and output conversion is done using the time-varying dynamic factor. The known $\bar{D}(t)$ will be added to the controlled system as the transformation coefficient, and then the magnification of the transformed system is around 1, so the adaptive predictor-corrector guidance method can use the all-coefficient adaptive theory [7]. Through analysis, the relationship between the predicted range error after the input conversion and the increment of lift characteristic variable can be described by the following first order difference equation with variable coefficients. It is different from the traditional second order characteristic model

$$y(k + 1) = f(k)y(k) + g(k)u(k) \tag{9}$$

where $y(k)$ represents the predicted range error after the input conversion, and $u(k)$ represents the increment of lift characteristic variable.

Based on the all-coefficient adaptive theory, the unknown time-varying parameters $f(k)$ and $g(k)$ are identified using gradient method, and then the estimated parameter values are projected into their respective ranges because there are estimated parameter values constraints using the adaptive control.

Define the regression vectors

$$\alpha(k) = [y(k - 1), u(k - 1)]^T \tag{10}$$

The parameter vectors

$$\mathbf{\beta}(k) = [f(k), g(k)]^T \tag{11}$$

The estimated parameter vectors are represented as $\hat{\mathbf{\beta}}(k) = [\hat{f}(k), \hat{g}(k)]^T$.

The gradient method to identify the time-varying parameters is

$$\hat{\mathbf{\beta}}(k) = \hat{\mathbf{\beta}}(k - 1) + \frac{\lambda_1 \boldsymbol{\alpha}(k)}{\boldsymbol{\alpha}^T(k) \boldsymbol{\alpha}(k) + \lambda_2} [y(k) - \boldsymbol{\alpha}^T(k) \hat{\mathbf{\beta}}(k - 1)] \tag{12}$$

where λ_1 and λ_2 are positive constants.

Then the values are projected into the following respective ranges

$$f(k) \in [f_{\min}, f_{\max}], \quad g(k) \in [g_{\min}, g_{\max}] \tag{13}$$

So the gradient method can be reformulated as

$$\hat{\mathbf{\beta}}(k) = \pi_D \left\{ \hat{\mathbf{\beta}}(k - 1) + \frac{\lambda_1 \boldsymbol{\alpha}(k)}{\boldsymbol{\alpha}^T(k) \boldsymbol{\alpha}(k) + \lambda_2} [y(k) - \boldsymbol{\alpha}^T(k) \hat{\mathbf{\beta}}(k - 1)] \right\} \tag{14}$$

where π_D represents the orthogonal projection on set D . The set D is obtained by the formula (13). And then the increment $u(k)$ of bank angle can be obtained by the following linear feedback control

$$u(k) = -L \cdot \hat{f}(k) y(k) / (\hat{g}(k) + \lambda) \tag{15}$$

where L is the parameter of controller, and the sign of λ is same as $g(k)$.

The adaptive predictor-corrector guidance method is a non-iterative method and the adaptive control is used only once during each guidance period.

Because the all-coefficient guidance method is the guidance method of integral type, the total control value $u_1(k)$ is obtained by

$$u_1(k) = u_1(k - 1) + u(k) \tag{16}$$

where $u_1(k - 1)$ is the bank angle of the last guidance period.

The lateral guidance isn't discussed too much in this paper. The sign of bank angle is determined by a lateral logic that the crossrange threshold is used.

3.2 The Predictor Corrector Guidance Method Based on Iteration

In this section, the predictor corrector guidance method based on iteration will be introduced simply, and the detailed content can be found in the paper [8]. In this method, the bank angle magnitude at any range-to-go is

$$|\sigma_s| = \sigma_f + \frac{S_s - S_f}{S_{togo} - S_f} (\sigma_c - \sigma_f) \tag{17}$$

where σ_f represents the bank angle of terminal time, S_f represents the range-to-go of terminal time, σ_s and S_s represent the bank angle and the range-to-go of the current interval point, and σ_c and S_{togo} represent the bank angle and the range-to-go of the current position.

Using the above bank angle strategy, the predicted range error can be obtained. And then the correction step is done. The traditional predictor corrector guidance method uses the iteration technique which can be achieved as follows:

$$\cos \sigma_{n+1} = \cos \sigma_n - \frac{\cos \sigma_n - \cos \sigma_{n-1}}{\Delta s(\sigma_n) - \Delta s(\sigma_{n-1})} \Delta s(\sigma_n) \tag{18}$$

where $\Delta s(\sigma_n)$ is the range error of the current guidance period.

4 Numerical Simulation

First of all, the all-coefficient intelligent adaptive predictor-corrector guidance method is compared with the traditional predictor-corrector guidance method based on iteration. The nominal values and ranges of variation are given in Table 1.

Given the same and biggest errors in the initial position, and aerodynamic coefficients uncertainty, such as the drag and lift coefficient increase by 20%, the simulation is done. The Figure 2 shows the simulation results. The bank angle range of all-coefficient intelligent adaptive method is smaller than the method based on iteration, and this can leave some margin for the bank angle. In addition, the bank angle of terminal time is zero in the all-coefficient intelligent adaptive method which can create good condition for opening the parachute.

In order to see the results clearly, the third figure gives the change of longitude and latitude for the final 30 s, the actual landing points and the expected landing point. We can see that the all-coefficient intelligent adaptive method has higher

Table 1 Initial states and dispersion ranges

States	Initial values	Ranges of variation
Height (km)	125	[-3.5, 3.5]
Velocity (m/s)	5505	[-100, 100]
Flight path angle	-14.5°	[-0.3°, 0.3°]
Longitude	-90.072°	[-1°, 1°]
Latitude	43.88°	[-0.5°, 0.5°]
Velocity azimuth angle	85.01°	[-0.2°, 0.2°]

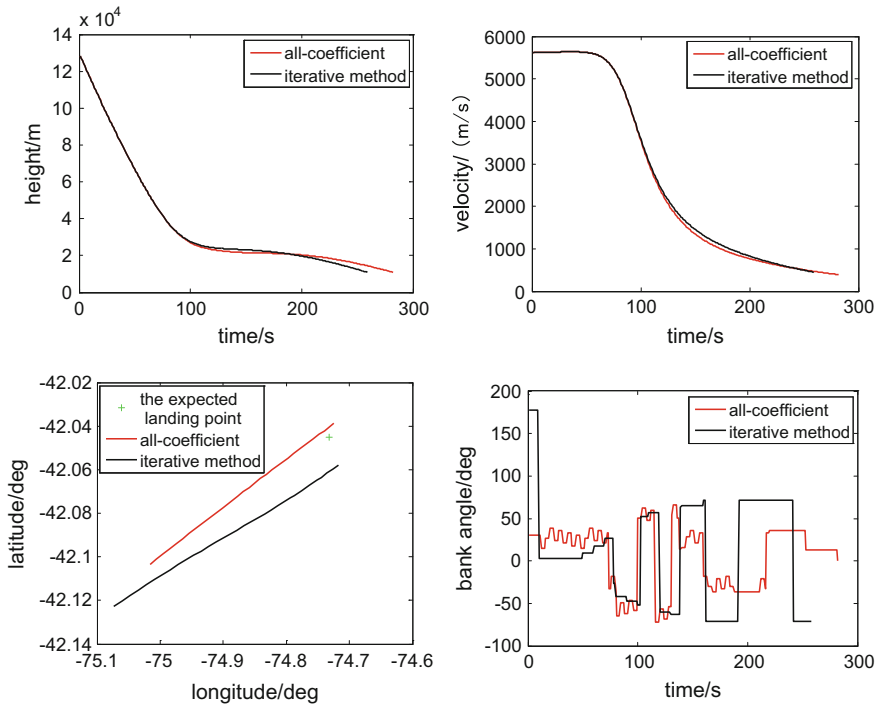


Fig. 2 The comparison charts of two predictor-corrector guidance methods

accuracy than the iterative method. Taking the above simulation condition as an example, the time of all-coefficient intelligent adaptive method is 0.31 s, and the time of iterative method is 1.19 s, so the former is easier to implement in the engineering and the property of the real time can be guaranteed.

Then the Monte Carlo simulation using the all-coefficient intelligent adaptive predictor-corrector guidance method is done with random variations in the initial states. The Fig. 3 shows the 40 guided trajectory charts of intelligent adaptive predictor-corrector guidance method. In the third chart of Fig. 3, the red points represent the entry points of detector and the cyan point represents the expected landing point. The method can guide the detector to the expected landing point accurately. The height and velocity can also satisfy the terminal conditions.

A 300-run Monte Carlo simulation is done. From Fig. 4 we can see that the range errors are all within 1.6 km, so the method has high accuracy.

Finally the two predictor-corrector methods are compared with the robust guidance method in [9]. The results show that the maximum point radius is not more than 1.5 km using the all-coefficient intelligent adaptive predictor-corrector guidance method, and the maximum point radius is not more than 3 km using the predictor-corrector guidance method based on iteration. If the initial states errors are big, the reference trajectory can't be tracked using the robust reference trajectory

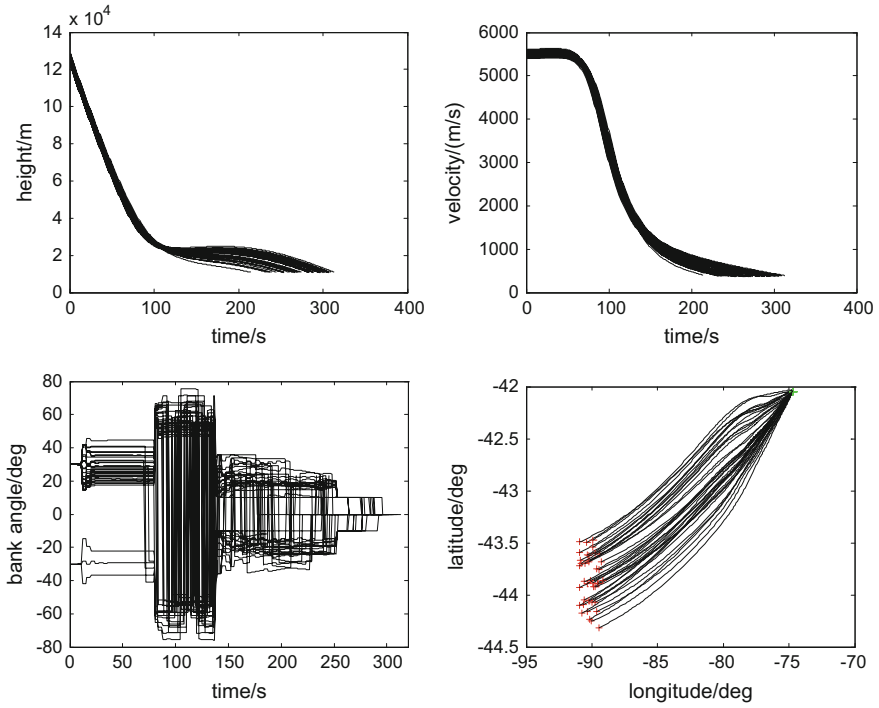


Fig. 3 The 40 guided trajectory charts of adaptive predictor-corrector guidance methods

Fig. 4 The range errors of 300-run Monte Carlo simulation

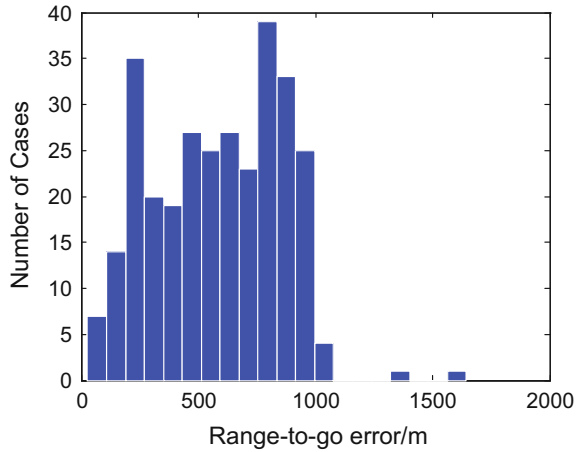


Table 2 Initial states errors

States	Initial values	Ranges of variation
Height (km)	125	[-1.2, 1.2]
Velocity (m/s)	5505	[-50, 50]
Flight path angle	-14.15°	[-0.1°, 0.1°]
Longitude	-90.072°	[-0.2°, 0.2°]
Latitude	43.898°	[-0.1°, 0.1°]
Velocity azimuth angle	85.01°	[-0.1°, 0.1°]

guidance method. Table 2 shows the initial states errors that the robust reference trajectory guidance method can be tolerant. We can see the initial errors are smaller than the ones in Table 1. The tracking reference trajectory guidance method is sensitive to the initial reentry conditions.

5 Conclusion

Aiming at the problem of Mars entry guidance, the all-coefficient intelligent adaptive predictor-corrector guidance method based on the first-order characteristic model is proposed. And it is compared with the predictor-corrector guidance method based on iteration and robust tracking reference trajectory method. It can be seen that the all-coefficient intelligent adaptive predictor-corrector guidance method is more suitable for the problem of Mars entry guidance.

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