# Hybrid Fx-NLMS Algorithm for Active Vibration Control of Flexible Beam with Piezoelectric Stack Actuator

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**Abstract.** Filtered-x Least Mean Square (FxLMS) algorithm is a meaningful adaptation algorithm used in the field of Active Vibration Control (AVC). Hybrid FxLMS algorithm, which is the combination of the feedforward structure and the feedback structure of FxLMS, has a better stability and could get the same performance with a lower filter order. In order to get a faster convergence speed, this paper adopts Normalized LMS (NLMS) algorithm to replace of LMS algorithm in the hybrid AVC system. To verify the Hybrid Fx-NLMS algorithm, this paper developed a simulation platform for active vibration control of a flexible beam with piezoelectric stack actuator using ADAMS and MATLAB SIMULINK. Simulation results show that the convergence speed and vibration suppression performance of the Hybrid Fx-NLMS algorithm are better than other traditional algorithms.

**Keywords:** Active Vibration Control · Fx-LMS algorithm · Hybrid Fx-NLMS algorithm · Flexible beam · Convergence analysis

## 1 Introduction

Active vibration control has received a great deal of attention due to its effectiveness at low frequencies and alternative passive absorption materials are relatively more expensive to deploy [1]. A AVC system is mainly classified into two categories: feed-forward AVC and feedback AVC. If combined the feed-forward with feedback control structure, we get a hybrid AVC method [2]. The hybrid AVC method can use a lower order filter to achieve the same performance [3]. The hybrid method is also clearly demonstrated an advantage over either simple feedforward AVC or feedback AVC method alone when there is significant plant noise.

In an AVC system, the secondary path is vital to the convergence of whole system and the control effect [4]. The FxLMS algorithm eliminates the effect of secondary path

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and then becomes one of the most commonly used algorithms in AVC also for its robust, low computational complexity, and easy to implement [5]. Although FxLMS algorithm is widely used but it has a slower convergence [6]. Numerous research works have been carried out on this problem. Various improvements were made to offset the defects of the FxLMS algorithm, such as the Filtered-x Normalized LMS (Fx-NLMS) [7], the Modified Filtered-x LMS (MFXLMS) [8], the Leaky FxLMS [9], etc.

In this paper, the hybrid filtered-x normalized LMS (Fx-NLMS) is applied to control the flexible beam with piezoelectric stack actuator. The hybrid FxLMS is presented in Sect. 2. The Sect. 3 briefs about the hybrid Fx-NLMS algorithm. The Sect. 4 shows the flexible beam module in ADAMS. The Sects. 5 and 6 give the simulation and conclusion.

## 2 Hybrid FxLMS Algorithm

In applications where adaptation is needed, the LMS algorithm is probably the most frequently used algorithm. The LMS algorithm, FxLMS algorithm, feedback FxLMS algorithm and hybrid FxLMS algorithm are briefly introduced as follow.

### 2.1 LMS Algorithm

As shown in Fig. 1, the adaptive filter **W** is fed with the input sequence $x(n)$. $y'(n)$ is the output of the filter $y(n)$ through the secondary path. The noise signal $d(n)$ compared with $y'(n)$, $e(n)$ is get. The goal is to adjusts the filter to minimize $e(n)$.

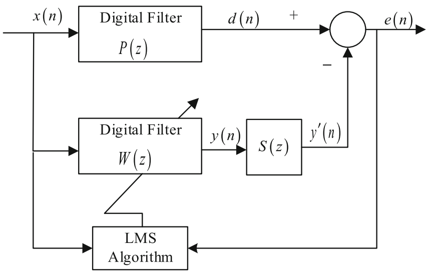


**Fig. 1.** The block diagram of LMS in feed-forward AVC system

From Fig. 1, $e(n)$ can be described as

$$e(n) = d(n) - S(n) * \left\{ \mathbf{W}^T(n)\mathbf{x}(n) \right\} \tag{1}$$

Where, $n$ is time index, and $S(n)$ is impulse response of secondary path $S(z)$. And the filter coefficients expressed as

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu x(n)e(n) \tag{2}$$

### 2.2 Feed-Forward FxLMS Algorithm

The block diagram for a single-channel feed-forward AVC system using the FxLMS algorithm is shown in Fig. 2 [1].

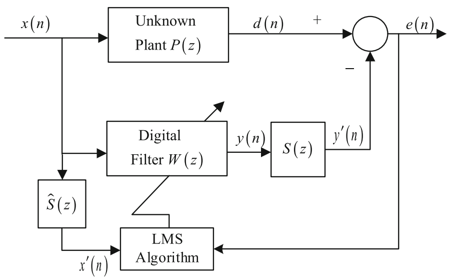


**Fig. 2.** The block diagram of feed-forward FxLMS

The unknown plant is represented by $P(z)$ that the vibration propagates. The vibration signal is represented by $d(n)$. The controller cancel this vibration signal through a signal $y'(n)$ that has a same amplitude but opposite phase to $d(n)$. After the $y'(n)$ mixed with $d(n)$, we get the error signal $e(n)$. Through updating the coefficients of filter, $e(n)$ is subsequently minimized.

The error signal can be expressed with Eq. (1), the same like LMS algorithm. Specifically, the weight updating equation is given by Eq. (3).

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu x'(n)e(n) \tag{3}$$

### 2.3 Feedback FxLMS Algorithm

In real system, the reference signal $x(n)$ cannot be obtained directly sometimes. Then, the feedback FxLMS algorithm was proposed [10]. A block diagram for a single-channel feedback AVC system is presented in Fig. 3.

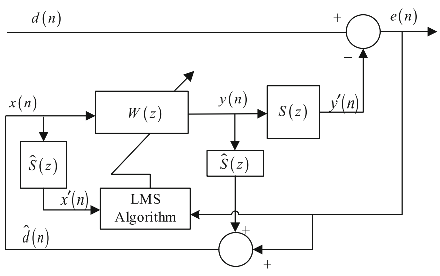


**Fig. 3.** The block diagram of feedback FxLMS

Compared to FxLMS algorithm, the obvious distinguish is to estimate the $x(n)$. From the diagram, it can be figured out that the estimated primary vibration signal $\widehat{d}(n)$ is generated by adding the output of filter $y(n)$ and the error signal $e(n)$. It is shown as:

$$\widehat{d}(n) = e(n) + y'(n) = e(n) + S(n) * \left\{\mathbf{W}^T(n)\mathbf{x}(n)\right\} \tag{4}$$

$$x(n) \equiv \widehat{d}(n) \tag{5}$$

Except $x(n)$, the other parameters of feedback FxLMS algorithm are the same as that in FxLMS algorithm. The coefficients of the filter updating also as Eq. (3).

## 2.4 Hybrid FxLMS Algorithm

In feed-forward AVC system, to function efficiently, the primary signal must be highly correlated with the reference signal. However, it is difficult in complex practical application. And in feedback AVC systems, the cancellation effect only efficient in narrow band, and may be unstable on some frequencies. A coordination of the FxLMS and feedback FxLMS algorithm is called the hybrid FxLMS, as in Fig. 4 [11].

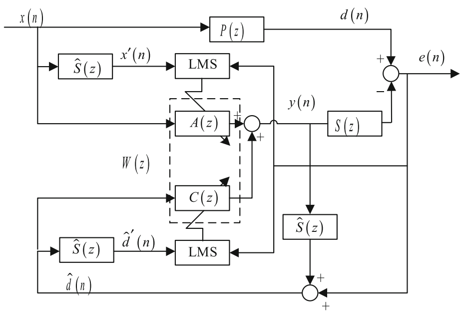


**Fig. 4.** The block diagram of hybrid FxLMS

In the hybrid FxLMS algorithm, there are two sensors to sample the corresponding signals, one is the reference sensor for FxLMS; the other one is error sensor to provide the error for synthesizing the reference signal. At the same time, the error signal is used for both the FxLMS algorithm and feedback FxLMS algorithm.

From Fig. 4, it can be figured out that the adaptive filter of FxLMS and feedback FxLMS were assumed as $A(n)$ and $C(n)$. And then, it is obvious that the control output is the sum of the output of $A(n)$ and $C(n)$. The length of $A(n)$ and $C(n)$ can be different. It is shown as

$$y(n) = \mathbf{A}^T(n)\mathbf{x}(n) + \mathbf{C}^T(n)\widehat{\mathbf{d}}(n) \tag{6}$$

And the error signal is:

$$e(n) = d(n) - S(n) * y(n) = d(n) - S(n) * \left\{ \mathbf{A}^T(n)\mathbf{x}(n) + \mathbf{C}^T(n)\widehat{\mathbf{d}}(n) \right\} \tag{7}$$

The hybrid FxLMS algorithm can get a lower order filter in the same performance with FxLMS algorithm or feedback FxLMS algorithm.

## 3 Hybrid Fx-NLMS Algorithm

In the process of convergence, a great step size may lead a fast convergence. But it may be divergent if the step-size is too large. Many variable step-size (VSS) LMS algorithms were proposed.

### 3.1 Normalized LMS Algorithm

The purpose of VSS-LMS is to adjust the step-size. In 1967, Nagumo proposed the Normalized LMS (NLMS) algorithm, which is considered as the first VSS modify [7].

The step-size of NLMS algorithm is shown as:

$$\mu(n) = \frac{\overline{\mu}}{\beta + R} = \frac{\overline{\mu}}{\beta + \mathbf{x}(n)^T \mathbf{x}(n)} \tag{8}$$

Where $\overline{\mu} > 0$ is a scalar, the maximum of $\overline{\mu}$ in some case is 2 [12]. The value of $\beta$ is made quite small as possible.

### 3.2 Hybrid Fx-NLMS Algorithm

In the hybrid AVC system, replacing the LMS algorithm with the NLMS algorithm, it is obtained a hybrid Fx-NLMS algorithm. This algorithm update the step-size by

$$A(n+1) = A(n) + \left( \frac{\overline{\mu}}{\beta + \mathbf{x}^T \mathbf{x}} \right) x'(n)e(n) \tag{9}$$

$$C(n+1) = C(n) + \left( \frac{\overline{\mu}}{\beta + \widehat{\mathbf{d}}^T \widehat{\mathbf{d}}} \right) \widehat{d}'(n)e(n) \tag{10}$$

Where $A(n)$ is the filter coefficients of the feedforward Fx-NLMS algorithm, and $C(n)$ is the filter coefficients of the feedback Fx-NLMS algorithm.

The hybrid Fx-NLMS algorithm can get a greater step-size $\mu$ when the error $e(n)$ is great. Then, the convergence of this algorithm is faster than the traditional one.

# 4 Module of the Flexible Beam

To verify the effectiveness of the above algorithms, a flexible beam with piezoelectric stack actuator is built in ADAMS as a test bed. As shown in Fig. 5, the length, width and thickness of the flexible beam is $L$, $W$ and $T$. Specifically, $L$ is 1500 mm, $W$ is 60 mm, and $T$ is 15 mm. The piezoelectric stack actuator installed at the bottom of flexible beam. $X_1$ is the distance between the fixed ends of the flexible and the piezoelectric stack actuator. The disturbance signal is a force which exert at the $X_2$ position from the top of flexible beam. The $X_1$ is 450 mm, and the $X_2$ is 1050 mm. Also, the reference sensor fixed at the $X_2$ position in the bottom of flexible beam, and the error sensor fixed at the barycenter of flexible beam.

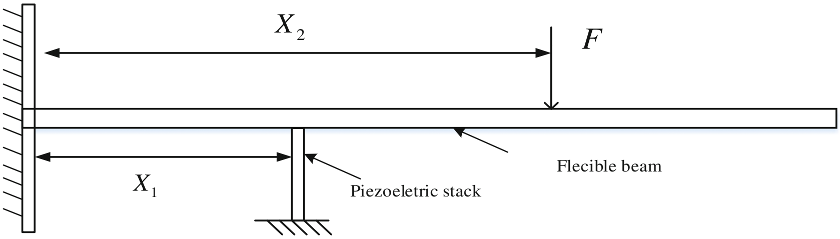


**Fig. 5.** Module of flexible beam with piezoelectric stack actuator

# 5 Simulation

To verify the effectiveness of the above algorithms, the flexible beam with piezoelectric stack actuator is setup as a test bed. In this section, both initial states of the flexible beam and the designed parameters of the algorithms are detailed for the realization. In what fellow, performance comparing between the six kind algorithms are designed to demonstrate the performances of the control schemes.

In the simulation of this paper, the disturbance force is a sinusoidal signal whose frequency is 99 Hz, and amplitude is 0.1 N. Figure 6 is the vibration observed at the beam without active control.

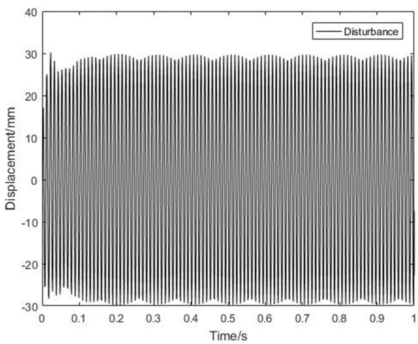


**Fig. 6.** Characteristics of disturbance signal

Figure 7(a) show the comparison of error signal with the control of FxLMS algorithm and Fx-NLMS algorithm; Fig. 7(b) show the comparison of error signal with the control of Feedback FxLMS algorithm and Feedback Fx-NLMS algorithm; Fig. 7 (c) show that the comparison of error signal with the control of Hybrid FxLMS and Hybrid Fx-NLMS. From the characteristics, it could be observed that the error signals of vibration by using NLMS algorithm have a better convergence speed than the traditional one and also have a satisfied steady-state error.

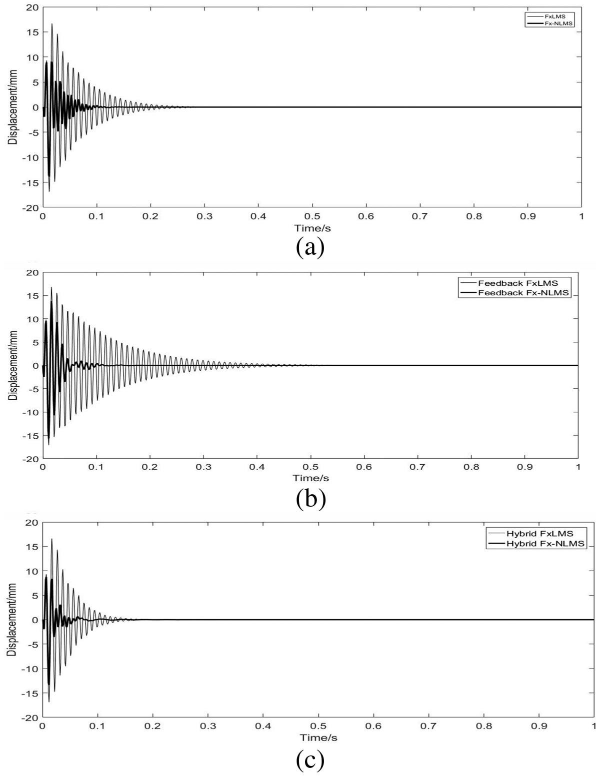


**Fig. 7.** (a) Characteristics of error signal comparing FxLMS with Fx-NLMS. (b) Characteristics of error signal comparing Feedback FxLMS with Feedback Fx-NLMS. (c) Characteristics of error signal comparing Hybrid FxLMS with Hybrid Fx-NLMS

To better analysis the effect of these above algorithms applying to the flexible beam with piezoelectric stack actuator, the error signals of vibration by using FxLMS algorithm, feedback FxLMS algorithm, hybrid FxLMS algorithm, Fx-NLMS algorithm, feedback Fx-NLMS algorithm and hybrid Fx-NLMS algorithm are shown in Fig. 8(a). For the comparison, original disturbance signal and error signals of vibration by using FxLMS is shown in Fig. 8(b).

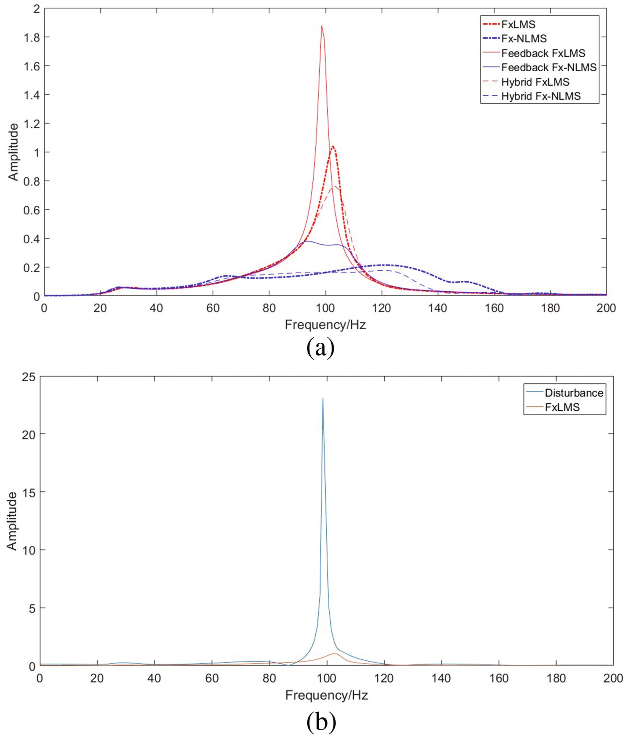


**Fig. 8.** (a) Characteristics of error signal with active control in frequency domain. (b) Characteristics of original disturbance signal and error signal with FxLMS algorithm in frequency domain

From Fig. 8(b), it could be observed that the effectiveness of the FxLMS is obvious. And in Fig. 8(a), it can be analysis based on the FxLMS. First, all the six adaptive algorithms are effective in the active vibration control of the simulation module of flexible beam in this paper. Second, the control performance of the above six algorithms are compared. Except feedback FxLMS algorithm, the amplitude of error signal with other four algorithms are all smaller than FxLMS'. And the amplitude of Fx-NLMS algorithm, feedback Fx-NLMS algorithm and hybrid Fx-NLMS algorithm are obvious smaller than FxLMS algorithm, feedback FxLMS algorithm and hybrid FxLMS algorithm as the normalized step size.

## 6 Conclusion

In this paper, the hybrid Fx-NLMS algorithm is adopted for vibration control of the flexible beam with piezoelectric stack actuator. The control effect between FxLMS algorithm, feedback FxLMS algorithm, hybrid FxLMS algorithm, Fx-NLMS algorithm, feedback Fx-NLMS algorithm and hybrid Fx-NLMS algorithm are compared and analyzed in terms of time and frequency domain.

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