Directed Graph-Based Adaptive Attitude Cooperative Control for Fractionated Spacecraft

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1 Introduction

The concept of fractionated spacecraft is conceived by optimally decomposing the functional units including payload, communication, energy, navigation, calculation process, etc. of a traditional spacecraft into multiple modular spacecrafts. Each modular spacecraft carries different functions and resources related to the tasks [1–3]. The fractionated spacecraft system finishes specific tasks by constructing a big virtual spacecraft through physical separation, free-flying cluster, wireless information exchange and wireless energy exchange in the mode of functional coordination and resource sharing. Cooperative attitude maneuver is an important way of attitude maneuver for fractionated spacecraft, and also is the basis for finishing payload tasks. Therefore, it is necessary to study the cooperative control of attitude maneuver among modular spacecrafts.

A kind of robust adaptive controller without decoupling was designed for certain small satellites [4]. However, this kind of controller is based on the consideration that the external disturbance is unknown constant quantity. An adaptive spacecraft attitude controller was designed by using back-stepping method [5] with the presentation of clear design steps. However, the influence of external disturbance is not taken into consideration. Both the abovementioned methods consider single spacecraft control rather than multi-spacecraft cooperative control. Ren et al. [6–8] investigated the distributed cooperative control strategy based on local information exchange for multi-agent formation systems. Then, they presented the second-order consensus protocol which considers the system state and differential

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state. Furthermore, aiming at the typical Euler–Lagrange system, they proposed the consistency algorithm, which considers various non-linear factors including saturation. To solve the problem of distributed spacecraft attitude synchronization, a method of attitude coordination and tracking control based on information consistency was presented [9]. This method is used to solve attitude synchronization problems among distributed satellites by designing information transmission models among satellites. However, this method also fails to take the influence of external disturbance into account.

By summarizing the above researches, this paper proposes a kind of distributed adaptive attitude cooperative controller based on the graph theory and the consistency theory so as to solve the attitude cooperative control problem of fractionated spacecraft. This method uses one parameter to comprehensively represent the uncertainty of moment of inertia of modular spacecraft, followed by the online evaluation using the designed adaptive law. Meanwhile, compensation term is designed in the controller to offset the influence of disturbance torque. The simulation results validate that this controller can realize the cooperative tracking of desired attitude.

2 Mathematical Models for Modular Spacecraft

There are various forms of attitude parameters for spacecraft. Since the modified Rodriguez parameter (MRP) has no nuisance parameter, it does not need to solve complex constraint equations and meanwhile can reduce the influence of singularity. Therefore, this paper adopts the MRP to describe attitude. The MRP of the ith modular spacecraft is defined as follows [10]:

$$r_i = (r_{i1}, r_{i2}, r_{i3})^{\mathrm{T}} = e_i \tan\frac{\Phi_i}{4} \tag{1}$$

where, e_i and Φ_i are the Euler rotation axis and Euler angle of the ith modular spacecraft, respectively. Rigid body dynamic equation is utilized to describe the attitude dynamics of the modular spacecraft whose attitude dynamics and kinematics equation are presented as follows:

$$\boldsymbol{J}_i \dot{\boldsymbol{\omega}}_i + \boldsymbol{\omega}_i^{\times} \boldsymbol{J}_i \boldsymbol{\omega}_i = \boldsymbol{T}_{iu} + \boldsymbol{T}_{id} \tag{2}$$

$$\dot{\boldsymbol{r}}_i = \boldsymbol{G}_i(\boldsymbol{r}_i)\boldsymbol{\omega}_i \tag{3}$$

where, $\boldsymbol{J}_i \in \mathbf{R}^{3\times3}$ is the moment of inertia of the ith modular spacecraft; $\boldsymbol{\omega}_i \in \mathbf{R}^3$ denotes the angular velocity; $\boldsymbol{T}_{iu} \in \mathbf{R}^3$ is the control torque and $\boldsymbol{T}_{id} \in \mathbf{R}^3$ presents the space disturbance torque. $\boldsymbol{G}_i(\boldsymbol{r}_i) = \frac{1}{2}\left[\left(\frac{1-\boldsymbol{r}_i^{\mathrm{T}}\boldsymbol{r}_i}{2}\right)\boldsymbol{I} + \boldsymbol{r}_i\boldsymbol{r}_i^{\mathrm{T}} + \boldsymbol{r}_i^{\times}\right]$, and $\boldsymbol{r}_i^{\times}$ is a cross-product matrix in the size of 3×3. $\boldsymbol{G}_i(\boldsymbol{r}_i)$ presents the following properties.

$$\boldsymbol{r}_i^{\mathrm{T}}\boldsymbol{G}_i(\boldsymbol{r}_i) = \left[\frac{1+\boldsymbol{r}_i^{\mathrm{T}}\boldsymbol{r}_i}{4}\right]\boldsymbol{r}_i^{\mathrm{T}} \tag{4}$$

Suppose that the desired MRP and the desired angular velocity of the ith modular spacecraft are $\boldsymbol{r}_{id}$ and $\boldsymbol{\omega}_{id}$ separately. The error of MRP and error of angular velocity are denoted as $\boldsymbol{r}_{ie} = \boldsymbol{r}_i - \boldsymbol{r}_{id}$ and $\boldsymbol{\omega}_{ie} = \boldsymbol{\omega}_i - \boldsymbol{C}_i\boldsymbol{\omega}_{id}$, respectively, in which $\boldsymbol{C}_i$ is the direction cosine matrix of transformation. By substituting these parameters into Formula (2), we can obtain the error dynamics and kinematics equations:

$$\boldsymbol{J}_i\dot{\boldsymbol{\omega}}_{ie} = -\boldsymbol{\omega}_i^{\times}\boldsymbol{J}_i\boldsymbol{\omega}_i + \boldsymbol{J}_i(\boldsymbol{\omega}_{ie}^{\times}\boldsymbol{C}_i\boldsymbol{\omega}_{id} - \boldsymbol{C}_i\dot{\boldsymbol{\omega}}_{id}) + \boldsymbol{T}_{iu} + \boldsymbol{T}_{id} \tag{5}$$

$$\dot{\boldsymbol{r}}_{ie} = \boldsymbol{G}_i(\boldsymbol{r}_{ie})\boldsymbol{\omega}_{ie} \tag{6}$$

Assumption 1 Since the precise value of the moment of inertia of the ith modular spacecraft is impossible to be obtained owing to the factors including rotation of payloads, $\boldsymbol{J}_i$ is considered as a constant in the range of $0 \le \|\boldsymbol{J}_i\| \le c_{iJ}$. Modular spacecraft is mainly subjected to disturbances including solar radiation torque and space environment disturbance torque. Suppose $0 \le \|\boldsymbol{T}_{id}\|^2 \le d_i$, c_{iJ} and $d_i \in \mathbf{R}$ are unknown constants.

3 Design of an Adaptive Cooperative Controller

Lemma 1 [11]: *If a positive definite function $V(t)$ satisfies the following in equation:*

$$\dot{V}(t) \le -\xi V(t) + \chi(t) \tag{7}$$

where, ξ presents a positive constant. For $\forall t > 0$, there exists $\chi(t) > 0$. If $\chi(t) = C$ is a constant, the system is considered to be globally uniform and boundedly stable.

The adaptive cooperative controller is designed to $\lim_{t\to\infty}\|\boldsymbol{r}_{ie}\| = 0$ and $\lim_{t\to\infty}\|\boldsymbol{\omega}_{ie}\| = 0$. To describe the communication topology among modular spacecrafts, define cooperative error variable as follows.

$$\begin{aligned}\boldsymbol{e}_i &= \boldsymbol{\omega}_{ei} + \lambda_i\boldsymbol{G}^{-1}(\boldsymbol{r}_{ei})\left[\sum_{j=1}^{n} a_{ij}(\boldsymbol{r}_i - \boldsymbol{r}_j) + \boldsymbol{r}_{ei}\right] \\ &= \boldsymbol{\omega}_{ei} + \lambda_i\boldsymbol{G}^{-1}(\boldsymbol{r}_{ei})\left[\sum_{j=1}^{n} a_{ij}(\boldsymbol{r}_{ei} - \boldsymbol{r}_{ej}) + \boldsymbol{r}_{ei}\right]\end{aligned} \tag{8}$$

where, $\lambda_i > 0 \in \mathbf{R}$ presents design parameters. By performing derivation to Formula (8), we acquire:

$$\dot{\boldsymbol{e}}_i = \dot{\boldsymbol{\omega}}_{ei} + \lambda_i \dot{\boldsymbol{G}}^{-1}\left[\sum_{j=1}^{n} a_{ij}(\boldsymbol{r}_{ei} - \boldsymbol{r}_{ej}) + \boldsymbol{r}_{ei}\right] + \lambda_i \boldsymbol{G}^{-1}\left[\sum_{j=1}^{n} a_{ij}(\dot{\boldsymbol{r}}_{ei} - \dot{\boldsymbol{r}}_{ej}) + \dot{\boldsymbol{r}}_{ei}\right] \tag{9}$$

Suppose $\boldsymbol{\xi}_i = \lambda_i \dot{\boldsymbol{G}}^{-1}\left[\sum_{j=1}^{n} a_{ij}(\boldsymbol{r}_{ei} - \boldsymbol{r}_{ej}) + \boldsymbol{r}_{ei}\right] + \lambda_i \boldsymbol{G}^{-1}\left[\sum_{j=1}^{n} a_{ij}(\dot{\boldsymbol{r}}_{ei} - \dot{\boldsymbol{r}}_{ej}) + \dot{\boldsymbol{r}}_{ei}\right]$, we gain the following formula after substituting $\boldsymbol{\xi}_i$ into Formula (9) and multiplying the two sides of the formula by $\boldsymbol{J}_i$.

$$\boldsymbol{J}_i \dot{\boldsymbol{e}}_i = -\omega_i^{\times} \boldsymbol{J}_i \boldsymbol{\omega}_i + \boldsymbol{J}_i(\boldsymbol{\omega}_{ei}^{\times} \boldsymbol{C}_i \boldsymbol{\omega}_d - \boldsymbol{C}_i \dot{\boldsymbol{\omega}}_d) + \boldsymbol{T}_{ui} + \boldsymbol{T}_{di} + \boldsymbol{J}_i \boldsymbol{\xi}_i \tag{10}$$

To simply Formula (10) and clarify the structure of the controller, let $\boldsymbol{\varepsilon}_i = \boldsymbol{\omega}_{ei}^{\times} \boldsymbol{C}_i \boldsymbol{\omega}_d - \boldsymbol{C}_i \dot{\boldsymbol{\omega}}_d + \boldsymbol{\xi}_i$ and then substitute it into Formula (10).

$$\boldsymbol{J}_i \dot{\boldsymbol{e}}_i = -\boldsymbol{\omega}_i^{\times} \boldsymbol{J}_i \boldsymbol{\omega}_i + \boldsymbol{J}_i \boldsymbol{\varepsilon}_i + \boldsymbol{T}_{ui} + \boldsymbol{T}_{di} \tag{11}$$

The matrix and parameter vector are defined as follows:

$$\boldsymbol{P}_i = \begin{bmatrix} \varepsilon_{i1} & \omega_{i2}\omega_{i3} & -\omega_{i2}\omega_{i3} & \varepsilon_{i2} + \omega_{i1}\omega_{i3} & \varepsilon_{i3} - \omega_{i1}\omega_{i2} & \omega_{i3}^2 - \omega_{i2}^2 \\ -\omega_{i1}\omega_{i3} & \varepsilon_{i2} & \omega_{i1}\omega_{i3} & \varepsilon_{i1} - \omega_{i2}\omega_{i3} & \omega_{i1}^2 - \omega_{i3}^2 & \varepsilon_{i3} + \omega_{i1}\omega_{i2} \\ \omega_{i1}\omega_{i2} & -\omega_{i1}\omega_{i2} & \varepsilon_{i3} & \omega_{i2}^2 - \omega_{i1}^2 & \varepsilon_{i1} + \omega_{i2}\omega_{i3} & \varepsilon_{i2} - \omega_{i1}\omega_{i3} \end{bmatrix} \tag{12}$$

$$\boldsymbol{\theta}_i = \begin{bmatrix} J_{i11} & J_{i22} & J_{i33} & J_{i12} & J_{i13} & J_{i23} \end{bmatrix}^{\mathrm{T}} \tag{13}$$

Then Formula (11) is converted to:

$$\boldsymbol{J}_i \dot{\boldsymbol{e}}_i = \boldsymbol{P}_i \boldsymbol{\theta}_i + \boldsymbol{T}_{ui} + \boldsymbol{T}_{di} \tag{14}$$

The uncertainty of the moment of inertia is mainly reflected by the parameter $\boldsymbol{\theta}_i$, therefore, adaptive online evaluation needs to be performed on this parameter. The control law and adaptive law are designed as follows:

$$\boldsymbol{T}_{ui} = -\boldsymbol{P}_i \hat{\theta}_i - \rho_i \boldsymbol{e}_i - \gamma_i \boldsymbol{e}_i \tag{15}$$

$$\dot{\hat{\theta}}_i = \beta_i \boldsymbol{P}_i^{\mathrm{T}} \boldsymbol{e}_i \tag{16}$$

where, ρ_i and $\gamma_i > 0 \in \mathbf{R}$ are design parameters and $\beta_i \in \mathbf{R}^{6\times6}$ shows the positive definite symmetric matrix. $\gamma_i e_i$ is designed to compensate the T_{id} in Formula (2) and Formulas (15) and (16) can be used to realize the control aim. The controller is proved to be stability, and the process of proof is omitted here due to the length of the paper.

4 Simulation Example and Result Analysis

To verify the validity of the designed controller in this paper, the cooperative attitude tracking control for fractionated spacecraft is simulated based on Matlab/Simulink. Suppose that there are four modular spacecrafts in the cluster, and the communication topology among the modular spacecrafts is shown in Fig. 1 where the values on the edges represent weights. Based on the abovementioned graph theory, we can obtain the corresponding information including the adjacent matrix.

Suppose the initial uncertainty of the moment of inertia of modular spacecraft is $\boldsymbol{\theta}_i = [5 \quad 10 \quad 10 \quad 0.05 \quad 0.05 \quad 0.05]^{\mathrm{T}}$, the simulation is performed for 30 s. Suppose the desired MRP is $\boldsymbol{r}_d = [0.02\sin 0.5t,\, 0.03\sin 0.3t,\, 0.06\cos 0.3t]^{\mathrm{T}}$. Figures 2, 3, 4 and 5 demonstrate the simulation results. It can be observed from the

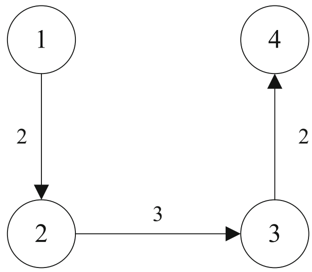

Fig. 1 Directed topology among the four modular spacecrafts

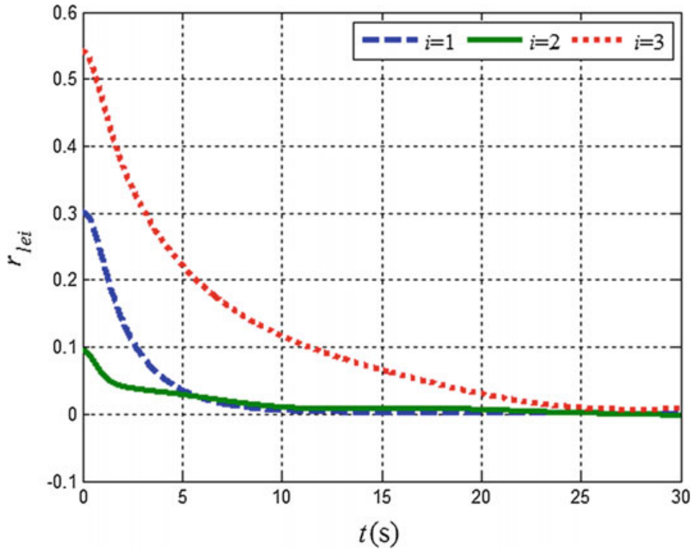

Fig. 2 The error of MRP of module 1

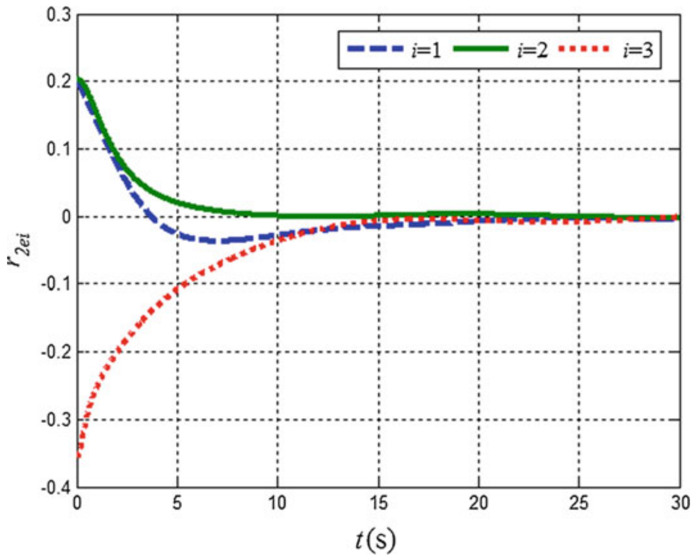

Fig. 3 The error of MRP of module 2

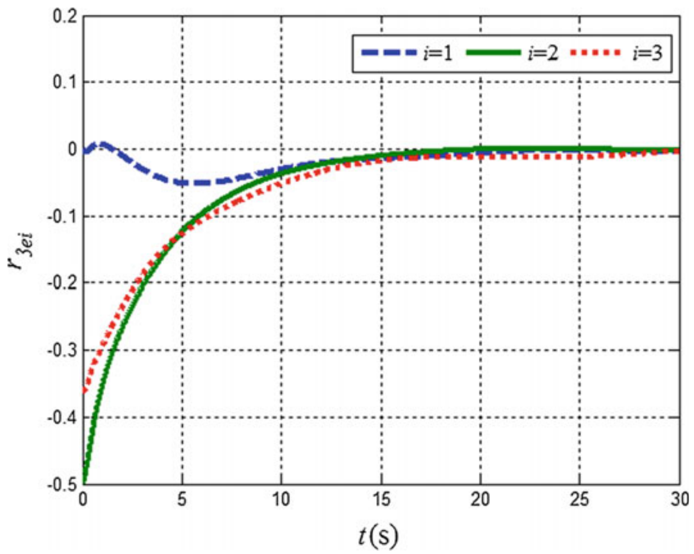

Fig. 4 The error of MRP of module 3

results that, the four modular spacecrafts cooperatively track the desired MRP at 25 s. As can be seen, the controller presents strong robustness to external disturbance and high tracking precision. By comparing the figures, the cooperativity of the modular spacecraft 1 is weaker than the other three. Meanwhile, the

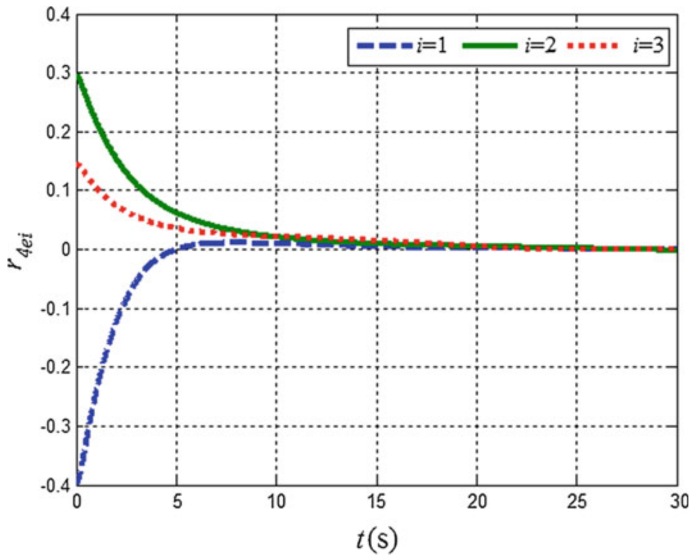

Fig. 5 The error of MRP of module 4

communication topology figure shows that the modular spacecraft 1 does not acquire attitude information from the other three modular spacecrafts, that is, the modular spacecraft 1 is located on the top of the sole directed spanning tree in Fig. 1.

5 Conclusions

Aiming at the attitude cooperative control of fractionated spacecraft, this paper designs a distributed adaptive attitude cooperative controller based on the graph theory and consistency theory. This controller is designed by considering the uncertainty of moment of inertia of modular spacecraft and the influence of external disturbance torque. Firstly, the MRP and Euler equation of motion are used to describe the attitude of modular spacecraft; meanwhile, the kinetic equation of attitude error of the modules is derived. Then, the directed graph is applied to demonstrate the communication topology among modular spacecrafts. Next, a distributed cooperative controller is designed based on the consistency theory, followed by the online evaluation of the unknown moment of inertia by using the designed adaptive law. The controller designed in this paper presents simple structure and can be operated easily. Moreover, the final simulation results indicate that this controller can realize cooperative maneuver of module attitude and shows certain robustness to the external disturbance.

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