Observability and Controllability Preservation for Multi-agent Systems with Time Delay and Time-Varying Topology

Xiangju Jiang, Zhijian Ji, Ting Hou and Fanggang Sun

1 Introduction

In recent years, the study of multi-agent systems has attracted more and more attention. It is known that controllability is a core concept of modern control, playing a fundamental role in analysis and synthesis of linear control systems [1–3]. For multi-agent systems, the controllability problem aims at driving follower agents to achieve any configurations from any initial states only through controlling a few leaders externally. The controllability of multi-agent systems is first proposed by Tanner [4], in which the Laplacian matrix was divided into submatrices and the controllability was acquired by taking advantage of the classical controllability concept. There are some related results e.g., leaders selection [5], switching topology [6], directed topology [7–9], time-delay [10], structural controllability [11], etc. However, the study on the preservation of controllability is at starting stage. Sufficient conditions are derived in [12] that guarantee controllability of a network system from a graph-theoretic perspective. Fraceschelli etc. presented a strategy in [13] to verify the controllability, the strategy is based on decentralized estimation of the spectrum of the Laplacian matrix. An energy function associated with the second-smallest eigenvalue of Laplacian matrix is presented in [14], that ensure the connectivity of multi-agent systems. In addition, The proposed methodology is based on a decentralized strategy presented in [15–17] for the maintenance of the connectivity of the multi-agent systems, that will be here exploited for guaranteeing structurally controllable systems.

X. Jiang · Z. Ji (✉) · F. Sun
The School of Automation Engineering, Qingdao University, Qingdao 266000, China
e-mail: jizhijian@pku.org.cn

T. Hou
Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, Shandong, China

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Compared with the concept of controllability, the concept of observability is related to the possibility of observing the whole state of the systems from only a subset of the agents. In the relevant studies about the observability, Ji etc. have studied the observability of the distributed sensors from the equivalence partition and they have gotten the requirement of the system observability [18]. The observability of the multi-agent system about choosing the leaders has been studied by the literature [19]. Lozano, R. and others have studied the observability of the second-order systems in the structure of the path graph and the circle graph.

Usually, a multi-agent system is interconnected through neighbor rules and there will exist a time delayed due to finite speed of communication between two coupled agents. From a graph-theoretic perspective, Sufficient conditions are derived in [20] that guarantee controllability of multi-agent systems with interaction time delays. Liu etc. studies the controllability of a leader-follower network of dynamic agents in the presence of communication delays in [21]. In this paper, the case of time delay is also studied.

2 Preliminaries

In this section, we present some fundamental terminologies on graph theory including the system model, the formulation of controllability problem that will be used in the proof of the main results.

2.1 Graph Preliminaries

An undirected graph G is composed of three parts including a vertex set, an edge set, and an adjacency matrix with weights. And it is marked as $G = (V, \varepsilon, A)$, where the nodes are marked as $1, 2, \ldots, n$, he edge set is marked as $\varepsilon = \{(i,j) \in V \times V\}$ and $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ is the adjacency matrix, where $a_{ij} > 0$ if the vertex i is adjacent with the vertex j. An edge of the graph G is denoted by $e_{ij} = (v_j, v_i)$, where the v_j is called as parent vertex of v_i and v_i is called as child vertex of v_j. We suppose there is no self-loops in this paper, namely $e_{ii} \notin \varepsilon$. If the vertex i is connected with the j by an edge e_{ij}, then those two vertexes are called the adjacency vertexes. The vertex i is adjacency directly with the edge e_{ij}. Therefore we take the adjacency vertexes set $\mathrm{N}_i = \{v_j \in V | (v_j, v_i) \in \varepsilon\}$ as the vertex set where the vertexes are all directly adjacency with the vertex i. $d(v_i) = \sum_j a_{ij}$ is on behalf of the degree of a vertex and the $D(G) = diag(d(v_i))$ is the corresponding degree matrix. The Laplacian matrix $L(G)$ of the graph G is defined as $L(G) = D(G) - A$. For an undirected graph, the Laplacian matrix is symmetrical and positive semi-definite.

2.2 Problem Formulation

With the leader-following structure of the linear system, the observability will be studied. A multi-agent system is composed of leaders and followers. The followers are under control by the neighbor agreement. But the leaders are free of such a constrain and are allowed to picked its control input arbitrarily. In this case, the leaders are regarded as control inputs and the followers are controlled by the leaders. Given leaders, the Laplacian matrix of the figure G is divided into $L = \begin{bmatrix} L_f & L_{fl} \\ L_{lf} & L_l \end{bmatrix}$ where $L_f \in \mathbb{R}^{n \times n}$ and $L_l \in \mathbb{R}^{l \times l}$ correspond to the indices of followers, leaders. L_{lf} represents the communication relation from followers to leaders. L_{fl} represents the communication relation from leaders to followers. We denote by $F \triangleq L_f, R \triangleq L_{fl}$. In the undirected graphs, $L_{lf} = L_{lf}^T$, namely $L_{lf} = R^T$. After having chosen the leaders, Laplacian matrices are completely decided by the communication topology. In this paper, the leaders are already set in advance.

3 Observability of Multi-Agent Systems with Time Delay and Time-Varying Topology

Considering $n + l$ multi-agents, the dynamic equation of the single integral is as follows:

$$\dot{x}_i(t) = u_i(t), i = 1, 2, \ldots, n + l \tag{3.1}$$

The corresponding delay control protocol is as follows:

$$u_i(t) = -\sum_{j \in N_i} a_{ij}\big(x_i(t) - x_j(t - \tau)\big) + \sum_{j \in N_i} a_{ij}\big(x_j(t) - x_i(t)\big) \tag{3.2}$$

where $x_i(t)$ represents state of the agent i. $u_i(t)$ represents the input of the system, and $\tau > 0$ represents the time delay, a_{ij} represents the weight of edge from agent v_j to agent v_i.

The state space equation of multi-agent systems with time varying topology is described as:

$$\dot{x}(t) = -(D + L)x(t) - Bx(t - \tau) \tag{3.3}$$

where

$$D = \begin{bmatrix} \sum_{j \in N_1} a_{1j} & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & 0 & \cdots & \vdots \\ \vdots & 0 & \sum_{j \in N_n} a_{nj} & 0 & \vdots \\ \vdots & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \sum_{j \in N_n + l} a_{(n+l)j} \end{bmatrix} L = [l_{ij}] \quad \text{with} \quad l_{ij} = \begin{cases} -a_{ij}, & j \neq i \,\text{and}\, j \in N_i \\ \sum_{j \in N_i} a_{ij}, & i = j \\ 0, & \text{otherwise} \end{cases} \quad \text{and } B = [b_{ij}] \text{ with}$$

$$b_{ij} = \begin{cases} -a_{ij}, & j \neq i \,\text{and}\, j \in N_i \\ 0, & \text{otherwise} \end{cases}$$

The dynamic equation of the followers is as follows

$$\dot{x}_f(t) = -(D_1 + F)x_f(t) - B_1 x_f(t - \tau) - R(x_l(t) + x_l(t - \tau)) \tag{3.4}$$

where

$$D_1 = \begin{bmatrix} \sum_{j \in N_1} a_{1j} & 0 & \cdots & 0 \\ 0 & \sum_{j \in N_2} a_{2j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sum_{j \in N_n} a_{nj} \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & 0 & \cdots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \cdots & 0 \end{bmatrix}$$

where $x_f(t) = [x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)]^T$ is the stacked vector of followers' position in the state of t. $x_f(t - \tau) = [x_1(t - \tau) \quad x_2(t - \tau) \quad \cdots \quad x_n(t - \tau)]^T$ is the stacked vector of followers' position in the state of $t - \tau$. $x_l(t) = [x_{n+1}(t) \quad x_{n+2}(t) \quad \cdots \quad x_{n+l}(t)]^T$ is the stacked vector of leaders' position in the state of t. $x_l(t - \tau) = [x_{n+1}(t - \tau) \quad x_{n+2}(t - \tau) \quad \cdots \quad x_{n+l}(t - \tau)]^T$ is the stacked vector of followers' position in the state of $t - \tau$.

According to literature [22], the controllability of multi-agent systems with time delay is usually determined by the algebraic criterion proposed for the associated auxiliary system without delay. In order to study the controllability of the system represented by (3.4), the following auxiliary equations are introduced:

$$\dot{x}_f(t) = -2Fx_f(t) - 2Rx_l(t) \tag{3.5}$$

Lemma 1 Hewer [22] If the system (3.5) is controllable, then system (3.4) is controllable for any $\tau > 0$.

Let $y \in \mathbb{R}^l$ respect the output vector, which contains all the state vectors measured by the leader. The output equation of the system is as follows:

$$y(t) = R^T x_f(t) \tag{3.6}$$

Then the equations of the time-varying multi-agent system can be written as described in the literature [14]:

$$\begin{cases} \dot{x}_f(t) = -2Fx_f(t) - 2Rx_l(t) \\ y(t) = R^T x_f(t) \end{cases} \tag{3.7}$$

Lemma 2 Dong et al. [23] For the multi-agent system (3.7), there exists all the eigenvalues can make the matrix $\begin{bmatrix} \lambda I - F \\ R^T \end{bmatrix}$ nonsingular. Namely, if $Lv = \lambda v$, then $R^T v \neq 0$, where v is the nonzero eigenvector of the L.

Theorem 1 *The system* (3.7) *is observable if and only if there is no eigenvector of L taking zero on the element corresponding to the leader.*

Proof The result will be shown by contradiction. From $Lv = \lambda v$, where $v = \begin{bmatrix} v_f \\ v_l \end{bmatrix}$, we can get: $Lv = \begin{bmatrix} F & R \\ R^T & E \end{bmatrix} \begin{bmatrix} v_f \\ v_l \end{bmatrix} = \lambda \begin{bmatrix} v_f \\ v_l \end{bmatrix}$

(Necessity) If the eigenvector of the matrix L corresponding to the leader node the element is zero, that is $v_l = 0$, we have

$$\begin{bmatrix} Fv_f \\ R^T v_f \end{bmatrix} = \lambda \begin{bmatrix} v_f \\ 0 \end{bmatrix} \tag{3.8}$$

The equation of (3.8) leads to $Fv_f = \lambda v_f, R^T v_f = 0$. Then, the matrix $[F, R^T]$ is not observable. So, if the system (3.7) is able to observe, the matrix L in the eigenvector and the corresponding element of the leader node is not a factor of 0.

(Sufficiency) If the system (3.7) is not observable, by the lemma 2, we see that the matrix F has a characteristic value of λ and its eigenvector is v_f, which satisfies $R^T v_f = 0$. (3.8) means that $v_l = 0$. The eigenvectors corresponding to the matrix and L and leader node elements are not 0 contradictory, so if the corresponding eigenvector matrix L and the leader node element is not 0, the system (3.7) can observe.

4 The Preservation of Controllability

For a multi-agent system with time-delay and time-varying topology, the number and lengths of the edges between different nodes will change as time goes on. The change further affects the Laplacian matrix of the system, which will give rise to a variation of algebraic connectivity that makes the whole system uncontrollable from the Theorem 1 and the reference [24]. Therefore, it will introduce a concept of structural controllability in order to ensure preservation of controllability of the systems

4.1 Structural Controllability

Definition 1 A multi-agent system is said to be structurally controllable if and only if it is controllable for almost all choices of the edge weights.
According to reference [24], a multi-agent system is structurally controllable if and only if the topology of the system is connected. It means that the algebraic connectivity $\lambda_2 > 0$.

In view of the definition of structural controllability and reference [24], Fig. 1 can be used to show the relationship between the structural controllability and the usual controllability for a multi-agent system.

4.2 The Strategy of Structural Controllable Preservation

Due to the change of the relationship between the agents, the connectivity of the corresponding topological graph of the system (1) may change at a certain time. As show in Fig. 2a, the node 1 represents the leader which is endowed an external control input. As the followers arrive at expected positions, the connection strength between the node 2 and the node 3 is weakened in Fig. 2b, because the distance

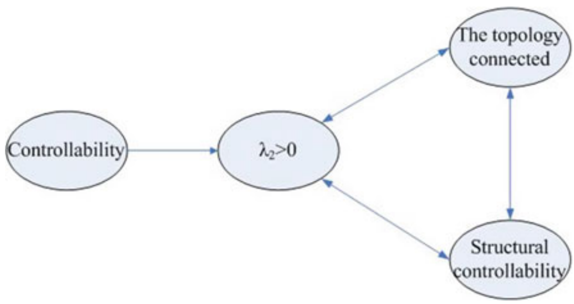

Fig. 1 Relationship between controllability and structural controllability

between them increases longer, which is inconsistent with the design requirements of the multi-agent systems.

In case of the aforesaid conditions, it will introduce a control strategy of preserving controllability:

$$u_i^c = -\frac{\partial V(\lambda_2(L_a(G)))}{\partial x_i} \tag{4.1}$$

where $L_a(G)$ is the Laplacian matrix of the graph G. The weights of the graph G are inversely proportional to the distances between agents i and j. From the formula (4.2), it can get the specific relationship between the weights and distances. It assumes the R is the maximum communication distance between any two agents, which means when $\|x_i - x_j\| \le R$, the two agents are still connected. Therefore

$$a_{ij} = \begin{cases} e^{-(\|x_i - x_j\|)/(2\sigma^2)} & if \|x_i - x_j\| \le R \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

the scalar σ is chosed by the threshold condition $e^{-((R^2))/(2\sigma^2)} = \Delta$, Δ is a minimum predetermined value.

This control protocol ensures that the value of the algebraic connectivity of the system will not be lower than the value of a given threshold ξ. From [4], the value of λ_2 will decrease with a_{ij} decreasing.

Definition 3 *Energy Function* $V(\lambda_2(\cdot))$

The energy function $V(\lambda_2(\cdot))$ has the following properties:

(P1) $\forall \lambda_2(\cdot) > \xi$ energy function is continuous and differentiable.

(P2) The energy function is non-negative.

(P3) With $\lambda_2(\cdot)$ increasing, the energy function converges to a constant value.

(P4) $\lim\limits_{\lambda_2(\cdot) \to \xi} V(\lambda_2(\cdot)) = \infty$. (P5) $\lim\limits_{\lambda_2(\cdot) \to \xi} \left\| \frac{\partial V(\lambda_2(\cdot))}{\partial \lambda_2(\cdot)} \right\| = \infty$.

According to the reference [9], the energy function is defined as follows:

$$V(\lambda_2(L_a(G))) = \frac{1}{(\lambda_2(L_a(G)))^3} \tag{4.3}$$

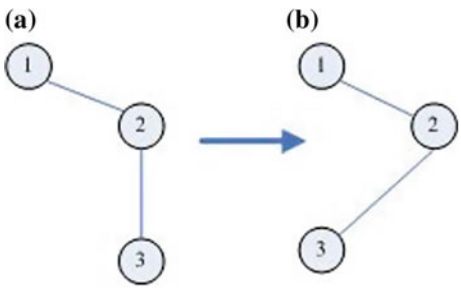

Fig. 2 Time varying topology

The dynamic model is considered by $\dot{x}_i = u_i^c$. The control protocol u_i^c is shown by (4.1).So dynamic functions are modified by the next forms:

$$\begin{cases} \dot{x}_i = -\sum_{j\in N_i} a_{ij}(x_i(t) - x_j(t-\tau)) \\ \quad + \sum_{j\in N_i} a_{ij}(x_j(t) - x_i(t)) + u_i^c & if\ i = 1,2,\ldots,n \\ \dot{x}_i = u_i + u_i^c & if\ i = n+1,\ldots,n+l \end{cases} \tag{4.4}$$

Theorem 2 *As for the multi-agent system decorated by* (4.4)*, it supposes the system is structurally controllable at the moment of* $t = 0$*.Then, the corresponding control strategy* $u_i^c = -\frac{\partial V(\lambda_2(L_a(G)))}{\partial x_i}$ *enables the multi-agent system preserving structural controllability.*

Proof At the moment of $t = 0$, it supposes the system is structurally controllable. So the topology is connected and $\lambda_2 > \xi$. Inspired by [17], we can get that the control strategy guarantees that, if a time-varying system is connected firstly, then the system is always connected with time varying. At this time, it is necessary to consider an energy function $V(\cdot)$ defined according to Definition 3, which does not increase with the varying of the systems. The dynamic functions decorated by the form (4.4) can be modified as the follows:

$$\dot{x}_i = u_i^c + u_i^\varepsilon \quad \forall\, i = 1,2,\ldots,n+l \tag{4.5}$$

where external input u_i^ε is bounded. It means $\exists u_M \in \mathbb{R}$ which cause:

$$\left\| u_i^\varepsilon \right\| \le u_M, \forall\, i = 1,2,\ldots,n+l \tag{4.6}$$

The derivative of the energy function is written as the following form:

$$\dot{V}(\cdot) = \nabla_x V(\cdot)^T \dot{x} = \sum_{i=1}^{n+l} \frac{\partial V(\cdot)^T}{\partial x_i} \dot{x}_i \tag{4.7}$$

According to the form (4.1) and (4.5), the derivative of the energy function can be transformed by follows:

$$\dot{V}(\cdot) = \sum_{i=1}^{n+l} \frac{\partial V(\cdot)^T}{\partial x_i} \left(-\frac{\partial V(\cdot)}{\partial x_i} + u_i^\varepsilon \right) \tag{4.8}$$

Considering the form (4.6) and $\frac{\partial V(\cdot)}{\partial x_i} = \frac{\partial V(\cdot)}{\partial \lambda_2(\cdot)} \frac{\partial \lambda_2(\cdot)}{\partial x_i}$. It can get the following inequality:

$$\dot{V}(\cdot) \le -\left\|\frac{\partial V(\cdot)}{\partial \lambda_2(\cdot)}\right\|^2 \sum_{i=1}^{N} \left\|\frac{\partial \lambda_2(\cdot)}{\partial x_i}\right\|^2 + \left\|\frac{\partial V(\cdot)}{\partial \lambda_2(\cdot)}\right\| u_M \sum_{i=1}^{N} \left\|\frac{\partial \lambda_2(\cdot)}{\partial x_i}\right\| \tag{4.9}$$

So, if the inequality

$$\left\|\frac{\partial V(\cdot)}{\partial \lambda_2(\cdot)}\right\| \sum_{i=1}^{N} \left\|\frac{\partial \lambda_2(\cdot)}{\partial x_i}\right\|^2 \ge u_M \sum_{i=1}^{N} \left\|\frac{\partial \lambda_2(\cdot)}{\partial x_i}\right\| \tag{4.10}$$

is right, then $\dot{V}(\cdot) \le 0$.

It assumes the following conditions are right:

$$\sum_{i=1}^{N} \left\|\frac{\partial \lambda_2(\cdot)}{\partial x_i}\right\|^2 \ne 0 \tag{4.11}$$

Therefore, the form (4.10) can be written by the following form:

$$u_M \frac{\sum_{i=1}^{N} \left\|\frac{\partial \lambda_2(\cdot)}{\partial x_i}\right\|}{\sum_{i=1}^{N} \left\|\frac{\partial \lambda_2(\cdot)}{\partial x_i}\right\|^2} \le \left\|\frac{\partial V(\cdot)}{\partial \lambda_2(\cdot)}\right\| < \infty \tag{4.12}$$

According to the Property (P5) in Definition 2, $\exists \bar{\lambda} > \xi$ such that, $\forall \lambda_2(\cdot) \le \bar{\lambda}$, the inequality (4.12) is right. This indicates that the value of $\lambda_2(\cdot)$ is always bounded away from ξ. Suppose that the form (4.11) is not satisfied, so $\dot{\lambda}_2(\cdot) = \frac{\partial \lambda_2(\cdot)}{\partial x_i} \dot{x}_i = 0$ therefore the value of the $\lambda_2(\cdot)$ is invariable $\lambda_2 > \xi$.

From the aforesaid conclusion, if the topology is connected at the initial moment, the second eigenvalue of the corresponding Laplacian matrix will forever be greater than ξ even though the topology has changed with time varying. At last, it can ensure the multi-agent be structurally controllable.

According the Fig. 3, if the structural controllable system is endowed a set of weights, then not only the structural controllability can preserve and $\lambda_2 > \xi$, but also the topology is connected.

To sum up, by the effect of the control strategy u_i^c, the controllability of the time-varying multi-agent systems can be preserved by assigning a set of weights to multi-agent systems.

5 Example Analysis

Consider a system of six agents with time-delay interaction topology as shown in Fig. 4. Let the sixth agent be the leader and the interconnection graph be defined by the matrices:

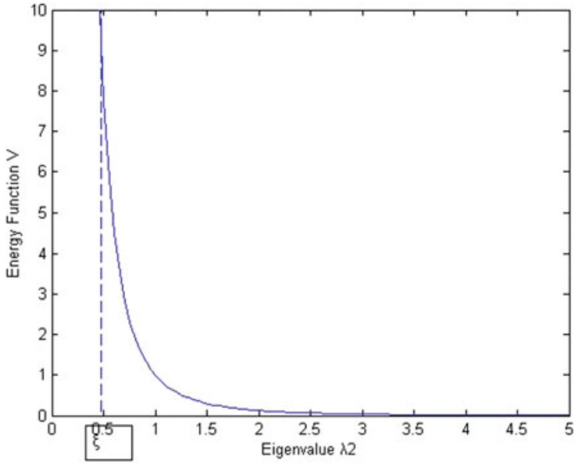

Fig. 3 The decreasing trend of energy function

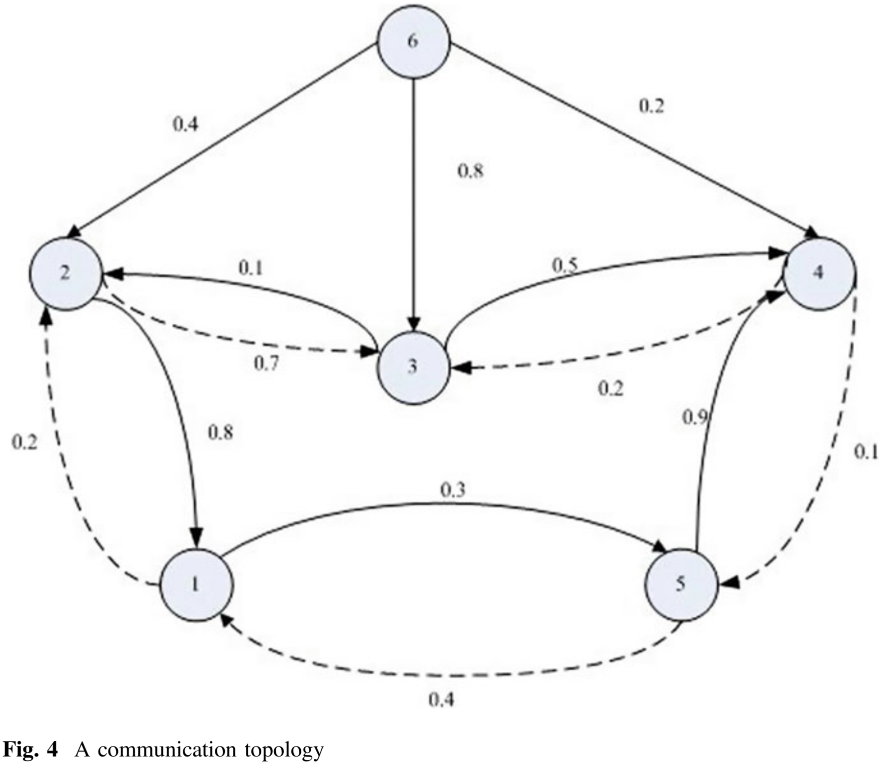

Fig. 4 A communication topology

$$D_1 = \begin{bmatrix} 1.2 & & & & \\ & 0.3 & & & \\ & & 0.9 & & \\ & & & 1.4 & \\ & & & & 0.4 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & -0.8 & 0 & 0 & -0.4 \\ -0.2 & 0 & -0.1 & 0 & 0 \\ 0 & -0.7 & 0 & -0.2 & 0 \\ 0 & 0 & -0.5 & 0 & -0.9 \\ -0.3 & 0 & 0 & -0.1 & 0 \end{bmatrix}$$

and

$$F = \begin{bmatrix} 1.2 & -0.8 & 0 & 0 & -0.4 \\ -0.2 & 0.3 & -0.1 & 0 & 0 \\ 0 & -0.7 & 0.9 & -0.2 & 0 \\ 0 & 0 & -0.5 & 1.4 & -0.9 \\ -0.3 & 0 & 0 & -0.1 & 0.4 \end{bmatrix} \quad R = \begin{bmatrix} 0 \\ -0.4 \\ -0.8 \\ -0.2 \\ 0 \end{bmatrix}$$

With the above parameters, we have verified the equation of the (5):

$$-D_1 - F + B_1 = -2F, R^T = \begin{bmatrix} 0 & -0.4 & -0.8 & -0.2 & 0 \end{bmatrix}$$

It is easy to verify that rank(Q) = 5. Therefore, the condition of Theorem 1 is satisfied and hence the given system is observable.

6 Conclusion

This paper mainly study the preservation of the controllability and observability of time delay and time-varying topology multi-agent systems with leader-following structure. We have gotten that the controllability of multi-agent systems with time-delay are related to the corresponding systems without time-delay. With respect to the selected leader agents, the observability of the multi-agent system with time delay and time-varying topology is equivalent to that of (F, R^T). In the future, it is meaningful to study the preservation of the controllability of time-varying topology and time-delay multi-agent systems with high-order dynamic agents.

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