Event-Triggered Communication and H_∞ Filtering Co-design for Networked Control Systems

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Abstract. In this paper, the issue of event-triggered H_∞ filtering for networked control systems (NCSs) with transmission delay is investigated. First of all, a strategy called event-triggered is introduced where tasks are generated only at the time that the event-triggering condition set before on the sampled measurements of the plant is satisfied. Then considering the double effects of the communication delay and the event-triggering technique, transform the filtering error system into a time-delay system the problem of which can be derived by the existing theory. A co-design method of event-triggered mechanism and H_∞ filtering which also guarantees the asymptotic stability of the NCSs is obtained by constructing a properly Lyapunov-Krasovskii functional and LMI technique based on the new model. Finally an example of verification is given to show the validity of the proposed method.

Keywords: H_∞ filtering design · Event-triggered communication scheme · LMI · NCSs · Lyapunov-Krasovskii functional

1 Introduction

In recent years, the networked control system (NCS) has a significant influence to control systems, especially in industrial process control and real-time control system, for its advantages [1]. However, it also brings a series of problems that reduce the performance of the system and even lead to the instability [2]. Therefore, it is necessary to take negative effects into account when design network-based systems with desired filtering performances. Kalman filtering is a useful way to solve the problem of filtering, but the external noise statistical information need to know in advance [3]. Different from the Kalman filtering approach, H_∞ filtering algorithm can successfully handle the uncertainties of external noises and minimize the infinity norm of the filtering error under bounded external noise interference.

For most NCSs, the measured outputs are sampled in a constant sampling period and all sampled signals will be sent to determine an H_∞ filtering, i.e., time-triggered. Modeling and analysis of NCSs based on this scheme are easy to implement. It is no

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necessity to send a signal which carries almost no fluctuation information compared the last measurement output. So this kind of triggering method will send "redundant" sampling signals, what is unsuitable from the perspective of network resource utilization and network load reduction [5, 6]. Unlike the time triggering, the event-triggered transport cycle is not fixed and the task is executed only when a prescribed event condition is met, such as, a signal exceeds a given threshold. Accordingly, sampled-data packets are broadcast only when needed and the amount of corresponding tasks will be reduced, which can overcome the drawback of time-triggering [7, 8]. However, after introducing the event triggering mechanism, the difficulty of modeling and analysis of the system will be improved.

In [9], A convex optimization problem with some LMI constraints is formulated to design the H_∞ filtering but based on time-triggered method. In [10], the optimal event-triggered filtering approach has been proposed in linear discrete time systems but based on Kalman filtering method. In [4], the problem of event-based H_∞ filtering for linear continuous system which is exponential stability is studied without considering lower bound delay. In [14], event-based H_∞ filtering for sampled-data systems is developed by an improved inequality for the integral term while maintain satisfactory closed-loop performances. However, there are a few studies on consideration of the network-induced transmission delay, H_∞ filtering problems, and the output signal-dependent event-triggered communication scheme simultaneously up to now, which is the dominant motivation leading to the present research.

The main advantages of this paper will be epitomized as follows: (i) a discrete event-triggered mechanism is proposed which only needs the measured output information at the sampling instants. Consequently, it is convenient for software implementation and the additional hardware devices can be avoided. (ii) a Lyapunov-Krasovskii functional, which can provide less conservatism, is applied to get the event H_∞ filtering under which both the robust stability of the system and a specified H_∞ disturbance rejection attenuation level is met combining with LMI.

2 Problem Statement

2.1 System Description

Consider the following linear time-invariant continuous-time system:

$$\begin{cases} \dot{x}(t) = Ax(t) + B\omega(t) \\ y(t) = Cx(t) + D\omega(t) \\ z(t) = Lx(t) \\ x(0) = x_0 \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $y(t) \in R^m$ is the measured output, $\omega(t) \in R^l$ is the external input disturbance which belongs to $L_2[0, \infty)$, and $z(t) \in R^p$ is the signal to be estimated; A, B, C, D and L are constant matrices; $x(t_0) = x_0$ is the initial condition.

Construct the filter for NCSs based on event-triggered mechanism with transmission delay. The structure is shown in Fig. 1. There is an intelligent sensor in the NCSs.

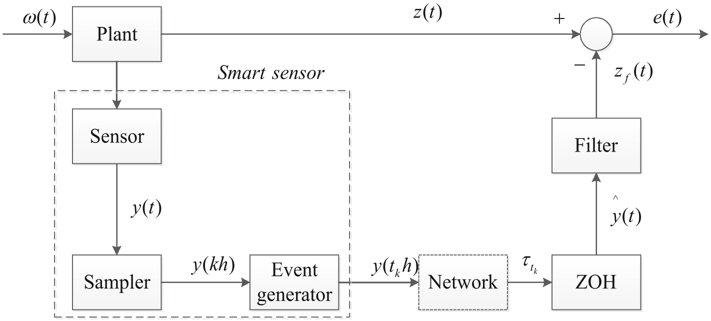

Fig. 1. The structure of event-triggered NCSs with time-varying and filter

The sensor and the sampler are time triggered. The controller and the ZOH are event-triggered. Once a signal data packet has been successfully sent over the network, it is will be continually hold in the ZOH until the next packet arrives. The signal transmission is a single packet, and no data packet loss or disorder. $\tau_{t_k} (k \in N)$ is network-induced delay, which is time-varying and bounded, i.e. $0 < \tau_m < \tau_{t_k} < \tau_M$. N denotes the natural number. The sampling period is h, the sampling instant is kh, and the sampled data $y(kh)$ is transmitted to the event generator directly. $y(t_k h)$ is a measurement signal which is transmitted successfully after the event generator.

The essence of the filtering problem is to estimate $z(t)$ through the system known measured outputs. Consider a full-order estimator, as following:

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f \hat{y}(t) \\ z_f(t) = C_f x_f(t) + D_f \hat{y}(t) \end{cases} \tag{2}$$

where $x_f(t)$ is the filter state vectors, $\hat{y}(t)$ is the filter input vectors, $z_f(t)$ is the estimation output of $z(t)$. A_f, B_f, C_f, D_f are matrices to be designed.

2.2 Event Trigger Mechanism

The event-triggered mechanism which transmits the signals on demand for NCSs with time-varying delay in Fig. 1 is designed as following:

$$\partial^T (t_k h + jh) \Omega \partial (t_k h + jh) \geq \sigma y^T (t_k h + jh) \Omega y(t_k h + jh) \tag{3}$$

where $y(t_k h + jh)$ is the current sampling data of $y(t)$; $y(t_k h)$ is the latest transmitted data of $y(t)$; $\partial(t_k h + jh)$ is the error between the current sampling data $y(t_k h + jh)$ and the latest transmitted sampling signal $y(t_k h)$, $j \in N$; Ω is a positive weighting matrix and enhances the feasibility of the linear matrix inequality compared by $\Omega = I$ [13]; σ is the threshold.

If the sampled data satisfies the given condition, it will be transmitted; otherwise, be neglected. This strategy does not require complex computing bounds of trigger interval and avoids Zeno phenomenon in principle [10]. The release instants of sampled data are $t_0h, t_1h, \ldots, t_kh \ldots$. And denote $\beta_k h = t_{k+1}h - t_kh$ is the transmission period of NCSs based on above event-triggered mechanism. The released sampled-data packets will reach the ZOH at the instants $t_0h + \tau_{t_0},\ t_1h + \tau_{t_1},\ \ldots, t_kh + \tau_{t_k}, \ldots$.

2.3 Filtering Error System

In combination with above analysis, take the bounded time-varying delay and the event-triggered mechanism (3) into consideration, the filter input can be converted to:

$$\hat{y}(t) = y(t_kh),\ t \in [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{4}$$

Combining (2) and (4), yields

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f y(t_kh) \\ z_f(t) = C_f x_f(t) + D_f y(t_kh) \\ t \in [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \end{cases} \tag{5}$$

Consult the method in [4], and then analyze and discuss the interval $[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$. Two cases are discussed as follows.

(1) If $t_{k+1}h + \tau_{t_{k+1}} > t_kh + h + \tau_M$, there must be a positive integer d, such that

$$t_kh + dh + \tau_M \le t_{k+1}h + \tau_{t_{k+1}} \le t_kh + (d+1)h + \tau_M \tag{6}$$

Then the interval can be divided into $d+1$ sub intervals and reconstructed as:

$$\left[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right) = \beta_1 \cup \beta_2 \cup \beta_3 \tag{7}$$

where

$$\beta_1 = [t_kh + \tau_{t_k}, t_kh + h + \tau_M),\ \beta_2 = \{\cup_{i=1}^{d-1}[t_kh + ih + \tau_M, t_kh + (i+1)h + \tau_M\},$$
$$\beta_3 = [t_kh + dh + \tau_M, t_{k+1}h + \tau_{t_{k+1}})$$

Define a piecewise function $\tau(t)$:

$$\tau(t) = \begin{cases} t - t_kh, t \in \beta_1 \\ t - t_kh - ih, t \in \beta_2, i = 1, 2, \ldots, d-1 \\ t - t_kh - dh, t \in \beta_3 \end{cases} \tag{8}$$

$\tau(t)$ satisfies $0 < \tau_m \le \tau(t) \le h + \tau_M$, $\dot{\tau}(t) = 1$. Define the error as

$$\partial_k(t) = \begin{cases} 0, t \in \beta_1 \\ y(t_kh + ih) - y(t_kh), t \in \beta_2, i = 1, 2, \ldots d-1 \\ y(t_kh + dh) - y(t_kh), t \in \beta_3 \end{cases} \tag{9}$$

(2) If $t_{k+1}h + \tau_{t_{k+1}} \le t_k h + h + \tau_M$, define a function:

$$\tau(t) = t - t_k h, t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{10}$$

In this case, the error can be defined as

$$\partial_k(t) = 0 \tag{11}$$

Combining (8) and (10), yields

$$\partial_k^T(t)\Omega\partial_k(t) \le \sigma y^T(t - \tau(t))\Omega y(t - \tau(t)), t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{12}$$

Form (8), (9) and (10) together with (11), the state space of the filter in the formula (5) can be represented as:

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f y(t - \tau(t)) - B_f \partial_k(t) \\ z_f(t) = C_f x_f(t) + D_f y(t - \tau(t)) - D_f \partial_k(t) \\ t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \end{cases} \tag{13}$$

Define $e(t) = z(t) - z_f(t)$, $\xi(t) = [x^T(t)\, x_f^T(t)]^T$, $v(t) = [\omega^T(t)\, \omega^T(t - \tau(t))]^T$, $\phi(t)$ is initial status of the resultant system. Combining (1) and (13), obtain the filtering error system model for NCSs based on event-triggered mechanism as following:

$$\begin{cases} \dot{\xi}(t) = \bar{A}\xi(t) + \bar{E}H\xi(t - \tau(t)) - \bar{B}_e\partial_k(t) + \bar{B}v(t) \\ t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \\ e(t) = \bar{C}\xi(t) + \bar{F}H\xi(t - \tau(t)) + D_f\partial_k(t) + \bar{D}v(t) \\ \xi(t) = \phi(t), t \in [t_0 - h_2, t_0 - h_1) \end{cases} \tag{14}$$

where $\bar{A} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}$, $\bar{E} = \begin{bmatrix} 0 \\ B_f C \end{bmatrix}$, $H = [I_n \quad 0]$, $\bar{B}_e = \begin{bmatrix} 0 \\ B_f \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B & 0 \\ 0 & B_f D \end{bmatrix}$, $\bar{C} = [L \quad -C_f]$, $\bar{F} = -D_f C$, $\bar{D} = [0 \quad -D_f D]$, $h_1 = \tau_m$, $h_2 = \tau_M + h$.

3 Event-Triggered H_∞ Filtering Performance Analysis

In this section, assuming that the filter parameters are known, an H_∞ filtering analysis criterion for the filtering error system (14) is established by applying Lyapunov-Krasovskii functional approach and LMI method. The stability criterion for the resultant filtering error system (14) is presented in Theorem 1.

Theorem 1. For given positive parameters A_f, B_f, C_f, D_f, Ω, h_1, h_2, γ and threshold parameter σ, under the event-triggered communication scheme (3), the closed-loop system (14) is asymptotically stable with an H_∞ performance index γ for the disturbance attention, if there exist matrixes $P = P^T > 0$, $\Omega > 0$, $Q_i = Q_i^T > 0$, $R_i = R_i^T > 0$ $(i = 1, 2)$, $S = S^T > 0$ with appropriate dimensions, such that

$$\begin{bmatrix} \Phi_{11} & * \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} < 0 \tag{15}$$

where

$$\Phi_{11} = \begin{bmatrix} \Upsilon_1 & * & * & * & * & * \\ R_1^T H & \Upsilon_2 & * & * & * & * \\ R_2^T H & 0 & \Upsilon_3 & * & * & * \\ \bar{E}^T P & S^T & S^T & \Upsilon_4 & * & * \\ \bar{B}_e^T P & 0 & 0 & 0 & -\Omega & * \\ \bar{B}^T P & 0 & 0 & \sigma[0 \quad D]^T \Omega C & 0 & \Upsilon_5 \end{bmatrix},$$

$$\Phi_{12}^T = \begin{bmatrix} h_1 R_1 H\bar{A} & 0 & 0 & h_1 R_1 H\bar{E} & -h_1 R_1 H\bar{B}_e & h_1 R_1 H\bar{B} \\ h_2 R_2 H\bar{A} & 0 & 0 & h_2 R_2 H\bar{E} & -h_2 R_2 H\bar{B}_e & h_2 R_1 H\bar{B} \\ \alpha SH\bar{A} & 0 & 0 & \alpha SH\bar{E} & -\alpha SH\bar{B}_e & \alpha SH\bar{B} \\ \bar{C} & 0 & 0 & \bar{F} & D_f & \bar{D} \end{bmatrix}$$

$\Upsilon_1 = P\bar{A} + \bar{A}^T P + H^T Q_1 H + H^T Q_2 H - H^T R_1 H - H^T R_2 H$, $\Upsilon_2 = -Q_1 - R_1 - S$,
$\Upsilon_3 = -Q_2 - R_2 - S$, $\Upsilon_4 = -2S + \sigma C^T \Omega C$, $\Upsilon_5 = \sigma_1 [0 \quad D]^T \Omega [0 \quad D] - \gamma^2 K^T K$,
$\Phi_{22} = diag\{-R_1, -R_2, -S, -I\}$, $K = [I_l \quad 0]$, $\alpha = h_2 - h_1$

The notation $X > 0$ denotes a real symmetric positive definite. Diag{} denotes the block-diagonal matrix. "*" is used as the ellipsis for terms induced by symmetry.

Proof. Select a Lyapunov-Krasovskii functional:

$$V(t, \xi(t)) = V_1(t, \xi(t)) + V_2(t, \xi(t)) + V_3(t, \xi(t)) \tag{16}$$

where

$$\begin{aligned} V_1(t, \xi(t)) &= \xi^T(t) P \xi(t) \\ V_2(t, \xi(t)) &= \int_{t-h_1}^{t} \xi^T(s) H^T Q_1 H \xi(s) ds + \int_{t-h_2}^{t} \xi^T(s) H^T Q_2 H \xi(s) ds \\ V_3(t, \xi(t)) &= h_1 \int_{-h_1}^{0} \int_{t+s}^{t} \dot{\xi}^T(v) H^T R_1 H \dot{\xi}(v) dv ds + h_2 \int_{-h_2}^{0} \int_{t+s}^{t} \dot{\xi}^T(v) H^T R_2 H \dot{\xi}(v) dv ds \\ &\quad + \alpha \int_{-h_2}^{-h_1} \int_{t+s}^{t} \dot{\xi}^T(v) H^T S H \dot{\xi}(v) dv ds \end{aligned}$$

Taking the derivative of $V(t, \xi(t))$ with respect to t along the trajectory of (14) obtains

$$\dot{V}(t, \xi(t)) = \dot{V}_1(t, \xi(t)) + \dot{V}_2(t, \xi(t)) + \dot{V}_3(t, \xi(t)) \tag{17}$$

where

$$\begin{aligned}
\dot{V}_1(t,\xi(t)) &= 2\xi^T(t)P\dot{\xi}(t)\\
\dot{V}_2(t,\xi(t)) &= \xi^T(t)H^TQ_1H\xi(t) - \xi^T(t-h_1)H^TQ_1H\xi(t-h_1)\\
&\quad + \xi^T(t)H^TQ_2H\xi(t) - \xi^T(t-h_2)H^TQ_2H\xi(t-h_2)\\
\dot{V}_3(t,\xi(t)) &= \dot{\xi}(t)[h_1^2H^TR_1H + h_2^2H^TR_2H + \alpha^2H^TSH]\dot{\xi}(t)\\
&\quad - h_1\int_{t-h_1}^{t}\dot{\xi}^T(s)H^TR_1H\dot{\xi}(s)ds - h_2\int_{t-h_2}^{t}\dot{\xi}^T(s)H^TR_2H\dot{\xi}(s)ds\\
&\quad - \alpha\int_{t-h_2}^{t-h_1}\dot{\xi}^T(s)H^TSH\dot{\xi}(s)ds + \partial_k^T(t)\Omega\partial_k(t) - \partial_k^T(t)\Omega\partial_k(t)
\end{aligned}$$

Applying Jensen's inequality to deal with the integral items in (17), to obtain:

$$\begin{aligned}
&-\alpha\int_{t-h_2}^{t-h_1}\dot{\xi}^T(s)H^TSH\dot{\xi}(s)ds\\
&= -\alpha\Big[\int_{t-\tau(t)}^{t-h_1}\dot{\xi}^T(s)H^TSH\dot{\xi}(s)ds + \int_{t-h_2}^{t-\tau(t)}\dot{\xi}^T(s)H^TSH\dot{\xi}(s)ds\Big]\\
&\leq -\left(\xi^T(t-h_1)H^T\ \xi^T(t-\tau(t))H^T\right)\begin{pmatrix} S & -S\\ -S & S\end{pmatrix}\begin{pmatrix} H\xi(t-h_1)\\ H\xi(t-\tau(t))\end{pmatrix}\\
&\quad -\left(\xi^T(t-\tau(t))H^T\ \xi^T(t-h_2)H^T\right)\begin{pmatrix} S & -S\\ -S & S\end{pmatrix}\begin{pmatrix} H\xi(t-\tau(t))\\ H\xi(t-h_2)\end{pmatrix}
\end{aligned} \tag{18}$$

$$\begin{aligned}
&-h_1\int_{t-h_1}^{t}\dot{\xi}^T(s)H^TR_1H\dot{\xi}(s)ds\\
&\leq -\begin{pmatrix}\xi^T(t) & \xi^T(t-h_1)H^T\end{pmatrix}\begin{pmatrix} H^TR_1H & -H^TR_1\\ -R_1H & R_1\end{pmatrix}\begin{pmatrix}\xi(t)\\ H\xi(t-h_1)\end{pmatrix}
\end{aligned} \tag{19}$$

$$\begin{aligned}
&-h_2\int_{t-h_2}^{t}\dot{\xi}^T(s)H^TR_2H\dot{\xi}(s)ds\\
&\leq -\begin{pmatrix}\xi^T(t) & \xi^T(t-h_2)H^T\end{pmatrix}\begin{pmatrix} H^TR_2H & -H^TR_2\\ -R_2H & R_2\end{pmatrix}\begin{pmatrix}\xi(t)\\ H\xi(t-h_2)\end{pmatrix}
\end{aligned} \tag{20}$$

Define

$$\eta^T(t) = [\xi^T(t)\ \xi^T(t-h_1)H^T\ \ \xi^T(t-\tau(t))H^T\ \ \xi^T(t-h_2)H^T\ \partial_k(t)\ v^T(t)] \tag{21}$$

Combining (12) and (16–21), by Schur complements, then we can get:

$$\dot{V}(t) \leq \eta^T(t)\Phi\eta(t) - e^T(t)e(t) + \gamma^2\omega^T(t)\omega(t) \tag{22}$$

where $\omega(t) = Kv(t)$, $\Phi = \Phi_{11} - \Phi_{12}\Phi_{22}^{-1}\Phi_{12}^T$, Φ_{11}, Φ_{12}^T and Φ_{22} have been defined in Theorem 1. By Schur complements and using the Lyapunov-Krasovskii functional (16) guarantee that $\dot{V}(t,\xi(t)) < 0$ in (17); derive the closed-loop system (14) with $\omega(t) \equiv 0$ is asymptotically stable and $\|e(t)\|_2 \leq \gamma\|\omega(t)\|_2$ under the zero initial condition.

4 Co-design of Event-Triggered Communication and H_∞ Filter

In this section, for closed-loop system (5), an approach to the co-design of the reasonable filter parameters and the threshold parameter Ω under the event condition is proposed based on the Theorem 1.

Theorem 2. For given positive parameters h_1, h_2, γ and threshold parameter σ, under the event-trigged communication scheme (3), the closed-loop system (14) is asymptotically stable with an H_∞ performance index γ for the disturbance attention, if there exist matrices $P_1 > 0$, $\Omega > 0$, $Q_i = Q_i^T > 0$, $R_i = R_i^T > 0\,(i = 1, 2)$, $S = S^T > 0$, $M > 0$, $\bar{A}_f$, $\bar{B}_f$, $\bar{C}_f$, $\bar{D}_f$ with appropriate dimensions, such that $P_1 - M > 0$ and the following matrix inequity holds

$$\begin{bmatrix} \Phi'_{11} & * \\ \Phi'^{\mathrm{T}}_{12} & \Phi'_{22} \end{bmatrix} < 0 \tag{23}$$

Parameters of the filter can be designed as:

$$A_f = M^{-1}\bar{A}_f,\ B_f = M^{-1}\bar{B}_f,\ C_f = \bar{C}_f,\ D_f = \bar{D}_f$$

or

$$A_f = \bar{A}_f M^{-1},\ B_f = \bar{B}_f,\ C_f = \bar{C}_f M^{-1},\ D_f = \bar{D}_f$$

where

$$\Phi'^{T}_{12} = \begin{bmatrix} h_1R_1A & 0 & 0 & 0 & 0 & 0 & h_1R_1B & 0 \\ h_2R_2A & 0 & 0 & 0 & 0 & 0 & h_2R_2B & 0 \\ \alpha SA & 0 & 0 & 0 & 0 & 0 & \alpha SB & 0 \\ L & -\bar{C}_f & 0 & 0 & -\bar{D}_fC & \bar{D}_f & 0 & -\bar{D}_fD \end{bmatrix}$$

$$\Phi'_{11} = \begin{bmatrix} \bar{\Upsilon}_1 & * & * & * & * & * & * & * \\ MA + \bar{A}_f^T & \bar{A}_f + \bar{A}_f^T & * & * & * & * & * & * \\ R_1^T & 0 & \Upsilon_2 & * & * & * & * & * \\ R_2^T & 0 & 0 & \Upsilon_3 & * & * & * & * \\ C^T\bar{B}_f^T & C^T\bar{B}_f^T & S^T & S^T & \Upsilon_4 & * & * & * \\ \bar{B}_f^T & \bar{B}_f^T & 0 & 0 & 0 & -\Omega & * & * \\ B^TP_1 & B^TM & 0 & 0 & 0 & 0 & -\gamma^2 I_l & * \\ D^T\bar{B}_f^T & D^T\bar{B}_f^T & 0 & 0 & \sigma D^T\Omega C & 0 & 0 & \sigma D^T\Omega D \end{bmatrix}$$

$\Phi'_{22} = diag\{-R_1 \quad -R_2 \quad -S \quad -I\}$, $\bar{\Upsilon}_1 = P_1A + A^TP_1 + Q_1 + Q_2 - R_1 - R_2$

Proof. Define $J_1 = diag\{I \quad P_2P_3^{-1}\}$, $J_2 = diag\{J_1 \quad I \quad I \quad I \quad I \quad I \quad I\}$, pre and post multiplying both sides of (15) with J and its transpose respectively. Here we define $M = P_2P_3^{-1}P_2^T$, $\bar{A}_f = P_2A_fP_3^{-1}P_2^T$, $\bar{B}_f = P_2B_f$, $\bar{C}_f = C_fP_3^{-1}P_2^T$, $\bar{D}_f = D_f$. It is easy to get (23) from (15). Consequently, system (14) is asymptotically stable and with an H_∞ performance index γ for the disturbance attention by Theorem 2.

$P = [P_{ij}]_{2\times2} > 0$ is equivalent to $P_1 - P_2P_3^{-1}P_2^T = P_1 - M > 0$ by applying Schur complement. According to the definition of $\bar{A}_f$, $\bar{B}_f$, $\bar{C}_f$, $\bar{D}_f$, we can obtain:

$$\begin{bmatrix} A_f & B_f \\ C_f & D_f \end{bmatrix} = \begin{bmatrix} P_2^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{A}_f & \bar{B}_f \\ \bar{C}_f & \bar{D}_f \end{bmatrix} \begin{bmatrix} P_2^{-T}P_3 & 0 \\ 0 & I \end{bmatrix} \tag{24}$$

This completes the proof.

5 An Illustrative Example

In this section, a simulation example is given to show the effectiveness of the proposed filter design method developed in this paper. The simulation platform adopts MATLAB, and the related parameters can be solved through the LMI toolbox.

Consider a quarter-car model depicted in Fig. 2. The state space expression and parameters of the quarter-car model in this example are the same as [11]. The purpose here is to identify a filter to estimate $\dot{z}_s(t)$.

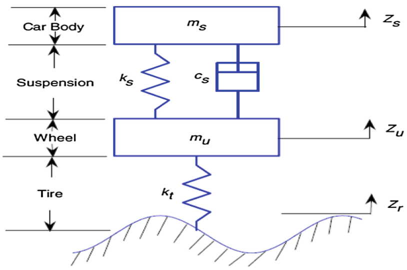

Fig. 2. The structure of active suspension system

Under the given performance index $\gamma = 0.2$, select the sampling period $h = 0.1$, $h_1 = 0.01$, $h_2 = 0.11$ and the threshold $\sigma = 0, 0.1, 0.2, 0.3, 0.4, 0.5$, respectively [4]. Applying Theorem 2 to get the different average transmission period $\bar{h}$ and the proportion of sensor sent data χ as shown in Table 1.

Table 1. Results of different σ_1 with given $\gamma = 0.2$

σ	0	0.1	0.2	0.3	0.4	0.5
Ω	53554	58.4531	32.7579	23.2908	17.7079	13.8569
$\bar{h}$	0.1	0.1406	0.1644	0.1803	0.2021	0.2256
χ	100%	69%	59%	53%	48%	43%

As shown in the Table 1, the event-triggered parameter Ω becomes smaller, the average transmission period h and the proportion of sensor sent data χ are larger when the threshold σ increases within certain range. Not all sampled signals are sent through a communication channel, so the event-triggering allows an effective reduction of the network burden and takes full advantage of the limited resources.

Then choose the parameters $\sigma = 0.2$, $\gamma = 0.9$, $h_1 = 0.01$, $h_2 = 0.11$ and obtain the event-triggered matrices and the filter parameter matrices by Theorem 2 as following:

$$A_f = \begin{bmatrix} 0.0 & 0.0 & -0.9 & 0.5 \\ 136.9 & -33.1 & -624.9 & 3391.1 \\ 44.0 & 0 & 2.8 & -1.6 \\ -368.5 & 887.0 & -57.7 & 196.3 \end{bmatrix}, \; B_f = \begin{bmatrix} 0.5 & -3213.8 & -1.4 & -161.0 \end{bmatrix}^T$$

$$C_f = \begin{bmatrix} -0.0003 & -0.0001 & 0.0017 & 0.0012 \end{bmatrix}, \; D_f = 0.1538, \; \Omega = 36.8407$$

Suppose there is an external disturbance input, choose $b = 0.2$, $d = 5m$, $h = 0.1s$.

$$\omega(t) = \begin{cases} \frac{b\pi v}{d} \sin(\frac{2\pi v}{d} t), & \text{if } 0 \leq t < 10 \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

The corresponding system estimation signal $z(t)$ and $zf(t)$ is clearly shown in Fig. 4. The time at which the estimation signal reaches equilibrium is less than in [12]. The system based on event-triggered method in this paper has good dynamic characteristics. A conclusion can be drawn that the H_∞ filter which has been designed is able to produce a good estimation on the signal $z(t)$ from Fig. 4.

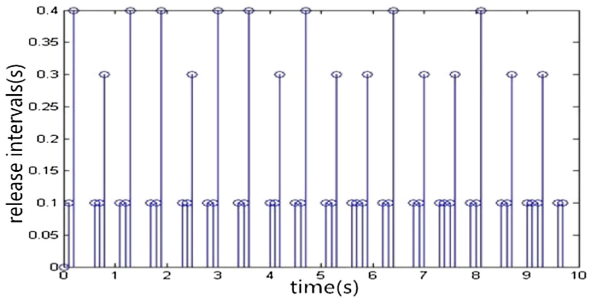

Fig. 3. The release instants and release intervals of sampling signals

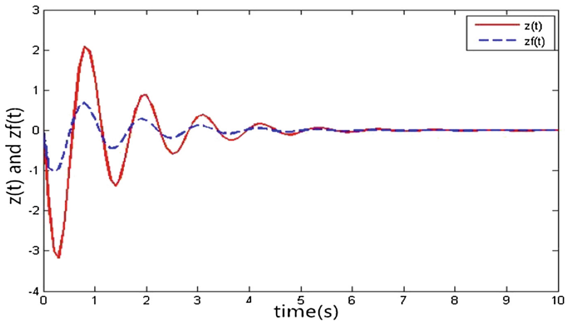

Fig. 4. The estimated signal $z(t)$ and $z_f(t)$

6 Conclusion Remarks

In this paper, the problem of event-triggered H_∞ filtering for NCSs has been studied. To save limited network resources, an event-triggered mechanism implemented by software is proposed through which the transmission data quantity can be reduced. Considering the influence of the time-varying transmission delay and the novel scheduling scheme, the filtering error system has been converted into an interval time-delay system. Based on the new model, the asymptotic stability of the NCSs filtering is studied by using Lyapunov stability theory and LMI technique. Since the proposed stability criterion, a cooperative design method of event-triggered mechanism and H_∞ filtering is obtained. However, there are other important research issues that need further study as well, such as the uncertainties or packet loss of the system (Fig. 3).

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