

Initial Attitude Estimation and Installation Errors Calibration of the IMU for Plane by SINS/CNS Integration

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1 Introduction

Strapdown inertial navigation system (SINS) has been widely used in various types of aircrafts, for the advantages of completely autonomy and high rate of output. The accuracy of initial alignment and calibration of the IMU installing errors greatly affect the overall navigation accuracy [1]. The initial alignment is the process of determining the initial values of attitude of the system. Installation errors mean that the IMU incorrectly installs on the plane, which can cause the existence of an additional interference acceleration in accelerator output and gyros drift in gyro output and finally lead to navigation errors.

Celestial navigation system (CNS) can provide accurate attitude information, whose navigation accuracy is not relate to time and distance, but strongly depends on the accuracy of measurements [2]. Research on the initial alignment has been carried on the past. Lei [3] proposed a fast initial alignment for SINS method which combines Extended States Observers (ESO) with Kalman filters. Using this method, the speed of initial alignment is more quickly and characters such as high accurate and robust are possessed. However, this fast initial alignment, with low observability, cannot calibrate the installation errors. Sun [4] proposed an online calibration method of marine SINS aided by CCD star sensor. This method could estimate the gyro drift and accelerometer bias quickly and accurately, but it does not take installation errors into account on the navigation result.

The initial alignment of the SINS and the calibration of the IMU installing errors are difficult problems under the environment of autonomous navigation. However, considering the complementary characteristics of SINS and CNS, the SINS initial alignment and calibration can be accomplished by combing both of them [5].

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A method using a single inflight star sighting to estimate the initial attitude and calibrate the IMU installing errors is presented in this paper. To make all state variables observable, the outputs of IMU are also added to the measurement variables, and an unscented Kalman filter is used to fuse the data of SINS and CNS.

2 Principles of SINS and CNS

Coordinate frames which including earth-centered inertial frame (i-frame), earth-fixed frame (e-frame), the navigation frame (n-frame), the plane body frame (b-frame), IMU coordinate frame (p-frame), and the star sensor frame (s-frame) used in this paper are shown in Fig. 1.

2.1 Principle of SINS and Influence of Installation Errors

A full inertial navigation system mainly consists of the corresponding SINS mechanization and an inertial measurement unit (IMU), which typically includes three orthogonal accelerometers and three orthogonal gyroscopes. Angular rates and linear accelerations measured by the IMU are transformed to the n-frame and are used to determine the position, velocity and attitude of the plane. SINS attitude initialization is called alignment, which is the process of determining the initial values of the coordinate transformation from the body frame to the navigation frame in SINS. In the SINS, the IMU is directly mounted on the plane, which is assumed to coincide with the body frame under ideal conditions. However, due to the existence of installation errors, the existence of an additional interferential acceleration in accelerator output and gyro drift in gyros output are induced, which can lead to navigation errors. The installation errors matrix of IMU can be represented by three small angles ϑ_x, ϑ_y and ϑ_z [6].

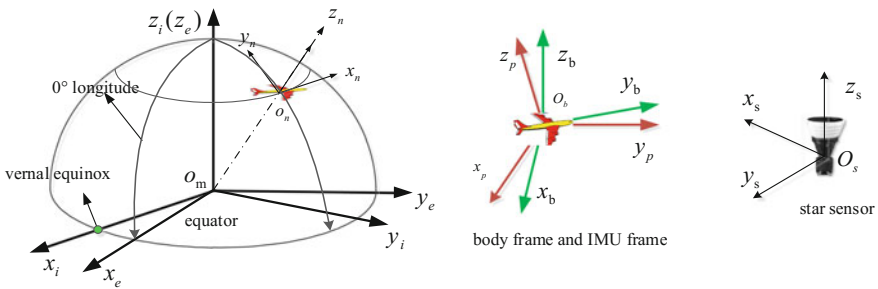


Fig. 1 Reference frame

$$\mathbf{C}_b^p = (\mathbf{C}_p^b)^T = \mathbf{I} + \Delta \mathbf{C}_b^p \approx \begin{bmatrix} 1 & \vartheta_z & -\vartheta_y \\ -\vartheta_z & 1 & \vartheta_x \\ \vartheta_y & -\vartheta_x & 1 \end{bmatrix} \quad (1)$$

In the IMU frame, $\tilde{\mathbf{w}}_{ib}^b, \tilde{\mathbf{f}}^b$ are the actual output value of the gyroscope and accelerometer respectively. $\mathbf{w}_{ib}^b, \mathbf{f}^b$ represent ideal output value of the gyroscope and accelerometer respectively. The measurement errors of gyro and accelerometer can be expressed as

$$\begin{cases} \delta \mathbf{w}_{ib}^n = \mathbf{C}_b^n \delta \mathbf{w}_{ib}^b = \mathbf{C}_b^n (\tilde{\mathbf{w}}_{ib}^b - \mathbf{w}_{ib}^b) = \mathbf{C}_b^n (\mathbf{C}_b^p - \mathbf{I}) \mathbf{w}_{ib}^b \\ \delta \mathbf{f}^n = \mathbf{C}_b^n \delta \mathbf{f}^b = \mathbf{C}_b^n (\tilde{\mathbf{f}}^b - \mathbf{f}^b) = \mathbf{C}_b^n (\mathbf{C}_b^p - \mathbf{I}) \mathbf{f}^b \end{cases} \quad (2)$$

where \mathbf{C}_b^n is the plane's attitude matrix, which can be defined using sequence of rotations 2-1-3 as

$$\begin{aligned} \mathbf{C}_n^b &= (\mathbf{C}_b^n)^T \\ &= \begin{bmatrix} \cos \theta \cos \psi - \sin \theta \sin \varphi \sin \psi & -\cos \varphi \sin \psi & \sin \theta \cos \psi + \cos \theta \sin \varphi \sin \psi \\ \cos \theta \sin \psi + \sin \theta \sin \varphi \cos \psi & \cos \varphi \cos \psi & \sin \theta \sin \varphi - \cos \theta \sin \varphi \cos \psi \\ -\sin \theta \cos \varphi & \sin \varphi & \cos \theta \cos \varphi \end{bmatrix} \end{aligned} \quad (3)$$

where θ, φ, ψ are pitch, roll and yaw angles, respectively.

2.2 Principle of CNS

Stars always move in the regular way, thus their positions at a specific time can be obtained exactly. CNS is mainly composed of the star sensor and the star image processing software. After preprocessing of the original star image captured by star sensor, the starlight unit vector in the s-frame \mathbf{s}_s can be obtained [7]. At the same time, When the number of stars observed is more than three, the transformation matrix (\mathbf{C}_b^i) from the b-frame to i-frame can be calculated. Also, the starlight unit vector expressed in the earth-centered inertial frame \mathbf{s}_i can be acquired after star image pattern recognition as follows [8].

$$\mathbf{s}_i = [\cos \Delta \cos R_A \quad \cos \Delta \sin R_A \quad \sin \Delta]^T \quad (4)$$

where Δ, R_A represent the declination and right ascension of the star respectively.

3 System Models

3.1 State Model

Due to the stationary of the plane, the altitude velocity component is ignored, and the SINS error equation in the navigation frame is used as state model, which can be written as [9]

$$\begin{cases} \dot{\boldsymbol{\phi}} = \boldsymbol{\phi} \times \boldsymbol{\omega}_{in}^n + \delta\boldsymbol{\omega}_{in}^n - \mathbf{C}_b^n \delta\boldsymbol{\omega}_{ib}^b - \mathbf{C}_b^n \boldsymbol{\varepsilon} \\ \delta\dot{\mathbf{v}}_n = -\boldsymbol{\phi} \times \mathbf{f}^n + \mathbf{C}_b^n \delta\mathbf{f}^b - 2\boldsymbol{\omega}_{ie}^n \times \delta\mathbf{v}_n + \mathbf{C}_b^n \nabla \\ \dot{\boldsymbol{\varepsilon}} = 0 \\ \dot{\nabla} = 0 \\ \dot{\boldsymbol{\vartheta}} = 0 \end{cases} \quad (5)$$

where $\boldsymbol{\phi} = [\phi_E \ \phi_N \ \phi_U]^T$ is the misalignment angle. $\delta\mathbf{v}_n = [\delta v_E \ \delta v_N]^T$ is the velocity error. $\boldsymbol{\varepsilon} = [\varepsilon_x \ \varepsilon_y \ \varepsilon_z]^T$ is the gyro drift. $\nabla = [\nabla_x \ \nabla_y \ \nabla_z]^T$ is the accelerometer bias. \mathbf{f}^n is the projection of the output of accelerometers in the n-frame.

$\boldsymbol{\omega}_{ie}^n = [0, \omega_{ie} \cos L, \omega_{ie} \sin L]^T$ is the earth rotation rate denoted in the n-frame. Assure state vector \mathbf{X} , Eq. (5) can be expressed as

$$\dot{\mathbf{X}} = f(\mathbf{X}) + \mathbf{W} \quad (6)$$

where \mathbf{W} is the process noise.

3.2 Measurement Model

To make all state variables observable, the horizontal velocity errors, starlight vector and outputs of IMU are chosen as the measurement variables.

(1) Horizontal Velocity Errors

When the plane is stationary, the horizontal velocities calculated by the SINS are velocity errors. Using this velocity errors as measurements, the measurement equation can be expressed as follows.

$$\delta \mathbf{v}_n = [\delta v_E, \delta v_N]^T = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{X} \quad (7)$$

(2) **Starlight Vector**

According to the principle of CNS, the starlight vector can be obtained in i-frame and s-frame. Therefore the measurement equation can be expressed as follows.

$$\mathbf{s}_s = \mathbf{C}_b^s \mathbf{C}_n^b \mathbf{C}_n^{n'} \mathbf{C}_e^n \mathbf{C}_i^e \mathbf{s}_i \quad (8)$$

(3) **Output of Accelerometers**

$$\tilde{\mathbf{f}}_{ib}^b = \mathbf{C}_b^p \mathbf{C}_n^b \mathbf{g}_n + \mathbf{V} \quad (9)$$

where $\mathbf{g}_n = [0 \ 0 \ g]^T$ is the local gravity vector. According to Eqs. (7), (8) and (9), the measurement using $\mathbf{Z} = [\delta \mathbf{v}, \mathbf{s}_s, \tilde{\mathbf{f}}_{ib}^b]^T$ can be expressed as

$$\mathbf{Z} = h(\mathbf{X}) + \mathbf{V} \quad (10)$$

where \mathbf{V} is the measurement noise. Because the measurement model (10) is non-linear, the Unscented Kalman Filter (UKF) [10, 11] is implemented to fuse SINS and CNS data in this study. The block diagram of alignment and installation errors calibration by SINS/CNS integration method based on UKF is shown in Fig. 2.

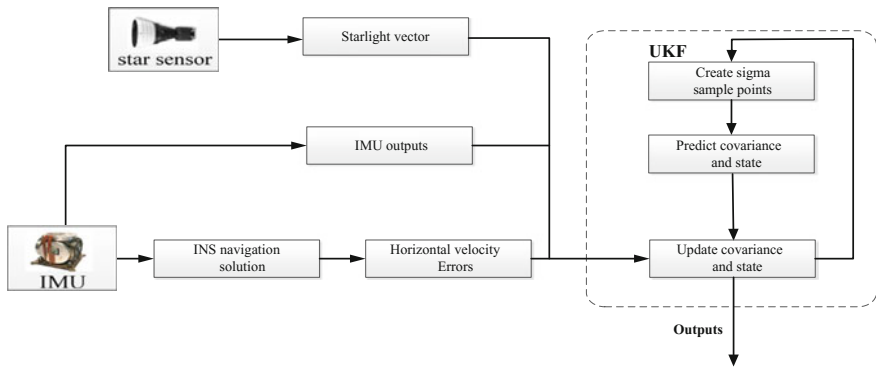


Fig. 2 Workflow of alignment and installation errors calibration by SINS/CNS integration algorithm

4 Results and Discussion

4.1 Simulation Condition

Simulations are performed to verify the feasibility of this method. In the following simulations, the plane keeps stationary, and the initial conditions are defined as: longitude 116°E, latitude 40°N. The initial pitch, yaw and roll angles are 0°, 20° and 0°, and its initial attitude errors of pitch angle, yaw angle and roll angle are all 10". The parameters of IMU are: the drifts and random drifts of each gyro are chosen as 0.005°/h and 0.001°/h, and the biases and random biases of each accelerometer are chosen as 50 and 10 μg , with its frequency as 100 Hz. The accuracy of the star sensor is 3" (1σ) and its frequency is 5 Hz. The total running time is 5 min and the filtering period is 1 s.

4.2 Simulation Results

The estimation of attitude and its error are shown in Fig. 3, it can be seen that the curve of estimated attitude converges rapidly from the start. During the filtering period, the mean estimation errors in pitch, yaw and roll are 0.8858", 0.9232" and 0.8369", respectively. The maximum error in attitude is 6.18", which is much better than that of tradition method. From this result, we can see that this method can accurately estimate the attitude of the plane.

Figure 4 shows the results of accelerometer and gyroscope errors estimated by this SINS/CNS integration method. The estimation curves of accelerometer and gyroscope converge quickly, which suggests that it can accurately estimate the gyroscope drifts and accelerometer biases. Figure 5 illustrates the estimation result of the IMU installing errors. As indicated in this figure, the estimated values of

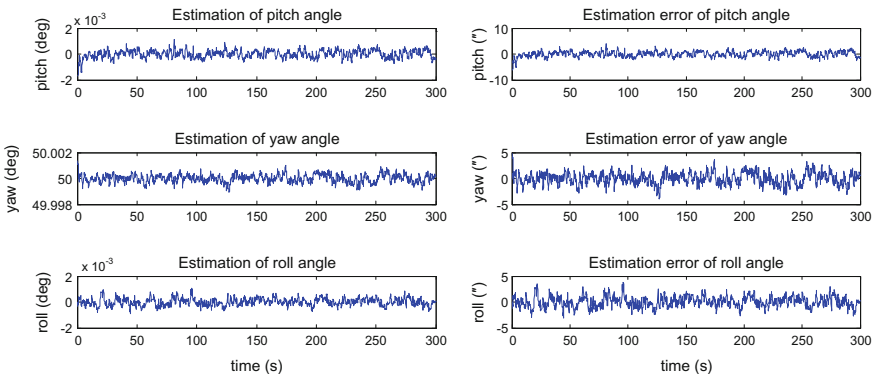


Fig. 3 The estimation of attitude and its error

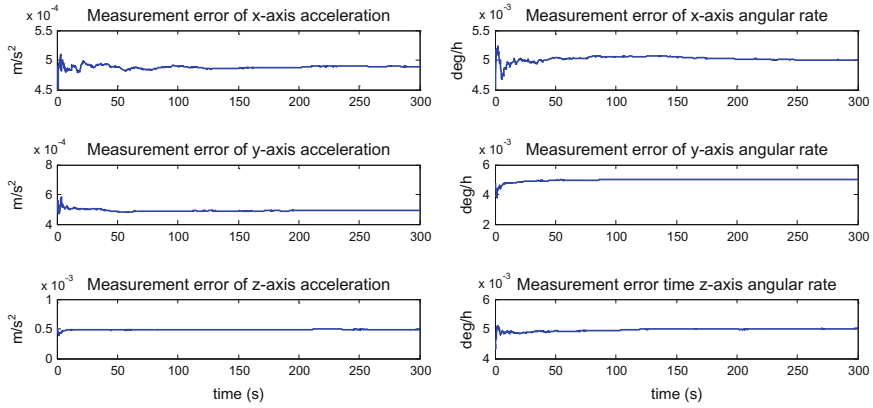


Fig. 4 The estimation of accelerometer and gyro errors

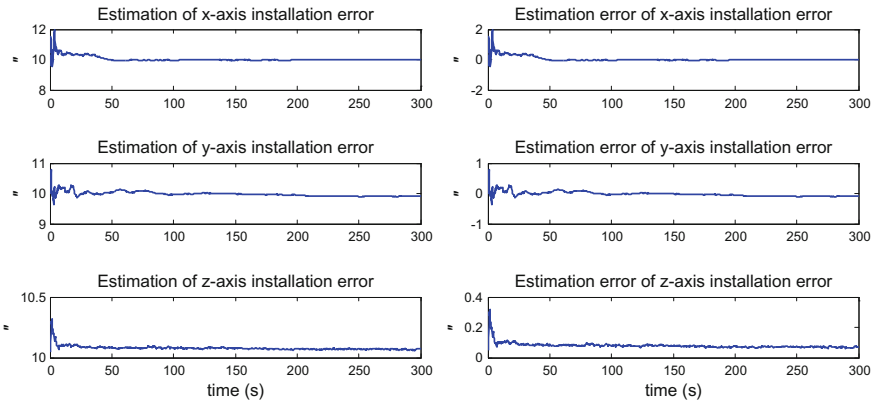


Fig. 5 The estimation of installation errors

installation errors in three directions converge to 9.98", 9.91", 10.07", which are very close to the true value 10", 10", and 10". The maximum estimation errors of installation errors is 2". From these figures, it is clear that the initial attitude estimation and installation errors calibration by SINS/CNS integration method developed in this paper is able to accurately estimate the accelerometer and gyroscope errors. Furthermore, it can also estimate the installation errors of the IMU effectively.

5 Conclusion

In the paper, an initial attitude estimation and installation errors calibration of the IMU for plane by SINS/CNS integration method is presented, which could solve the problems of SINS initial alignment and IMU calibration at the same time. The IMU installing errors are considered and its corresponding state equation is established. To make all state vectors observable, all original information provided by the sensors of the SINS and the CNS are used as measurements. The maximum error in attitude is 6.18", which indicates that both the inertial sensors' biases in the SINS and the installation errors of the IMU are estimated effectively. It should be noted that the proposed approach can be also applied to other carrier with a series of sensors.

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