

A New Tuning Approach to Second-Order Active Disturbance Rejection Control

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1 Introduction

Active disturbance rejection control (ADRC) is proposed by Han in 1998 [1]. It is able to actively estimate and compensate total disturbance in real time. ADRC has been applied in numerous industrial processes and achieved desired performance, such as superconducting cavity control [2], flywheel energy storage system [3], tracking of IPSRU [4], hysteresis compensation [5], piezoelectric beam control [6], pneumatic force control system [7] and internal permanent-magnet synchronous motor control [8].

But in practice, parameter tuning of ADRC is somewhat difficult. Reasons are as follows. Firstly, many parameters have to be determined. Secondly, experience is indispensable. In order to reduce the difficulty of determining parameters, numerous approaches have been proposed. Genetic algorithms [9], neural networks [10] have been utilized to optimize the ADRC's parameters and tuning processes, but those algorithms are time-consuming, which may not be practical in engineering. In general, n th-order controllers are designed to control n th-order plants, and bandwidth parameterization approach is proposed for linear ADRC (LADRC) [11]. However, it is costly to identify the exact order of a given plant and bandwidth parameterization approach is also not optimized for a specified performance index.

Generally, for simplification, second order controller, including velocity and acceleration information, is enough for control engineering. For getting a simple, effective and optimal controller parameter tuning approach and for applying ADRC to a wider range, just like PID, it is necessary to fix the order of ADRC and optimize the parameters of ADRC. Researchers have discussed cases that second order ADRC controllers for non-second-order plants [12], but it is only used for

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first to third order systems and authors did not give out a clear parameter tuning approach.

Integral of time-multiplied absolute-value of error (ITAE), one of the general performance indexes for evaluating the performance of a closed-loop system, is taken for single parameter optimal control and adaptive control [13–15]. Since ITAE is an index that describes system performance from the point of fast and accuracy, a set of normalized transfer function coefficients have been obtained by minimizing ITAE values [16]. Scholars have taken such idea to optimize PID tuning rules [17].

In this paper, we also use ITAE index to refine the second order ADRC tuning rule, and an ITAE optimal bandwidth parameterization method is proposed. For the approach proposed in this paper, only three parameters are needed to be tuned, which does make the tuning processes easier and guarantee fast and accurate system response.

2 Second Order Active Disturbance Rejection Control

In this paper, following second order system is considered

$$\ddot{y} = f + b_0u \tag{1}$$

where y is the system output, f is the total disturbance of the system, u is the control signal, and b_0 is the coefficient of u .

A closed-loop system by second order ADRC is shown in Fig. 1.

where r is the set value, y is the output, u represents the control input, k_p and k_d are control parameters, G_p represents the controlled plant.

Extended state observer (ESO) takes the form as follows

$$\begin{cases} \dot{z}_1 = z_2 + \beta_1(y - z_1) \\ \dot{z}_2 = z_3 + \beta_2(y - z_1) + b_0u \\ \dot{z}_3 = \beta_3(y - z_1) \end{cases} \tag{2}$$

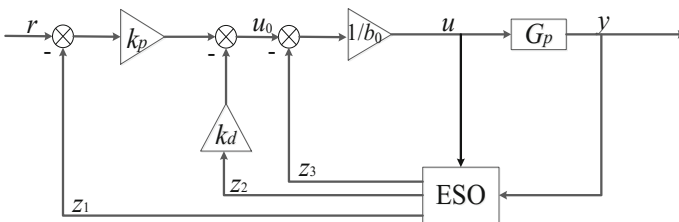


Fig. 1 Structure of closed-loop system by second order ADRC

where $\beta_1, \beta_2, \beta_3$ are observer parameters, b_0 is the coefficient of control input, z_1 is the estimation of system output, z_2 and z_3 represent the estimation of the velocity and total disturbance of the system, respectively.

Control law can be designed as

$$\begin{cases} u_0 = k_p(r - z_1) - k_d z_2 \\ u = (u_0 - z_3)/b_0 \end{cases} \tag{3}$$

If ESO works well, i.e. $z_1 \rightarrow y, z_2 \rightarrow \dot{y}, z_3 \rightarrow f$, we have

$$\ddot{y} = f + b_0 u \approx u_0 \tag{4}$$

i.e.

$$\ddot{y} \approx k_p(r - y) - k_d \dot{y} \tag{5}$$

Then we have the transfer function of the closed-loop system

$$G_{cl}(s) = \frac{y(s)}{r(s)} = \frac{k_p}{s^2 + k_d s + k_p} \tag{6}$$

where s is the Laplace operator.

3 ITAE Optimal Bandwidth Parameterization Approach

Here, ITAE optimal bandwidth parameterization approach is proposed. Firstly, the bandwidth parameterization approach is briefly introduced and then the new approach will be described.

Let controller bandwidth be ω_c and define [11]

$$k_p = \omega_c^2, \quad k_d = 2\omega_c \tag{7}$$

Also, let the bandwidth of ESO be ω_o and define

$$\beta_1 = 3\omega_o, \quad \beta_2 = 3\omega_o^2, \quad \beta_3 = \omega_o^3 \tag{8}$$

One can choose ω_c, ω_o and b_0 to get desired system response.

Based on above bandwidth parameterization approach, ITAE optimal bandwidth parameterization approach can be described as follows.

Let $k_p = \omega_n^2$, $S = s/\omega_n$, then Eq. (6) becomes

$$G_{clopt}(s) = \frac{\omega_n^2}{s^2 + k_d s + \omega_n^2} = \frac{1}{S^2 + \frac{k_d}{\omega_n} S + 1} \quad (9)$$

According to Graham [16], the second-order ITAE optimal transfer function can be written as

$$G_{opt}(s) = \frac{1}{S^2 + 1.41S + 1} \quad (10)$$

Then, we have

$$k_p = \omega_n^2, \quad k_d = 1.41\omega_n \quad (11)$$

For the parameters of ESO, we also take Eq. (8).

Therefore, for the ITAE optimal bandwidth parameterization approach, its adjustable parameter becomes ω_n, ω_o, b_0 . By such approach, we can adjust the un-damped natural oscillation frequency and obtain the ITAE optimal response.

4 Simulation Results

Assuming that step signal is the desired output. Four typical plants have been considered. Numerical results are shown in Table 1.

Table 1 shows the system responses by bandwidth parameterization approach and ITAE optimal bandwidth parameterization approach. Both time and frequency domain responses have been given to show the difference. From the time domain responses, it can be seen that the responding speed of the ITAE optimal bandwidth parameterization approach is faster than the bandwidth parameterization approach and such fact is also confirmed by the comparison between frequency responses. In order to depict the difference much clearer, closed-loop bandwidth ω_b , gain margin G_m , phase margin P_m and ITAE values have been listed in Table 2.

From Table 2, we can see that ω_{b2} is greater than ω_{b1} in all cases. It means that the system response got by ITAE optimal bandwidth parameterization approach is faster. Furthermore, G_{m1} and G_{m2} are generally close to each other, P_{m1} is slightly bigger than P_{m2} , which means that a small part of the stability is sacrificed by taking ITAE optimal method. Obviously, $ITAE_2$ is less than $ITAE_1$, which confirms the proposed approach is optimal in ITAE.

In summary, although sacrificing a little stability margin, the new tuning approach is able to improve the dynamic performance of the system.

Table 1 Comparisons of system responses by two tuning approaches

Plants		Bandwidth parameterization	ITAE optimal bandwidth parameterization
$G_{p1}(s) = \frac{e^{-5s}}{(s+1)^3}$	Parameters	$b_0 = 12, \omega_c = 1, \omega_o = 4$	$b_0 = 1, \omega_n = 0.8, \omega_o = 1$
	Time domain response		
	Frequency domain response		
$G_{p2}(s) = \frac{1-2s}{(s+1)^3}$	Parameters	$b_0 = 4.5, \omega_c = 1, \omega_o = 4$	$b_0 = 6, \omega_n = 200, \omega_o = 1$
	Time domain response		
	Frequency domain response		
$G_{p3}(s) = \frac{1}{s(s+1)^3}$	Parameters	$b_0 = 0.58, \omega_c = 0.4, \omega_o = 1.6$	$b_0 = 6, \omega_n = 0.3, \omega_o = 60$
	Time domain response		
	Frequency domain response		
$G_{p4}(s) = \frac{1}{(s+1)^6}$	Parameters	$b_0 = 8, \omega_c = 1, \omega_o = 4$	$b_0 = 1, \omega_n = 1.1, \omega_o = 1$
	Time domain response		
	Frequency domain response		

Table 2 Comparisons of indexes by two tuning approaches

Plants	Bandwidth parameterization				ITAE optimal bandwidth parameterization			
	ω_{b1}	G_{m1}	P_{m1}	ITAE ₁	ω_{b2}	G_{m2}	P_{m2}	ITAE ₂
$G_{p1}(s) = \frac{e^{-5s}}{(s+1)^3}$	0.642	92	180	160	0.801	94.5	171.180	110
$G_{p2}(s) = \frac{1-2s}{(s+1)^3}$	0.642	72.5	180	35	200.360	83.7	171.152	31
$G_{p3}(s) = \frac{1}{s(s+1)^3}$	0.257	92	180	40	0.301	76.7	171.077	35
$G_{p4}(s) = \frac{1}{(s+1)^6}$	0.642	92	180	88	1.102	91	171.152	58

5 Conclusion

In this paper, a new tuning approach, i.e. ITAE optimal bandwidth parameterization approach, has been proposed for second order active disturbance rejection control. Four different controlled plants are considered to confirm the proposed approach. Both bandwidth parameterization tuning approach and ITAE optimal bandwidth parameterization approach are taken in the simulations. By comparing time domain responses, the frequency domain responses and the ITAE values, we can arrive at that the new tuning approach is able to improve the system dynamic performance effectively. It provides another practical tuning approach for LADRC.

Acknowledgements This work is supported by National Science Technology Support Plan Projects 2015BAK36B04.

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