Consensus of Heterogeneous Multi-agent Systems Based on Event-Triggered

Zhiqiang Yan and Ronghao Wang

1 Introduction

During the past years, the consensus control of MASs has attracted many researchers' attention. In the beginning, the consensus problems of MASs with first-order and second-order or high-order dynamics have been investigated from severally different directions, such as communication delay, switching topology, external disturbances in [1–3].

Afterwards researchers have worked out some results about heterogeneous MASs. In [4], Wang discussed linear consensus protocol and input saturated consensus protocol and obtained a sufficient condition of the system. In [5], the consensus of heterogeneous MASs under bounded communication delays were studied. The orther studies directions about consensus of heterogeneous MASs have been investigated in [6]. In view of MASs is limited to computing ability, communication ability, energy reserves and other restrictions, research on MASs based on event-triggered mechanism has attracted considerable attention. In [7], Lemmon applied event-triggered mechanism to distributed networked control systems. Meanwhile, research directions of the MASs about event-triggered mechanism have been investigated in [8, 9].

In fact, there are several research on solving the cooperation of heterogeneous MASs by event-triggered control. In [10], Huang investigated consensus problems for two different dynamics: first and second-order integrators, but zeno behavior is

Z. Yan
School of Automation, Nanjing University of Science and Technology,
Nanjing, China

R. Wang (✉)
College of Defense Engineering, PLA University of Science and Technology,
Nanjing, China
e-mail: wrh@893.com.cn

Z. Deng (ed.), *Proceedings of 2017 Chinese Intelligent Automation Conference*,
Lecture Notes in Electrical Engineering 458,
https://doi.org/10.1007/978-981-10-6445-6_42

not excluded theoretically. In [11], Yin and Yue investigated the consensus problem for a set of discrete-time heterogeneous MASs.

Inspired by the above results, we consider a heterogeneous MASs, which compose of first-order and second-order agent, and solve consensus problem based on event-triggered mechanism. The contributions of this work are listed as follows. (i) This work propose a novel event-triggered condition different from have published articles in [10, 11] about the consensus problem of heterogeneous MASs, which can reduce the number of communication. (ii) Control protocols based on event-triggered mechanism are respectively designed for both leaderless and leader-follower heterogeneous systems. Meanwhile, the zeno phenomenon is excluded and a positive lower triggered bound will be found between two consecutive actuation updates.

2 Problem Formulation

Consider a heterogeneous MASs consisting of a group of n identical agents with a communication graph G. Assume that there are $m(m<n)$ first-order agents and $n-m$ second-order agents. The dynamics of the first-order agents can be described as follows:

$$\dot{x}_i(t) = u_i(t), i = 1, 2, \ldots, m, \tag{1}$$

where $x_i \in R, u_i \in R$, denote the position state and control input of the ith first-order agent and the second-order agents can be described as follows:

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases}, i = m+1, \ldots, n, \tag{2}$$

where $x_i \in R, v_i \in R, u_i \in R$, denote the position state, velocity state and control input of the ith second-order agent.

In a leader-follower consensus problem, the dynamics of the leader is described by the following frist-order differential equation:

$$\dot{x}_0(t) = v_0 \tag{3}$$

where $x_0(t) \in R$,$v_0 \in R$ are the position state and velocity state.

3 Leaderless Consensus with Event-Triggered Control

An event-triggered heterogeneous MASs (1) and (2) consensus protocol of agent i is designed as follows:

$$\begin{cases} u_i(t) = \sum_{j\in 1}^{n} a_{ij}(x_j(t_k) - x_i(t_k)), i = 1, 2, \ldots, m \\ u_i(t) = -2\mu v_i(t_k) + \mu \sum_{j\in 1}^{n} a_{ij}(x_j(t_k) - x_i(t_k)), i = m+1, \ldots, n \end{cases} \tag{4}$$

for $t \in [t_k, t_{k+1}), k = \{0, 1, \ldots\}$. Where $\mu > 0$ is the control gain parameter and will be determined in sequel. The system (1) and (2) with the protocol (4) can be rewritten

$$\begin{gathered} \dot{x}_i(t) = \sum_{j\in 1}^{n} a_{ij}(x_j(t_k) - x_i(t_k)), i = 1, 2, \ldots, m \\ \begin{cases} \dot{x}_i(t) = v_i(t),\ i = m+1, \ldots, n \\ \dot{v}_i(t) = -2\mu v_i(t_k) + \mu \sum_{j\in 1}^{n} a_{ij}(x_j(t_k) - x_i(t_k)) \end{cases} \end{gathered} \tag{5}$$

Suppose that the event-times of all agents are modeled as a sequence t_k for $k = \{0, 1, \ldots\}$. For agent i with dynamics (1) and (2), define measurement errors

$$\begin{aligned} e_{i,x}(t) &= x_i(t_k) - x_i(t) \\ e_{i,\tilde{v}}(t) &= \tilde{v}_i(t_k) - \tilde{v}_i(t) \end{aligned} \tag{6}$$

where $\tilde{v}_i(t) = v_i(t)/\mu + x_i(t)$. The event-times t_k are defined by the function

$f(e(t_k), x(t_k), \tilde{v}(t_k)) \geq 0$, which will be determined in sequel. Denote

$$\begin{cases} \mathrm{x_f(t) = col(x_1(t), x_2(t), \ldots .x_m(t)), x_s(t) = col(x_{m+1}(t), x_{m+2}(t), \ldots .x_n(t))} \\ \mathrm{\tilde{v}_s(t) = col(\tilde{v}_{m+1}(t), \tilde{v}_{m+2}(t), \ldots, \tilde{v}_n(t)), e_{x,f}(t) = col(e_{1,x}(t), e_{2,x}(t), \ldots e_{m,x}(t))} \\ \mathrm{e_{x,s}(t) = col(e_{m+1,x}(t), e_{m+2,x}(t), \ldots e_{n,x}(t)), e_{\tilde{v},s}(t) = col(e_{m+1,\tilde{v}}(t), e_{m+2,\tilde{v}}(t), \ldots e_{n,\tilde{v}}(t))} \end{cases} \tag{7}$$

Let $\psi(t) = col[x_f(t), x_s(t), \tilde{v}_s(t)]$,$e(t) = col[e_{x,f}(t), e_{x,s}(t), e_{\tilde{v},s}(t)]$, then system (1) and (2) can be transformed into the closed-loop system as follows:

$$\dot{\psi}(t) = -F\psi(t) + Je(t) \tag{8}$$

$$F = \begin{bmatrix} L_{11} & L_{12} & 0 \\ 0 & \mu I_{n-m} & -\mu I_{n-m} \\ L_{21} & L_{22} - \mu I_{n-m} & \mu I_{n-m} \end{bmatrix}, J = -\begin{bmatrix} L_{11} & L_{12} & 0 \\ 0 & 0 & 0 \\ L_{21} & L_{22} - \mu I_{n-m} & \mu I_{n-m} \end{bmatrix} \tag{9}$$

$L_{11} \in R^{m\times m}, L_{12} \in R^{m\times(n-m)}, L_{21} \in R^{(n-m)\times m}$ and are parts of $L_{22} \in R^{(n-m)\times(n-m)}$. Laplacian matrix is $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$.

Assume an undirected graph $G(V, E, A)$ is connected, mean that $L1_n = 0_n$, then we can get

$$\begin{aligned} 1_{2n-m}^T(-F\psi(t) + Je(t)) &= 0_{2n-m} \\ 1_{2n-m}^T\dot{\psi}(t) &= 0_{2n-m} \end{aligned} \tag{10}$$

Denote $\xi(t) = \psi(t) - \omega 1_{2n-m}$, where $\omega = (1/(2n-m))\sum_{i=1}^{2n-m}\psi_i$. From (10), it is clear that ω is a constant. Then the system (10) becomes

$$\dot{\xi}(t) = -F\xi(t) + Je(t) \tag{11}$$

We can get that $\lim_{t\to\infty}\psi(t) = \omega 1_{2n-m}$ when $\lim_{t\to\infty}\xi(t) = 0_{2n-m}$, that is mean $\lim_{t\to\infty} x_i = \omega, i = 1, 2, \ldots, n, \lim_{t\to\infty}\tilde{v}_i = \omega, i = m+1, m+2, \ldots, n.$ Because of $\tilde{v}_i(t) = v_i(t)/\mu + x_i(t)$, we can get $\lim_{t\to\infty} v_i = 0, i = m+1, m+2, \ldots, n$, the consensus problem of MASs (8) can be changed into the stability issue of system (11).

Theorem 1 *Assume that the interconnection topology G associated with multi-agent system (1)–(2) is connected. If there exist a symmetric positive definite matrix $P \in R^{(2n-m)\times(2n-m)}$, and constants $\mu > 0$ such that*

$$(F_1 + \mu F_2)^T P + P(F_1 + \mu F_2) \geq 0 \tag{12}$$

event-triggered condition $f(e(t_k), x(t_k), \tilde{v}(t_k)) = \|e\| - \sigma\frac{\lambda_{\min}(Q)\|\xi\|}{2\lambda_{\max}(P)\|J\|} \geq 0$, and the event-triggered consensus protocol (4), heterogeneous MASs (1) and *(2) can achieve consensus, where $F_1 = \begin{bmatrix} L_{11} & L_{12} & 0 \\ 0 & 0 & 0 \\ L_{21} & L_{22} & 0 \end{bmatrix}, F_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I_{n-m} & -I_{n-m} \\ 0 & -I_{n-m} & I_{n-m} \end{bmatrix}$.*

Proof For system (1) and (2), construct following Lyapunov function candidate

$$V(\xi) = \xi^T P\xi \tag{13}$$

where P is a positive definited matrix.

Consider the derivative of $V(\xi)$ and along the trajectory of (13) we can get

$$\begin{aligned} \dot{V}(\xi) &= \dot{\xi}^T P\xi + \xi^T P\dot{\xi} \\ &= -\xi^T(F^T P + PF)\xi + 2\xi^T PJe \\ &= -\xi^T Q\xi + 2\xi^T PJe \end{aligned} \tag{14}$$

where $Q = F^T P + PF$, $F = F_1 + kF_2$. Make the measurement error e to satisfy $\|e\| \leq \sigma \frac{\lambda_{\min}(Q)\|\xi\|}{2\lambda_{\max}(P)\|J\|}$, with $\sigma \in (0,1)$, from (12), Q is positive definite with k given in (13), therefore

$$\begin{aligned}\dot{V}(\xi) \leq & -\lambda_{\min}(Q)\|\xi\|^2 + 2\lambda_{\max}(P)\|J\|\|\xi\|\|e\| \\ = & -\|\xi\|(\lambda_{\min}(Q)\|\xi\| - 2\lambda_{\max}(P)\|J\|\|e\|) \\ \leq & -(1-\sigma)\lambda_{\min}(Q)\|\xi\|^2 \leq -\frac{(1-\sigma)\lambda_{\min}(Q)}{\lambda_{\max}(P)} V(\xi)\end{aligned} \tag{15}$$

From (15), we conclude that $\lim\limits_{t\to 0} \xi(t) = 0$. In other words, under the event-triggered consensus control protocol (4), MASs (1) and (2) achieves consensus.

Theorem 2 *In MASs* (1) and (2) *based on event-trigger mechanism of consensus control protocol* (4), *and trigger function $f(e(t_k), x(t_k), \tilde{v}(t_k))$, let K satisfy the following expression $K = \sigma \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)\|J\|}$, the triggered time interval $t_{k+1} - t_k$ are bounded by the time*

$$\Delta t_{\min} = \frac{\ln(\|F\|) - \ln\left(\left\|\frac{\|F\| + K\|J\|}{1+K}\right\|\right)}{\|F\| - \|J\|} \tag{16}$$

Proof be similar to Tabuada [12] proofs that calculate the interval lower bound. Based on measurement error $\|e\|$, we can written as follows:

$$\frac{\|e\|}{\|\xi\|} \leq \sigma \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)\|J\|} \tag{17}$$

Let $\phi(t) = \|e(t)\|/\|\xi(t)\|$, we can get that when $\phi(t) = \sigma \frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)\|J\|}$, system will be triggered, transfer state information and update control inputs. Take derivative of $\phi(t)$, when the derivative of $\phi(t)$ become bigger, the inter event times become shorter. So we can get the minimum inter-event times, when the derivative of $\phi(t)$ reach its maximum.Take the derivative of $\phi(t) = \|e(t)\|/\|\xi(t)\|$, we can get

$$\begin{aligned}\dot{\phi} &= \frac{d}{dt}\frac{\|e(t)\|}{\|\xi(t)\|} = \frac{d}{dt}\frac{(e^T e)^{1/2}}{(\xi^T \xi)^{1/2}} = -\frac{e^T \dot{\xi}}{\|e\|\|\xi\|} - \frac{\xi^T \dot{\xi}}{\|\xi\|^2}\frac{\|e\|}{\|\xi\|} \\ &\leq \frac{\|e\|\|\dot{\xi}\|}{\|e\|\|\xi\|} + \frac{\|\xi\|\|\dot{\xi}\|\|e\|}{\|\xi\|\|\xi\|\|\xi\|} = (1 + \frac{\|e\|}{\|\xi\|})\frac{\|\dot{\xi}\|}{\|\xi\|} = (1+\phi)(\|F\| + \|J\|\phi)\end{aligned} \tag{18}$$

From the above analysis, the derivative of $\phi(t)$ reach maximum when $\dot{\phi} = (1+\phi)(\|F\| + \|J\|\phi)$. Solving the differential equations $\phi(t)$, assume $\phi(t, \phi_0)$ is the solution differential equations, we can get the solution as follows:

$$\phi(t,\phi_0)=\frac{e^{(t+C)(\|J\|-\|F\|)}-\|F\|}{\|J\|-e^{(t+C)(\|J\|-\|F\|)}} \tag{19}$$

where e is natural constant, ϕ_0 is the initial value of differential equations, C is the constant of indefinite integral solutions. Because the system start first triggered, so we can get $\phi_0=0$, and get into the equation, $C=\frac{\ln(\|F\|)}{\|J\|-\|F\|}$, we can get

$$\phi(t,\phi_0)=\frac{e^{(t+\frac{\ln(\|F\|)}{\|J\|-\|F\|})(\|J\|-\|F\|)}-\|F\|}{\|J\|-e^{(t+\frac{\ln(\|F\|)}{\|J\|-\|F\|})(\|J\|-\|F\|)}} \tag{20}$$

Let $K=\sigma\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)\|J\|}$, solving the differential equation $\phi(t,\phi_0)$ leads to

$$\Delta\tau_{\min}=\frac{\ln(\|F\|)-\ln(\left\|\frac{\|F\|+K\|J\|}{1+K}\right\|)}{\|F\|-\|J\|}=\frac{\ln(\frac{(1+K)\|F\|}{\|F\|+K\|J\|})}{\|F\|-\|J\|}>0 \tag{21}$$

which is a positive bound due to the fact that $\|F\|>\|J\|$, the proof is now finished.

4 Leader-Following Consensus with Event-Triggered Control

Assumption 1 The state information of the leader can be measured in continuous-time by the followers. An undirected graph $G(V,E,A)$ is connected and at least one agent is connected with the leader.
A control protocol for heterogeneous MASs (1)–(3) with an active leader is designed

$$\begin{cases} u_i(t)=\sum\limits_{j\in 1}^{n}a_{ij}(x_j(t_k)-x_i(t_k))+a_{i0}(x_0(t)-x_i(t_k))+v_0(\mathrm{t}), i=1,2,\ldots,m \\ u_i(t)=\mu(v_0(t)-v_i(t_k))+\sum\limits_{j\in 1}^{n}a_{ij}(x_j(t_k)-x_i(t_k)) \\ +a_{i0}(x_0(t)-x_i(t_k)), i=m+1,\ldots,n \end{cases} \tag{22}$$

denote the measurement errors

$$\begin{aligned} e_{i,x}(t)&=x_i(t_k)-x_i(t) \\ e_{i,v}(t)&=v_i(t_k)-v_i(t) \end{aligned} \tag{23}$$

for $t\in[t_k,t_{k+1}), k=\{0,1,\ldots\}$. The t_k are defined by function $f(e(t_k),x(t_k),v(t_k))\geq 0$.

Let $H = L + B$, $\tilde{x}_i(t) = x_i(t) - x_0(t)$, $\tilde{v}_i(t) = v_i(t) - v_0(t)$, based on the fact that $H\tilde{x}(t) = Hx(t) - B\vec{1}x_0(t), H\tilde{v}(t) = Hv(t) - B\vec{1}v_0(t)$, MASs (1)–(3) can changed

$$\begin{pmatrix} \dot{\tilde{x}}_f(t) \\ \dot{\tilde{x}}_s(t) \\ \dot{\tilde{v}}_v(t) \end{pmatrix} = -U \begin{pmatrix} \tilde{x}_f(t) \\ \tilde{x}_s(t) \\ \tilde{v}_s(t) \end{pmatrix} + W \begin{pmatrix} e_{x,f}(t) \\ e_{x,s}(t) \\ e_{v,s}(t) \end{pmatrix} \tag{24}$$

$$U = \begin{pmatrix} L_{11} + B_f & L_{21} & 0 \\ 0 & 0 & -I_{n-m} \\ L_{21} & L_{22} + B_s & \mu I_{n-m} \end{pmatrix}, W = -\begin{pmatrix} L_{11} + B_f & L_{21} & 0 \\ 0 & 0 & 0 \\ L_{21} & L_{22} + B_s & \mu I_{n-m} \end{pmatrix}$$

$B_f = diag(a_{10}, a_{20}, \ldots, a_{m0}), B_s = diag(a_{(m+1)0}, a_{(m+2)0}, \ldots, a_{n0})$

Let $\xi(t) = col(\tilde{x}_f(t)\,\tilde{x}_s(t)\,\tilde{v}_s(t)), e(t) = col(e_{x,f}(t)\,e_{x,s}(t)\,e_{v,s}(t))$, (23) expressed as

$$\dot{\xi}(t) = -U\xi(t) + We(t) \tag{25}$$

Theorem 3 *Assume that the leader-follower topology G associated with MASs* (1)–(3) *is connected. If there exist a symmetric positive definite matrix* $P_L \in R^{(2n-m)\times(2n-m)}$, and *constants* $\mu > 0$ *such that*

$$U^T P_L + P_L U \geq 0 \tag{26}$$

under the event-triggered condition

$$f(e(t_k), x(t_k), v(t_k)) = \|e\| - \sigma \frac{\lambda_{\min}(Q_L)\|\xi\|}{2\lambda_{\max}(P_L)\|W\|} \geq 0 \tag{27}$$

and protocol (22), *heterogeneous MASs* (1)–(3) *can achieve consensus.*

Theorem 4 *In MASs* (1)–(3) *based on event-trigger mechanism of consensus control protocol* (22), *and trigger function* (27), *the triggered time interval* $t_{k+1} - t_k$, *K satisfy the following expression* $K = \sigma \frac{\lambda_{\min}(Q_L)}{2\lambda_{\max}(P_L)\|W\|}$, *the inter-event times are bounded by the time* $\Delta t_{\min} = \frac{\ln(\|U\|) - \ln(\left\|\frac{\|U\| + K\|W\|}{1+K}\right\|)}{\|U\| - \|W\|}$.

5 Simulations

A leaderless heterogeneous MASs is showed on Fig. 1a. The communication topology G has a globally vertex 5, 1 and 2 denote the first-order agents, 3, 4 and 5 denote the second-order agents. Each agent is governed by the controller (4). The

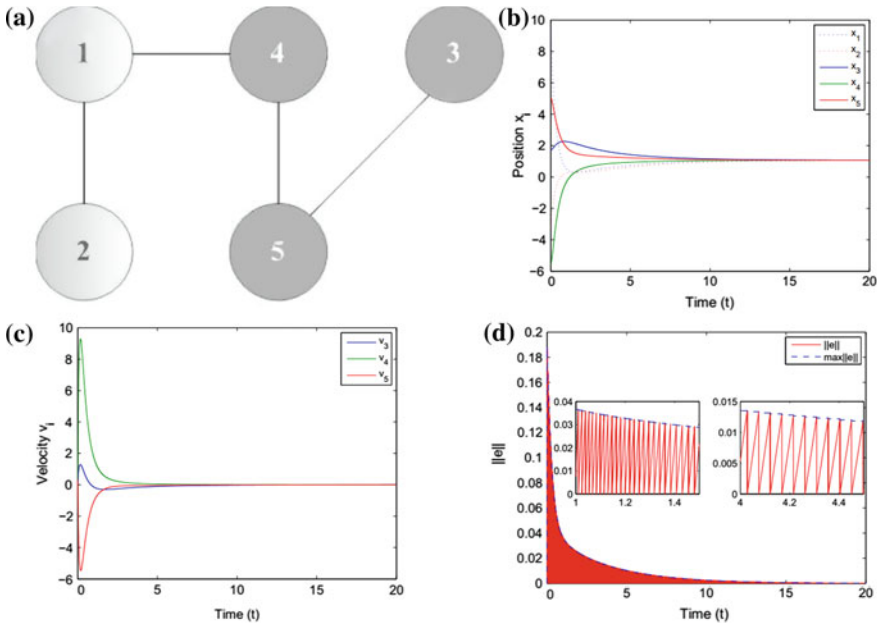

Fig. 1 **a** Leaderless heterogeneous MASs. **b** The trajectories of position x_i. **c** The trajectories of velocity v_i. **d** Evolution of error signals for agent

parameters are given by $k = 6$ and $\sigma = 0.8$ for all agents. The initial positions and velocities are randomly chosen within [−10, 10] and [0, 0.5].

Figure 1 we know that the leaderless heterogeneous MASs (1) and (2) with consensus protocol (4) can solve consensus problem. Figure 1d shows that curve $\|e\|$ is always in the bottom of curve max $\|e\|$,which means that the error $\|e\|$ is bounded by the threshold $\sigma \frac{\lambda_{\min}(Q)\|\xi\|}{2\lambda_{\max}(P)\|J\|}$. Figure 1d also shows consensus controllers are illustrated for the four agents, whose event-driven update frequencies are decreasing as time evolves, mean that the triggered time is getting longer as time goes by.

6 Conclusion

An event-triggered consensus problem of heterogeneous MASs with leaderless and leader-following was considered in this paper. The heterogeneous consensus controllers have been proposed for all agents based on an event-triggered control strategy. Some sufficient conditions for heterogeneous MASs consensus have been solved. Numerical examples have been presented to verify the efficiency of proposed controls. The consensus of heterogeneous MASs based on event-driven under external disturb and communication delay will be researched in the future.

Acknowledgements This work is supported by the National Natural Science Foundation of China under Grant No. 61603414.

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