Teaching and Learning for Numeracy Competence

Maura Sellars

Introduction

The mathematics that has been the focus of education in schooling, and which has been referred to previously as 'institutionalised' mathematics, is the Mathematics of abstract thinking, symbolic representation and formal logical thinking. It is frequently identified in texts, policies and other documents as Mathematics with a capital M. The pure and applied forms of this learning domain represent huge social, cultural and economic capital in globalised countries. The challenge for teachers of younger students is how best to adequately prepare young learners, despite the increasing degree of diversity in Australian classrooms, to develop their understanding of the foundational concepts of this domain in practical ways, which, in turn, facilitates access to the most advanced, complex areas of mathematical thinking and problem solving. As always, it is important to understand the developmental stages of the students, and to respect the prior knowledge and experiences of mathematics and numeracy that they bring to their classrooms (Bruner, 1997; Cole & Wertsch, 1996; Duncan, 1995).

Mathematics

Pure Mathematics is a discipline area. As a formal area of study, it has rules, standardised procedures, correct and incorrect procedures and strategies which can be used to obtain correct and incorrect answers. Whilst the contexts in which mathematical activities might be undertaken can be very diverse, social and cultural diversity is woven together across contextual differences by some commonalities in

M. Sellars (✉)
University of Newcastle, Callaghan, NSW, Australia
e-mail: Maura.Sellars@newcastle.edu.au

M. Sellars (ed.), *Numeracy in Authentic Contexts*,
https://doi.org/10.1007/978-981-10-5736-6_2

practice (Bishop, 1998). However, this does not imply that numeracy and mathematics are not able to be practically and conceptually linked. One way in which this can be achieved is to provide supportive teaching and learning environments which promote acceptance of different ideas and strategies and collaborative problem solving, in which all students have opportunities to actively engage in explanation and reasoning around solving real problems using diverse perspectives, without the imposition of any particular viewpoint being 'more correct' or more acceptable than others (Burgh, Field, & Freakley, 2006). The teacher in these contexts interacts as a mentor, guiding rather than dominating the discussion (Dyson, 2004; Mockler, 2011; Pilling-Cormick, 1997; Prestridge & Watson, 2004). The exact nature of the interactions and learning tasks facilitated by teachers in and out of the classroom can determine the students' capabilities to see relevance and value in their mathematical learning and also establish students' competencies in linking formal mathematics to their everyday numeracy needs and interests.

Macmillan (2009) discusses categories of social contexts within which mathematical thinking and reasoning are important. Using Bishop's (1988) six mathematical actions, Macmillan opines that any type of social context can be analysed in terms of these six universal actions. These actions were labelled as 'universal' because Bishop found, in his study of communities interacting in diverse contexts, that these actions were undertaken in all cultures and social conventions. A more detailed summary of these universal mathematical actions, and the categories of social contexts which control or influence the ways in which mathematical actions are effected, is presented as Table 1.

The information contained in Table 1 not only demonstrates the relationship that mathematic and numeracy having in mutually informing each other, but it also illustrates very clearly how teachers can make learning in mathematics more enjoyable and purposeful for students. It explicitly clarifies how formal, mathematical learning can contribute meaning to everyday actions and contexts, irrespective of the age or stage of students, or of their diversity of social experiences, customs and backgrounds (Deed, Pridham, Prain, & Graham, 2012). The initial findings of a study by Beswick, Watson, and Brown (2006), which focussed on students in middle school mathematics classrooms, endorses the need for students to understand and appreciate mathematics activities not only as relevant but as personally purposeful. Beswick et al. (2006) found that, in traditional classrooms, although mathematics were considered to be important, students struggled with the complexity of the conceptual knowledge and found it difficult to identify the learning as personally purposeful.

What does this mean for you as a teacher of numeracy?

- Students make meaning and see relevance in their mathematical learning when it is associated with social contexts with which they are familiar, so new learning is more easily supported when it is contextualised.
- Every category of social context has embedded within it components that require mathematical understanding or actions, so knowing about

Table 1 A socio cultural perspective of mathematical activity (Bishop, 1998; Macmillan, 2009 p. 21)

Universal mathematical activities	Universal social mathematical contexts
Counting: determining quantities, ordering objects, distinguishing one from another in concrete or abstract terms. May also apply to frequency, events or episodes	**Political: Examples:** examining and interpreting data or statistics for political purposes. For example: election results, constituent representation, data relating to political objectives
Measuring: using formal and informal measures to determine quantities or conceptual mathematical notions that cannot be counted	**Economic: Examples**: contexts that involve mathematical skills used to determine trade deals, budget buys, budgets, monetary transactions, savings and spending
Locating: determining place and position in spatial terms either in relation to oneself, other individuals or specified objects	**Physical: Examples**: interpreting data relating to physical phenomena such as information about tides, rainfall, temperatures, fire and storm warnings in terms of social and personal safety and comfort
Designing: employing higher order thinking skills to conceptualise a plan or strategy which is abstract and symbolic and record it by various means	**Scientific: Examples**: understanding the impact of scientific data relating to nutrition, exercise, medicines, monitoring dietary requirements, intake balance and medical information relating to scientific information on personally relevant issues
Playing: the capacities to recreate or imitate social and cultural actions that have content which requires mathematical exploration. These can be imaginative, supported with concrete materials and conducted within social interaction with others. May be non-goal orientated	**Social: Examples**: determining the protocol of seating at social events or simply deciding where to sit, positioning team members in social sports in order to make the most impact for the team. Cultural issues of personal space, bodily contact and proximity
Explaining: using the symbolic, shared understandings of language to verbally explain, justify, evaluate and communicate the facts, examine the logic or conceptual understandings of mathematical ideas, experiences, events, relationships and questions	**Emotional: Examples**: deciding personal choices, choosing a team to support or someone to dance with, standards of personal dress and adornment, selection of goods and services for personal use

students' lives and experiences, which initially appears to have little or no mathematical content, facilitates a different perspective and allows teachers to deconstruct social events and teach the embedded mathematical concepts, knowledge and strategies, in order to formalise the mathematical learning.

- Students can attribute value and develop interest in mathematics that are useful in their everyday lives, so students' interests and experiences are a constant resource for teachers.
- Developing an identity as an individual who is numerate is not necessarily (or often) a linear process in the ways in which syllabus and curriculum documents are delivered.

- The importance of tracking students' mathematical progress and their capacities to use their skills and concepts to make meaning in social contexts cannot be overstated. It not only provides information about what students are currently thinking and internalising, it also gives teachers some clear information about the next stage of learning.
- Conferencing, as a one-to-one interaction with individual students about their thinking and learning, is vital because it provides teachers with rich authentic knowledge of each student in ways that exams, tests, worksheets and other pen and paper activities cannot.
- Class discussions in which participants feel safe to contribute and can engage meaningfully are important aspects of mathematical learning, as they can be mutually supportive. Other social activities such as paired tasks, group problem solving and collaborative learning assignments give students opportunities to develop their skills in adaptive reasoning, which is an area of mathematics learning which is reported to be rather neglected in Australian classrooms.
- Matching the six universal mathematical actions with the curriculum not only offers the prospect of engaging with the syllabus requirements in a novel way, it also provides a framework from which teachers can work in planning for mathematical activities and numeracy experiences.
- Pedagogical approaches are important.
- *For example, there are many lessons throughout Section Two of this work that allow students to learn within the contexts of their interests as the lessons investigate the numeracy embedded in all the areas of the primary curriculum. A popular example which is both useful in terms of students' learning for their everyday lives and for their practical use of mathematical skills and thinking is the Media Year one lesson which investigates growth. The technology used to capture the life cycle of the plant is within reach of the young learners and the pedagogy, which includes the initial organisation of the onions, also facilitates rich discussion in mathematical comparative language and mathematical thinking, as the students can see what is usually hidden under the soil in a regular garden bed.*

Supporting the Development of Personal Mathematical Capacities

If students are to reach their full potential and learn to manage both the ethnological practice of mathematics in their personal lives and in formal institution mathematics, then the teaching and learning strategies employed in identifying numeracy practices and making significant links to formal mathematics must be robust, equitable and inclusive. There are, as in other disciplines, a number of perspectives regarding the early teaching of formal mathematics, and these and others are evolving in diverse Australian classrooms (Artigue, 2010; Bobis, 2013; Bobis et al., 2005; Bobis & Mulligan, 2010; Booker, 2011; Buschman, 2001; Clarke, 2001; Clarke & Clark, 2004; Hennessey, Higley, & Chesnut, 2011; Mukhopadhyay & Greer, 2010; Mulligan, 2010; Schoenfeld, 2002; Stacey, 2010; Suh, 2007; Watson, 2008). For this writing however, it is sufficient to note that mathematics in the primary, and indeed the early, years of secondary schooling needs to be based on the symbolic representation of practical mathematics and their use as numeracy competencies (Sullivan, 2011a, 2011b). The considerations that need to be made are mainly in terms of developmental capacities (Bobis, 2013; Bobis et al., 2005; Sullivan, 2011a, 2011b) and presenting mathematics in ways that allow students to understand, 'read' and use symbolic representations as a literacy that is deeply embedded in patterns and relationships, one which can be utilised to record and explain everyday events (Siemon et al., 2013). An important aspect of teaching mathematics, therefore, is to develop and explore numbers and other mathematical constructs in relation to each other and in relation to how they inform everyday life and the other subject areas in the school curriculum (Baker, Goesling, & Letendre, 2002; Baker, Street, & Tomlin, 2003; Ferme, 2014; Fox & Surtees, 2010).

The model developed and presented below has been designed specifically to highlight components of teaching and learning in numeracy that are important considerations but are frequently neglected when discussing the ways in which numeracy capacities can be supported and strengthened. The very heart of the integrated framework reflects the personal nature of numeracy that is cultivated in the specific, particular social and cultural circumstances and situations which each individual inhabits, whilst the entire paradigm is enclosed in the wider context of the civilisation and culture to which individuals belong. In addition to the work of Baker, Street and Tomlin (2006) the work of other theorists has been incorporated to illustrate some of the relationships and tensions between mathematics and numeracy. These tensions include the ways in which mathematics proficiencies can be developed to support activities in numeracy (Sullivan, 2011a, b) and the principal actions that teachers must incorporate into their lessons to support the application of mathematical notions and strategies in mathematical activities becoming accessible and useful in numeracy (Watson, 2011). It also includes the Four Resource Critical Numeracy Model (Watson, 2009), based on (Freebody & Luke, 1990, 2003) four roles of the reader. The parallel model developed from this is designed to be used to encourage students to make meaning of learning in mathematics in ways that support numeracy across the wide range of curriculum subject areas and in relation to students' own ethical decision-making (Fig. 1).

Five Strands of Mathematical Actions

Sullivan's (2011a, b) five strands of mathematical actions are based on the work of Kilpatrick and associates in 2001 (National Research Council (U.S.) Mathematics Learning Study Committee, 2001) and the subsequent refinement of these by

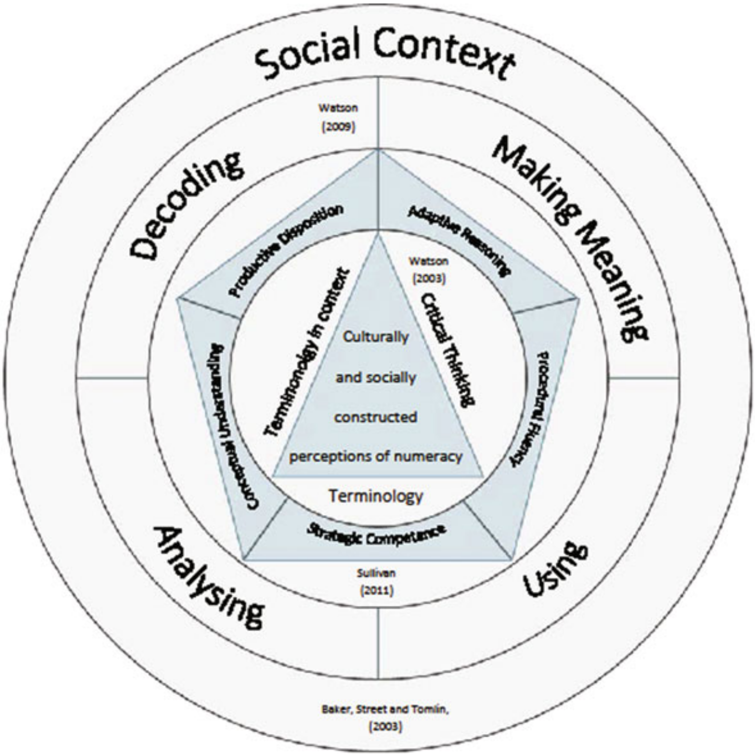

Fig. 1 A Framework for supporting personal numeracy

Watson and Sullivan in 2008. They describe the types of actions that support effective student learning in mathematics. Conceptual understanding, the first of the mathematical strands, highlights the need for students to be given the opportunities to understand not only what they are required to do in mathematics and *how* to do the tasks, but also to be able comprehend the mathematical concepts that are being engaged with, the structure and purpose of the operations being utilised and the relationships that are simultaneously being investigated, explored and discovered. The foundational theories that underpin this strand of mathematical action are the cognitive theories that support the development of '*robus*t' knowledge in the learners' neural networks. This is the type of well-understood knowledge that is connected to other learning in the brain's learning networks and, because of these strong neural links, is able to be accessed and to be 'transferred' into new learning contexts and experiences (Blakemore & Frith, 2005; Suarez-Orozco & Sattin-Bajaj, 2010). Knowledge that is learned and remembered without full understanding is considered to be *'inert'* knowledge because it is not linked meaningfully to other knowledge and is not able to be transferred easily into new learning contexts. This type of knowledge is primarily used in the same ways and in learning tasks and contexts that are overtly similar to the tasks in which it is first learned.

The second of these strands is identified as procedural (National Research Council (U.S.) Mathematics Learning Study Committee, 2001), or mathematical fluency (Watson & Sullivan, 2008). The term that is preferred by Watson and Sullivan is probably most useful, as it refers not only to the capacity to carry out mathematical procedures accurately, efficiently and correctly, but also to the capacity to recall factual knowledge and concepts readily as they are required to complete mathematical tasks. This strand may appear to be somewhat contradictory at first glance, as the essential skill is rapid recall of mathematical learning components. However, this fluency is not based on rote learning; it is achieved in the practice of repeated rehearsal, during which students have opportunities to explore and implement their learning in familiar and in new contexts and tasks, and during which they can deepen their understanding. This strand of mathematical learning is not linked to rote learning.

Strategic competence is the strand of activity that is most easily engaged with when students have achieved some conceptual understanding, and it is supported by mathematical fluency because it is the strand that is focussed on problem solving. The capacity to solve problems involves students engaging with analytical cognitive processes. To solve problems, students must first identify the problem and then strategically use the procedures that they have learned and understand well to solve it, or strategically use their mathematical knowledge to invent or devise a series of actions to resolve the issue. To do this effectively, students need to understand what they have learned about mathematical concepts, procedures and relationships, and then use this knowledge in new or unforeseen contexts and circumstances. This process can be considerably facilitated by a high degree of mathematical fluency. It is also made more accessible by the fourth strand of mathematical actions, adaptive reasoning.

Adaptive reasoning requires students to engage with higher order thinking. It involves developing competencies in logical reasoning, explanation, justification and reflection. Students need to be able to explain how they devise mathematical plans for problem solving. In order to do this effectively, students need opportunities to work creatively and constructively during their mathematical learning tasks. They need to participate in leaning tasks that promote discussion, conjecture and sharing of ideas and strategies, as this is the means by which students' mathematical thinking and reasoning can become a shared experience and provide the occasions for reflection, justification and evaluation that cannot be experienced during solitary, routine mathematics tasks that are concluded with an acknowledgement that students have or have not found a correct answer.

The final strand of the mathematical actions that are discussed by Sullivan (2011a, 2011b) is productive disposition (Watson, 2008), or a habitual inclination (Watson & Sullivan, 2008) to perceive mathematics as useful, productive and worthwhile. This strand acknowledges the impact of emotion, most especially positive emotion, motivation and positive engagement. The impact of emotion on learning is well documented (Gardner, 1993a, 1993b; Goleman, 1995; Souza, 2010) but not extensively explored by Sullivan (2011a, 2011b). This is interesting because of the links to the personal, social and cultural aspects and expectations of students, the diversity of learning preferences (Gardner, 1993b; Sternberg et al., 2000; Sternberg, Jarvin, & Griforenko, 2000; Sternberg & Kaufman, 2006) and the unique wiring of each individual brain (Coch, Fischer, & Darwin, 2010; Medina, 2010), which result in teachers having to know their students and their learning preferences, understand different ways of interpreting, understanding and using numeracy (Street, Rogers, & Baker, 2006), and then design programs of work based on the students' prior learning and with which the students can engage. This mathematical strand may not have been thoroughly extrapolated by Sullivan because he felt it was not as 'mathematical' as the other strands; however, it is argued here that this strand would be vital to any authentic learning context as, without some degree of interest (Reese, 1998; Sellars, 2008), students do not engage positively or meaningfully, nor do they work productively, in any area of learning. Productive disposition may be especially important in mathematics because of the specific difficulties that are experienced by some learners such as dyscalculia (Landerl, Bevan, & Butterworth, 2004; Munro, 2003) and maths anxiety (Sheffield & Hunt, 2006/2007).

The perceptions relating to the importance of this strand are compounded by the mathematics curriculum document proficiencies (ACARA, 2009 p. 6). These are obviously developed from the same source (ACARA, 2009) as Sullivan's strands of mathematical actions, and this is noted by ACARA. However, the strand identified as productive disposition does not appear, despite the importance placed, in all teaching and learning contexts, on positive attitudes to support successful learning. The proficiencies, as adapted by for the National Curriculum in Mathematics (ACARA, 2009), are as follows:

Understanding, which includes the building of robust knowledge of adaptable and transferable mathematical concepts, the making of connections between related concepts, the confidence to use the familiar to develop new ideas, and the understanding of the 'why' as well as the 'how' of mathematics.

Fluency, which includes skill in choosing appropriate procedures; carrying out procedures flexibly, accurately, efficiently and appropriately; and recalling factual knowledge and concepts readily.

Problem solving, which includes the ability to make choices, interpret, formulate, model and investigate problem situations, and to communicate solutions effectively.

Reasoning, which includes the capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising (ACARA, 2009).

What does this mean for you as a teacher of numeracy?

- Ensuring that students have sound conceptual understanding of mathematical language, procedures and numbers allows students to develop new ideas based on previous learning experiences.
- Making explicit linkages and connections and discovering and exploring number and other conceptual relationships helps students to develop robust knowledge that is easily transferred from familiar to unfamiliar contexts and problems.

- Basing mathematical fluency on deep understanding and avoiding rote learning allows students to respond to appropriate cues and prompts because they then have the capacity to work out what may have been forgotten.
- Facilitating social interaction and dialogue in mathematical problem solving allows students access to each other's thinking, investigations and processes for problem solving.
- Having the expectation that all students participate in the activities, including the discussions, sharing and reflections, prompts students to develop a community of learners when all ideas are acceptable and are discussed without prejudice or favour (Burgh et al., 2006).
- Avoiding asking students to complete tasks that are overly simplistic or unnecessarily repetitive, and not requiring students to explain their procedures, justify the answers or explain their work in discussion.
- Providing adequate time for sharing ideas, discussing, debating, collaborating and reflecting.
- Engaging with students in conferences, discussing with them their individual strategies, justifications and processes.
- Giving regular feedback that is meaningful for the students in terms of what is progressing well, discussing areas that are problematic or may be the next step in their learning and providing plans that address these; so students develop confidence and receive support with their learning and develop positive attitudes to learning in this area.
- Eliminating consumable materials that require students to fill in boxes with answers and texts that necessitate working through complete pages of operations that have the same degree of complexity, the same strategies and little differentiation in presentation.
- Using materials, resources and assessment designs that allow students to individually show what they know, not what they do not know.
- Creating assessment items and everyday tasks that resonate with students' own numeracy practices; that require personal, cultural and social responses; and that engage students in the logical skills described in adaptive reasoning.
- Providing safe, non-competitive learning environments in which students gain an understanding and an appreciation that mathematics is relevant, useful and interesting, where mathematical competencies are not judged on pages of ticks and crosses.
- *For example, the History lesson designed for Year Three, 'Using an abacus to make a personal time line' integrates some very understandings about data representation, yet is reliant on the students' own understanding of chronology, and their capacities to organise their personally selected events successfully as an interactive data display from which other mathematical information can be retrieved.*

Mathematical Thinking

Siemon et al. (2013) indicate that there are at least four types of mathematical thinking and activity. Although these are not, in practice, isolated one from the other, but rather are integrated in a wide diversity of tasks, it is important for educators to specifically plan for activities which engage each type of thinking. First, and most popular, are exercises. These are activities like algorithms, which have no context or limited context. Pages of these are given to students under the guise of developing fluency. Many worksheets and pages in mathematics text books have exactly this type of activity and little else. Second is problem solving, one of the most difficult tasks for some students as the contexts and construction of these is often not readily able to accessed by specific groups of learners. There are two types of activity to be identified here. They both have the same characteristic in that their answers are not obvious and there are many ways by which their problems can be solved. Word problems are commonly written in ways which describe a common experience for students at the various stages, for example sharing at parties, going shopping, etc. In order for the students to engage with the thinking required here, they need to be able to identify the appropriate operations (not be told that these are all division problems, for example) and apply them accurately and meaningfully to contribute to solving the problem, which may require the students to complete more than one step in order to solve the problem (Iii & Ford, 1991). There are also problems that are all numerical, without words. An example may be to find the radius of a circle, but the only information supplied means the students have to complete several operations in order to solve it. Neither type can engage student thinking successfully if the steps or strategies are thought through for them, not by them independently. Both these types need the students to be proficient in reasoning, especially adaptive reasoning, because they have to adapt their known strategies to the specific contexts of the new problems to be solved. Neuroscientific findings have demonstrated that the impact of verbal instruction is very limited in terms of students transferring what they know from one context to another. The critical component of this learning is that students know what the example is illustrating so they can identify the problem solving rules independently (Lee, Fincham, Betts, & Anderson, 2014).

The third, investigating, is possibly the most neglected of the mathematical thinking and activities, yet it is critical to the development of understanding relationships and patterns in mathematics. These are the types of tasks that do not have questions to be solved. They are activities that promote the search for patterns and relationships, and encourage students to find these for themselves; using the specialising-to-generalising thought process in order to establish rules or relationships for themselves and not being actually told these by the teacher. Finally, there are modelling tasks and thinking, which are essentially embodied learning. These are not the same as modelled activities in Literacy. Mathematically, these are the activities that are 'modelled' by using algebraic expression to express the relationships between the components of the problem to be solved. This is the most cognitively complex of the activities outlined. Engaging with these activities requires complex understanding of the generalized nature of algebra, the use of pronumerals and the capacity to solve problems using abstract terms.

What does this mean for you as a teacher of numeracy?

- Mathematical thinking and learning is an extremely complex cognitive activity.
- Students with robust, developmentally appropriate executive function skills have increased potential to achieve mathematically.
- Electronic games and board games that engage students with non-symbolic and symbolic representations of larger and smaller quantities engage both the inherent capacities relating to awareness of magnitude that babies are born with, and the representation of precise numbers as symbols.
- Students from different language backgrounds may have difficulties with the ways in which numbers are expressed in words in the English language.
- The four different types of mathematical activities and thinking are of equal importance in mathematical competence. They are neither exclusive nor exhaustive domains. For example, it may be impossible to solve a modelled activity without using algorithmic and algebraic knowledge, and the same task may involve investigating patterns and relationships in several mathematical areas.
- It is important not to dwell exclusively on exercises. This is because the creation of problem-solving strategies in these divergent tasks engages parts of the brain that are not ever required to function together as neural circuits in the less creative, more convergent contexts of exercises.
- *For Example, the Year Six lesson in Technology, Designing an Interactive Game, requires students to use their mathematical thinking and knowledge in combination with their creative capacities to design an interactive games per the criteria that is stipulated. This activity can be implemented with the minimum language difficulties as the criteria can be visually or concretely represented and the mathematical language modelled and investigated in this way. The problem solving and mathematical thinking for this activity requires students to not only to know and understand the mathematical content but also to have the executive function skills of taking initiative, persevering with the task and checking for accuracy, and monitoring their own thinking and emotional responses. This is a task which requires complex mathematical and creative thinking. The very detailed lesson in Media for Year Six, Moveable Triangles, provides a good introduction to this type of interactive construction and emphasises the importance of angles in the movement embedded in these tasks.*

Conclusion

This chapter has focussed on the development of sound mathematical skills, knowledge and concepts that formalise and symbolically represent students' learning experiences, both in and out of school environments. It explored the complexity of influences, social, neural and psychological that impact students' concepts of both the nature of mathematics and the nature of the learning process that is required for gaining mathematical proficiencies. It has suggested what these notions might mean for teachers in classrooms, their pedagogical practices in relation to mathematics and their opportunities to engage all students positively in the tasks that support successful learning in mathematics. The complexity of learning in mathematics is explored through the various models of mathematical actions, mathematical activity and the brain; and also the investigation of the various types of mathematical thinking and activities. However, this is just one side of the coin. The other is the numeracy: mathematics in action in everyday personal life and activity.

References

ACARA. (2009). *Shape of the Australian curriculum: Mathematics*. Barton ACT: Commonwealth of Australia.

Artigue, M. (2010). The future of teaching and learning mathematics with digital technologies: Mathematics education and technology-rethinking the Terrain. In C. Hoyles & J. Lagrange (Eds.), (Vol. 13, pp. 463–475), US: Springer.

Baker, D., Goesling, B., & Letendre, G. (2002). Socioeconomic status, School quality, and national economic development: A Cross-national analysis of the 'Heyneman-Loxley Effect' on mathematics and science achievement. *Comparative Education Review, 46*(3), 291.

Baker, D., Street, B., & Tomlin, A. (2003). Mathematics as social: Understanding relationships between home and school numeracy practices. *For the Learning of Mathematics, 23*(3 (Nov)), 11–15.

Beswick, K., Watson, J., & Brown, N. (2006). Teachers' confidence and beliefs and their students' attitudes to mathematics. From http://www.merga.net.au/documents/RP42006.pdf.

Bishop, A. (1998). *Mathematical enculturation*. Dordreht: Kluwer.

Blakemore, S., & Frith, U. (2005). *The learning brain—Lessons for education*. Uta, Malden: Blackwell Publishing.

Bobis, J. (2013). *Mathematics for children: Challenging children to think mathematically* (4th ed.). Frenchs Forest, N.S.W.: Pearson Australia.

Bobis, J., Clarke, B., Clarke, D., Thomas, G., Wright, B., Young-Loveridge, J., et al. (2005). Supporting teachers in the development of young children's mathematical thinking: Three large scale cases. *Mathematics Education Research Journal, 16*(3), 27–57. doi:10.1007/bf03217400.

Bobis, J., & Mulligan, J. (2010). *Teaching primary mathematics* (4th ed.). Frenchs Forest, N.S.W.: Pearson Australia.

Booker, G. (2011). *Building numeracy: Moving from diagnosis to intervention*. South Melbourne, Vic.: Oxford University Press.

Bruner, J. (1997). Celebrating divergence: Piaget and Vygotsky. *Human Development, 40*(2), 63–73.

Burgh, G., Field, T., & Freakley, M. (2006). *Ethics and the community of enquiry: Education for deliberative democracy*. South Melbourne: Cengage.
Buschman, L. (2001). Using student interviews to guide classroom instruction: An action research project. *Teaching Children Mathematics, 8*(4), 222–227.
Clarke, D. (2001). *Understanding, assessing, and developing young children's mathematical thinking: Research as a powerful tool for professional growth*. Paper presented at the 24th Annual MERGA Conference, Sydney. http://www.merga.net.au/documents/Plenary_Clarke.pdf.
Clarke, D., & Clark, B. (2004). Mathematics teaching in Grades K-2: Painting a picture of challenging, supportive and effective classrooms. In R. Runenstein & G. Bright (Eds.), *Perspectives on the Teaching of Mathematics* (Vol. 66th Yearbook of the National Council of Teachers of Mathematics, pp. 67–81). Reston, VA: NCTM.
Coch, D., Fischer, K., & Darwin, G. (Eds.). (2010). *Human Behaviour, learning and the developing brain*. New York: Guildford Press.
Cole, M., & Wertsch, J. (1996). Beyond the individual-social antimony in discussions of Piaget and Vygotsky. *Human Development, 39,* 250–256.
Deed, C., Pridham, B., Prain, V., & Graham, R. (2012). Drawn into mathematics: Applying student ideas about learning. *International Journal of Pedagogies & Learning., 7*(1), 99–108.
Duncan, M. (1995). Piaget and Vygotsky revisited: Dialogue or assimilitation? *Developmental Review, 15,* 458–472.
Dyson, M. (2004). A journey to transformism in Australian teacher education. In U. Monash (Ed.). Clayton Vic: Monash University.
Ferme, E. (2014). What can other areas teach us about numeracy? *The Australian Mathematics Teacher, 70*(4), 28–34.
Fox, S., & Surtees, L. (2010). Mathematics across the curriculum. New York: Continuum.
Freebody, P., & Luke, A. (1990). Literacies programs: Debates and demands in cultural context. *Prospect, 5,* 7–16.
Freebody, P., & Luke, A. (2003). Literacyu as engagiung with new forms of life: the 'four roles' model. In G. Bull & M. Anstey (Eds.), *The literacy lexicon* (pp. 51–66). Australia: Prentice Hall.
Gardner, H. (1993a). *Frames of mind* (Tenth Anniversary ed.). New York: Basic Books.
Gardner, H. (1993b). *Multiple intelligences. The theory in practice*. New York: Basic Books.
Goleman, D. (1995). *Emotional intelligence: Why it can matter more than IQ*. New York: Bantam Books.
Hennessey, M., Higley, R., & Chesnut, S. (2011). Persuasive pedagogy: A new paradigm for mathematics education. *Educational Psychology Review, 24*(2), 187–204.
Iii, J., & Ford, D. (1991). Identifying and nurturing the promise of gifted black american children. *The Journal of Negro Education, 60*(1), 3–18.
Kilpatrick, J., Swafford, J., & Findell, B. (2001). Adding It Up: Helping Children Learn Mathematics. Washington, US: National Academies Press.
Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9-year-old students. *Cognition, 93*(2), 99–125. doi:10.1016/j.cognition.2003.11.004.
Lee, H. J., Fincham, J., Betts, S., & Anderson, J. (2014). An FMRI investigation of instruction in mathematical problem solving. *Trends in Neuoscience and Education, 3,* 50–62.
Macmillan, A. (2009). *Numeracy in early childhood*. South Melbourne: Oxford University Press.
Medina, J. (2010). *Brain rules for learning*. (Vol. Pear Press): Seattle.
Mockler, N. (2011). Beyond 'what works': Understanding teacher identity as a practical and political tool. *Teachers and Teaching, 17*(5), 517–528. doi:10.1080/13540602.2011.602059.
Mukhopadhyay, S., & Greer, B. (2010). Modelling with purpose: Mathematics as a critical tool. In B. Atweh, H. Forgasz, & B. Nebrers (Eds.), *Sociocultural research on mathematics education: An international perspective* (pp. 295–312). New York: Routledge.

Mulligan, J. (2010). *Reconceptualising early mathematics learning* Paper presented at the ACER Research Conference, Melbourne.

Munro, J. (2003). Dyscalculia: A unifying concept in understanding mathematics learning disabilities. *Australian Journal of Learning Disabilities, 8*(4), 25–32. doi:10.1080/19404150309546744.

National Research Council (U.S.) Mathematics learning study committee. (2001). Adding it up: Helping children learn mathematics. In J. Kilpatrick, J. Swafford & B. Findell (Eds.), *Kilpatrick, J Swafford, J.*

Pilling-Cormick, J. (1997). Transformative and self-directed learning in practice. *New Directions for Adult & Continuing Education*.

Prestridge, S., & Watson, G. (2004). Developing classroom teachers' understanding of multiliteracies: The role of reflection. *Australian Journal of Educational Technology, 19*(2), 227–240.

Reese, A. (1998). Implications of results from cognitive science research for medical education. *Medical Education Online, 3*(1).

Schoenfeld, A. H. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher, 31*(1), 13–25. doi:10.3102/0013189x031001013.

Sellars, M. (2008). *Using students strengths to support learning outcomes: A study of the development of gardner's intrapersonal intelligence to support increased academic achievement for primary school students.* Saarbrucken, 97: VDM Verlag.

Sheffield, D., & Hunt, T. (2006/2007). How does anxiety influence maths performance and what can we do about it? *MSOR Connections, 6*(4), 19. http://journals.heacademy.ac.uk/doi/pdf/10.11120/msor.2006.06040019.

Siemon, D., Beswick, K., Brady, K., Clark, J., Faragher, R., & Warren, E. (2013). *Teaching mathematics: Foundations to middle years*. South Melbourne: Oxford University Press.

Souza, D. (2010). *Mind, brain and education: Neuroscience implications for the classroom*. Bloomington IN: Solution Tree Press.

Stacey, K. (2010). *Mathematics teaching and learning to reach beyond the basics.* Paper presented at the ACER Research Conference. http://research.acer.edu.au/cgi/viewcontent.cgi?article=1082&context=research_conference.

Sternberg, R., Forsythe, G., Hedlund, J., Horvath, J., Wagner, R., Williams, W., et al. (2000a). *Practical intelligence in everyday life*. Cambridge: The Cambridge University Press.

Sternberg, R., Jarvin, L., & Griforenko, E. (2000b). *Wisdom, intelligence, Creativity and Success*. Corwin: Thousand Oaks.

Sternberg, R., & Kaufman, A. S. (2006). Human abilities. *Educational Theory, 56*(1).

Street, B., Rogers, A., & Baker, D. (2006). Adult teachers as researchers: Ethnographic approaches to numeracy and literacy as social practices In South Asia. *Convergence, xxxix*(1).

Suarez-Orozco, M., & Sattin- Bajaj, C. (2010). *Educating the whole child for the whole world: The ross school model and education for the global era*. New York: New York University Press.

Suh, J. (2007). Tying it all together: Classroom practices that promote mathematical proficiency for all students. *Teaching Children Mathematics, 14*(3), 163–169.

Sullivan, P. (2011). Teaching mathematics: Using research-informed strategies. Victoria.

Sullivan, P. (2011). Teaching mathematics: Using research-informed strategies: *Australian Education Review* (pp. 1–72): ACER.

Watson, J. (2008). Developing critical numeracy across the curriculum. From http://www.simerr.educ.utas.edu.au/numeracy/critical_numeracy/critical_numeracy.htm#The_Four_Resource_Model_for_Critical_Numeracy.

Watson, J. (2009). Critical numeracy: Developing critical numeracy across the curriculum. Retrieved 17th January, 2014, from http://www.simerr.educ.utas.edu.au/numeracy/critical_numeracy/critical_numeracy.htm.

Watson, J. (2011). Personal and professional numeracy: A unit for pre-service teachers at the University of Tasmania. *Numeracy, 4*(1). doi:http://dx.doi.org/10.5038/1936-4660.4.1.2.

Watson, J., & Sullivan, P. (2008). Teachers learning about tasks and lessons. In D. Tiosh & T. Woods (Eds.), *Tools and resources in mathematics teacher education* (pp. 109–135). Rotterdam: Sense Publishers.

Author Biography

Dr. Maura Sellars graduated from the Froebel Institute in London (now part of the University of Roehampton) She has almost 30 years experience as a classroom teacher in primary school settings. She currently teaches mathematics, numeracy and pedagogy at the University of Newcastle, NSW. She is particularly interested in developing an equity pedagogy, belonging and inclusion, critical and creative thinking and literacy and numeracy as social practice.