The Research on the Dynamic Performance Test Method Based on SPHS

Jun Xiao Li, Xue Mei Wang, Zhe Xu and Tong Wu

1 Introduction

Dynamic performance testing is the application of modern science and technology. The dynamic performance of the system is tested by signal acquisition, transformation, transmission and real-time processing. It combines modern sensor technology, computer technology, signal analysis and processing technology and other disciplines and applied to the test system and test process. Different from the traditional testing process, the latter is just a simple comparison of measurement. But dynamic performance testing uses signal acquisition, signal conversion and signal processing and analysis to get a more comprehensive system characteristics. The technique used to analyze the dynamic characteristics and performance index of the system is called the modern dynamic testing technology. The dynamic test is different from the simple numerical correspondence between the output and the input in the traditional test, and the corresponding relationship between the output signal and the input signal is obtained. In the dynamic test, the waveform of the input signal which can output the waveform without distortion is required [1].

This paper introduces the advantages of SPHS in system dynamic testing. Taking multi harmonic phase difference signal as an example, this paper introduces the simulation analysis of the system by MATLAB, and adopts the mirror mapping method to obtain the frequency characteristic of the signal processing. It can avoid the problem of digital ill posed by traditional least square method and reduced the requirements for the test object.

J.X. Li (✉) · X.M. Wang · Z. Xu · T. Wu
Xi'an High Tech Research Institute, Xi'an 710025, China
e-mail: ljx_22@163.com

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2 SPHS

SPHS (Schroeder Phased Harmonic Sequence) is a kind of special multi frequency signal, which was put forward by Schroeder M.R in 1970. It is a kind of periodic multi frequency signal which is formed by the superposition of several sine waves with power, period and initial phase. By adjusting the initial phase angle of the positive (or residual) components of the signal to form its own characteristics [2]. General mathematical expression is:

$$x(t) = \sum_{k=1}^{N} \sqrt{2p_k} \cos(\frac{2\pi k}{T} t + \theta_k), \quad k = 1, 2 \cdots N \tag{1}$$

In Formula (1), N is the number of harmonics contained in the signal; p_k is the relative power of the second harmonic of k; Total signal power is $P = \sum_{k=1}^{N} p_k$; T is the fundamental period of the signal; θ_k is the first phase of the first harmonic of K.

The SPHS signal is encoded by Schroeder, and its phase can be obtained by the following method:

$$\theta_k = \pi[\sum_{i=1}^{k-1} (k-i)p_i], \quad p_i = \frac{P_i}{P} \tag{2}$$

The [•] said the operation rounded to zero, When the harmonic power is evenly distributed, $P_k = \frac{P}{N}, \quad (k = 1, 2 \ldots N)$, its phase $\theta_k = \pi\left[\frac{k^2}{2N}\right]$.

Set parameter N = 30,T = 0.5 s, P_k = 1 W. The waveform of the SPHS signal is shown in Fig. 1.

In the system dynamic characteristic test, the SPHS signal excitation has the following advantages:

(1) Compared with the widely used sine(cosine) point by point scanning method, the SPHS signal excitation method can be used to excite the mode of each frequency point in front, and greatly shorten the test time [3].
(2) With a low peak to peak ratio, the peak factor is relatively small, it can be used to test the system to apply a smooth, uniform excitation, especially suitable for inertial devices, servo devices and other equipment testing [4].
(3) Signal parameters can be set according to the needs, the form is more flexible, can adapt to the needs of a variety of system testing, the application field is more extensive.
(4) SPHS is a periodic signal, if the FFT method is used to analyze the data, it can be cut off for the whole week and avoid spectrum leakage [5].

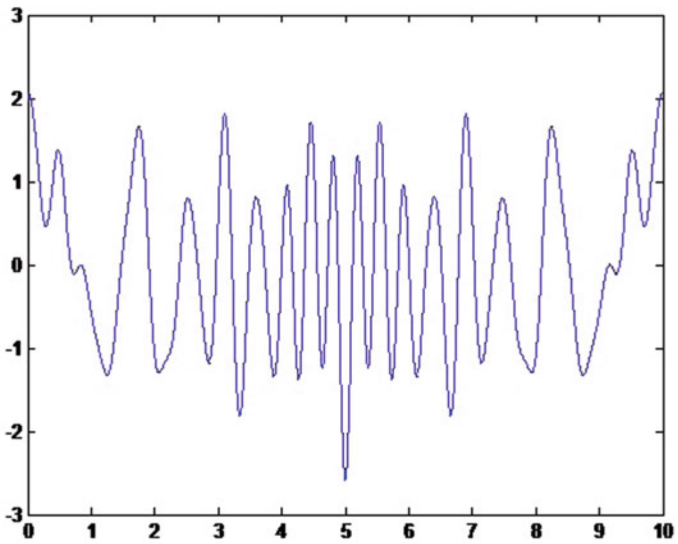

Fig. 1 SPHS signal waveform

3 System Response Signal Testing and Processing Method

3.1 Testing Process

In the process of testing, the SPHS signal is applied to the measured system firstly, and then the dynamic response of the system is obtained. Excitation signal for SPHS signal, immediate satisfaction $x(t) = \sum_{k=1}^{N} a_k \cos(\omega_k t + \theta_k)$, $\omega_k = \frac{2\pi k}{T}$, $a_k = \sqrt{2p_k}$. In general, the measured system is a linear system. According to the superposition principle and the frequency invariance of the linear system, the response of the system to $y(t)$ should satisfy the following formula [6]:

$$y(t) = \sum_{k=1}^{N} b_k \cos(\omega_k t + \theta_k + \varphi_k), \quad k = 1, 2\ldots N \tag{3}$$

In formula (3), a_k is the amplitude of the first harmonic of the excitation signal k, θ_k is the first phase of the first harmonic of k, b_k is the response amplitude of the system, phase shift generated by φ_k as input signal of excitation signal. The amplitude φ_k and phase of each harmonic b_k in the system response can be calculated by the mathematical method. According to frequency characteristics, the amplitude ratio of the input and output harmonics to $L(\omega_k) = 20\lg\frac{b_k}{a_k}$ is about the amplitude frequency characteristic of the measured system, which reflects the gain or attenuation of the excitation signal through the angular frequency ω_k [7]. Phase difference A and the function curve of the angular frequency B are the phase frequency characteristics of the system under test, which reflects the hysteresis or advance of the excitation signal of the system. Amplitude frequency characteristic and phase frequency characteristic are the frequency domain index to reflect the dynamic performance of the system [8–10].

The formula (3) on the right in accordance with the cosine function and the angular formula can be obtained:

$$\begin{aligned} y(t) &= \mathrm{b}_1\cos(\omega_1 t+\theta_1+\varphi_1)+\cdots\mathrm{b}_N\cos(\omega_N t+\theta_N+\varphi_N) \\ &= \mathrm{b}_1\cos(\omega_1 t+\theta_1)\cos(\varphi_1)-\mathrm{b}_1\sin(\omega_1 t+\theta_1)\sin(\varphi_1) \\ &\quad+\cdots\mathrm{b}_N\cos(\omega_N t+\theta_N)\cos(\varphi_N)-\mathrm{b}_N\sin(\omega_N t+\theta_N)\sin(\varphi_N) \\ &= \begin{bmatrix} \cos(\omega_1 t+\theta_1) \\ -\sin(\omega_1 t+\theta_1) \\ \vdots \\ \cos(\omega_N t+\theta_N) \\ -\sin(\omega_N t+\theta_N) \end{bmatrix}^T \times \begin{bmatrix} b_1\cos(\varphi_1) \\ b_1\sin(\varphi_1) \\ \vdots \\ b_N\cos(\varphi_N) \\ b_N\sin(\varphi_N) \end{bmatrix} \end{aligned} \tag{4}$$

If the sampling frequency is f_s, then the time interval of sampling data is $t_s = \frac{1}{f_s}$, that is $Y^T = [y(0), y(t_s), y(2t_s), \ldots, y((n-1)t_s)]_{1\times n}$,so the corresponding expansion should be:

$$\mathrm{Y}_{n\times 1} = \begin{bmatrix} \cos(\omega_1 0+\theta_1) & \cos(\omega_1 t_s+\theta_1) & \cdots & \cos(\omega_1(n-1)t_s+\theta_1) \\ -\sin(\omega_1 0+\theta_1) & -\sin(\omega_1 t_s+\theta_1) & \cdots & -\sin(\omega_1(n-1)t_s+\theta_1) \\ \vdots & \vdots & \vdots & \vdots \\ \cos(\omega_N t+\theta_N) & \cos(\omega_N t_s+\theta_N) & \cdots & \cos(\omega_N(n-1)t_s+\theta_N) \\ -\sin(\omega_N t+\theta_N) & -\sin(\omega_N t_s+\theta_N) & \cdots & -\sin(\omega_N(n-1)t_s+\theta_N) \end{bmatrix}_{2N\times n}^T \times \begin{bmatrix} b_1\cos(\varphi_1) \\ b_1\sin(\varphi_1) \\ \vdots \\ b_N\cos(\varphi_N) \\ b_N\sin(\varphi_N) \end{bmatrix}_{2N\times 1}$$

The right side of the equation is denoted as matrix $A_{2N\times n}$ and $C_{2N\times 1}$ respectively, By the formula (4) was obtained:

$$Y_{n\times 1} = A^T_{n\times 2N} C_{2N\times 1} \tag{5}$$

3.2 Data Processing Based on Mirror Mapping Method

The traditional method of solving the matrix equation is the least squares method, but this method requires AA^T non-singular, and easy to encounter digital morbid problem, using the image mapping method can effectively avoid this problem. The image mapping method is characterized by the use of Householder transform to find a suitable orthogonal array $H_{n\times n}$, so that $H_{n\times n}A^T_{n\times 2N}$ is converted to the upper triangular matrix. Let the observation equation be $L = Y + \Delta$, then the residual equation $V = A^T C - L$, the sum of squared residuals is $J = ||V||^2 = ||A^T C - L||^2$, so that the sum of the squares of the residuals is the least squares solution of the contradictory Eq. (5).

Let exist in the orthogonal matrix $H_{n\times n}$, so that:

$$H_{n\times n}A^T_{n\times 2N} = \begin{bmatrix} R_{2N\times 2N} \\ 0_{(n-2N)\times 2N} \end{bmatrix} \quad (6)$$

The first 2N elements in $H_{n\times n}L$ are m, the remaining elements are denoted as g, and the residual equation is orthogonal transform:

$$HV = HA^T C - HL = \begin{bmatrix} R \\ 0 \end{bmatrix} C - \begin{bmatrix} m \\ g \end{bmatrix} = \begin{bmatrix} RC - m \\ -g \end{bmatrix} \quad (7)$$

From the formula (7) can be obtained by the sum of squares:

$$J = ||V||^2 = ||HV||^2 = ||\begin{bmatrix} RC - m \\ -g \end{bmatrix}||^2 = \sqrt{(RC - m)^T (RC - m) + g^T g} \quad (8)$$

It can be seen that in Eq. (8), $C = R^{-1}m$ reaches the minimum value $\sqrt{g^T g}$. Thus the least squares solution of the contradictory Eq. (5) is $C = R^{-1}m$. According to this, the system response amplitude ratio $L(\omega_k)$ and the phase difference $\phi(\omega_k)$ corresponding to the angular frequency ω_k of the system are:

$$L(\omega_k) = 20\lg\frac{b_k}{a_k} = 20\lg\frac{\sqrt{c^2_{2k-1} + c^2_{2k}}}{a_k} \quad (9)$$

$$\phi(\omega_k) = -\arctan\frac{c_{2k}}{c_{2k-1}} \quad (10)$$

4 Matlab Simulation

In order to verify the effectiveness of the method in the dynamic performance test of the system, a SISO system is selected as the test object, and the simulation experiment is carried out using Matlab. The system open-loop transfer function model is:

$$G(s) = \frac{U(s)}{I(s)} = \frac{b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{11}$$

The main parameters of the system transfer function are:

$$a_3 = 0.0148,\ a_2 = 0.1701,\ a_1 = 0.74,\ a_0 = 1,\ b_1 = 0.282,\ b_0 = 2$$

The dynamic performance test is carried out in the frequency range of the system. The parameters of the SPHS signal are N = 30, T = 10 s, and the sampling interval Td = 0.002 s. Use Matlab to generate the SPHS signal shown in Fig. 2, discretize it into Simulink.

In Simulink as shown in Fig. 4 to build the system transfer function model, the output of the sampling system to get the response shown in Fig. 3.

For Eq. (5), the matrix Y (t) is obtained by sampling the system response, and the matrix A is obtained from the excitation SPHS signal parameters. Since the matrix Y (t) and A are known, the least squares fit can be obtained by using the mirror mapping method. According to Eqs. (9) and (10), the system response amplitude ratio $L(\omega_k)$ and the phase difference $\phi(\omega_k)$ corresponding to the angular frequency ω_k are obtained. As shown in Fig. 5, the abscissa is set to the frequency of every ten times the angle, and the system amplitude and frequency characteristic

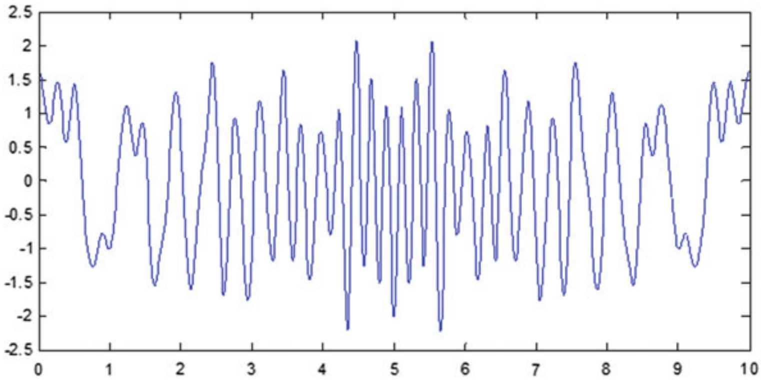

Fig. 2 SPHS excitation signal waveform

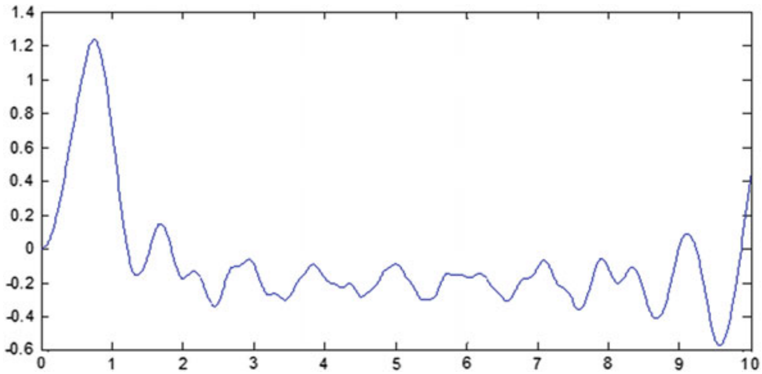

Fig. 3 System response signal waveform

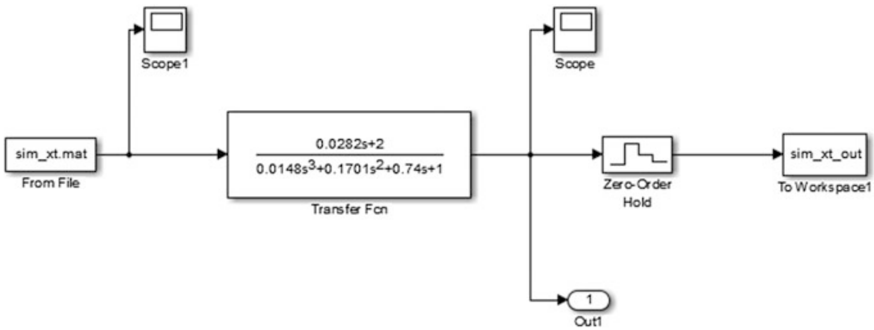

Fig. 4 Simulink build system model

graph and phase frequency characteristic graph are obtained by plotting $L(\omega_k)$ and $\phi(\omega_k)$ as the ordinate respectively. The dynamic performance of the system is analyzed and analyzed. It is necessary to pay attention to the result of the discretization of the response signal in the matrix data before the drawing. It is necessary to smooth the processing by the interpolation method before the drawing to obtain the system frequency characteristic curve.

In the above figure, the blue curve is a frequency characteristic curve directly obtained according to the transfer function of the system, and the green line is the frequency characteristic curve obtained according to the SPHS excitation. Contrast shows that the test results can better reflect the dynamic performance of the system.

According to the amplitude margin and the phase angle margin, the amplitude margin is the difference between the amplitude and the 0 dB at the phase angle crossing frequency. The phase margin is the difference between the phase angle and

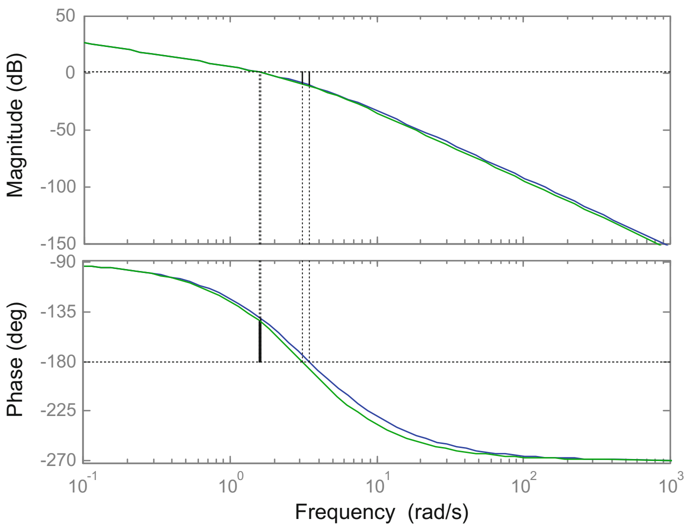

Fig. 5 System frequency characteristic curve

the −180° of the amplitude crossing frequency. From the analysis of Fig. 5 available, the system amplitude margin K_g=3.5633, phase angle margin γ=38.8237°, cutoff frequency $\omega_c = 1.6108\ \text{rad/s}$. And the original dynamic performance parameters of the system as follows:

	Amplitude margin	Phase margin (°)	Cut-off frequency (rad/s)
System parameters	3.5620	38.8229	1.7047
Test results	3.5633	38.8237	1.6108

5 Conclusion

Through the simulation experiment of SISO system, it can be seen that the dynamic performance test method based on SPHS signal can achieve high test precision. Using the image mapping method for data processing, effectively avoiding the digital morbid problem, expanding the scope of application of test methods. At the same time SPHS signal relative to the sine point by point scanning faster, and can be generated by the computer. Technology is easier to achieve than traditional methods.

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