Gaussian Mixture Model Based Image Denoising Method with Local Constraints

Min Li[1], Yuhui Zheng[1(✉)], Shunfeng Wang[2], and Jin Wang[3]

[1] School of Computer and Software, Nanjing University of Information Science and Technology, Nanjing 210044, China
`limin_5l3@l26.com`, `zheng_yuhui@nuist.edu.cn`
[2] College of Math and Statistics, Nanjing University of Information Science and Technology, Nanjing 210044, China
`wsfnuist@l26.com`
[3] College of Information Engineering, Yangzhou University, Yangzhou 215127, China
`jinwang@yzu.edu.cn`

Abstract. Recently, the image denoising methods based on patch priors have received extensive attention. Among these methods, expected patch log likelihood (EPLL) has achieved great success, using Gaussian mixture priors by the Gaussian mixture model (GMM). In the paper, we observe that GMM model requires the estimation of a global parameter λ, rather than locally adaptive parameters. Based on this, we propose a modification of the GMM model which is imposed the local constraints on partition of the image. The experimental results illustrate that our proposed method performs comparatively well.

Keywords: Image denoising · Gaussian mixture model · Expected patch log likelihood · Image patch · Local constraint

1 Introduction

Among many image restoration methods, patch-based methods [1–3] have offered effective ways, for example, log likelihood (log L) probability [4] and maximum a posteriori (MAP) [5] method. We usually use the Markov random field (MRF) [6] for the whole image processing directly. At this point, log L probability and MAP are difficult to calculate accurately. Thus, a general optimization framework based on patch prior has been widely put forward, the most representative, Field of experts (FOE) [7] framework.

Expected log patch likelihood (EPLL) [8] is also an optimization framework using Gaussian mixture priors [9] learned by the Gaussian mixture model (GMM) [10, 11]. We find that it is a global parameter λ in the GMM model, instead of local adaptive parameters. The influence is that the denoising performance of different image regions is inconsistent. For avoiding this effect, we propose a novel GMM model with local constraints. We use a set of constraints λ_i and each λ which permits to satisfy the constraint for one region does not serve for other ones. Certainly, a different selection of the regularization parameters [12] may give better results in some region, but no

single parameter can give an improvement for all regions at the same time. In the Lagrange multiplier formulations, we need several the Lagrange multipliers to be able to impose locally that noise variance is given by δ^2. Therefore, parameter is now spatial adaptive.

2 Proposed Method

2.1 Background of Expected Patch Log Likelihood

Expected patch log likelihood (EPLL) is a general optimization framework based on patch priors for image restoration. Given a natural image u and known priori p, EPLL can be defined as:

$$EPLL\,(u) = \log p(u) = \sum_i \log p(P_i u) \tag{1}$$

Where P_i denotes an operator for extracting image patch u_i from image u. The patch prior used in the joint conditional density with the EPLL is a Gaussian Mixture Model (GMM) given by:

$$p(u) = \prod_{i=1}^{N} \sum_{j=1}^{K} \pi_j N(u_i \mid \mu_j, \Sigma_j) \tag{2}$$

Where π_j are the mixing coefficients, μ_j and $\sum_j$ are the corresponding mean and covariance matrix.

Given a noise image u_0, the degradation model can be expressed by $\|u - u_0\|^2$. In order to achieve good results for restoration, we should maximize log likelihood (log L) probability of the image patch, while keeping the u and u_0 as consistent as possible. Therefore, EPLL model based on priori p is represented as follows:

$$\min_u \left\{ \frac{\lambda}{2} \|u - u_0\|^2 - \sum_i \log p(P_i u) \right\} \tag{3}$$

Where λ is a regularization parameter. The equation can be solved by Half Quadratic Splitting [8] which introduces a set of auxiliary variables z^i and changes the cost function into the following form:

$$\min_{u,\{z_i\}} \left\{ \frac{\lambda}{2} \|u - u_0\|^2 + \sum_i \left\{ \frac{\beta}{2} (\|P_i u - z_i\|^2) - \log p(z_i) \right\} \right\} \tag{4}$$

Where β is the penalty parameter which often is set to be large enough to ensure that the solution of (4) is close to that of (3). Then formula (4) can be minimized by alternatively updating z^i and u_i.

2.2   Proposed Method with Local Constraints

Suppose that $\{O_1, \ldots, O_r\}$ is a partition of image. Given $\lambda_1, \ldots, \lambda_r$, we consider the following problem:

Whether there are values $(\lambda_1, \ldots, \lambda_r)$ satisfying local constraints [13, 14] as follows

$$\frac{1}{|O_i|} \|u - u_0\|^2 = \delta^2, \forall i = 1, \ldots, r. \tag{5}$$

We proposed to solve the following constrained problem:

$$\max_u \sum_i \log p(P_i u) \tag{6}$$
$$s.t. \|u - u_0\|^2 = \delta^2 |o_i|$$

In case that we answer the above question in the affirmative sense, the solution of (6) would give a solution of the problem:

$$\min_u \left\{ \frac{\lambda(x)}{2} \|u - u_0\|^2 - \sum_i \log p(P_i u) \right\} \tag{7}$$

Where $\lambda(x) = \sum_{i=1}^{r} \lambda_i \chi o_i$. For simplicity we shall write $\vec{\lambda} = \{\lambda_1, \ldots \lambda_r\}$ and $\vec{\lambda} \geq 0$ if $\lambda_i \geq 0$ for all $i = 1, \ldots, r$. The novel model that we propose is an extension of GMM model, where the parameter λ takes different values for different regions. To solve (7) we use the same numerical approach we used to solve (3). The Eq. (7) is equivalently transformed into the following function:

$$\min_{u, \{z_i\}} \left\{ \frac{\lambda(x)}{2} \|u - u_0\|^2 + \sum_i \left\{ \frac{\beta}{2} (R\|Ru - z_i\|^2) - \log p(z_i) \right\} \right\} \tag{8}$$

For solving (8), at first, we choose the most likely Gaussian mixing weight j_{max} for each patch $R_i u$. Then Eq. (8) is minimized by alternatively updating z_i and u:

$$z_i^{n+1} = \left(\Sigma_{j_{max}} + \frac{1}{\beta} I \right)^{-1} \cdot \left(R_i u^n \Sigma_{j_{max}} + \frac{1}{\beta} \mu_{j_{max}} I \right) \tag{9}$$

$$u^{n+1} = u^n + \Delta t [\lambda(x)(u_0 - u^n) - \sum_i \beta R_i^T (R_i u^n - z_i^n) \tag{10}$$

Where I is the identity matrix, Δt is the time step. In practice, for updating the parameters λ_i, we use Uzawa's method [15]. In summary, the algorithm can be implemented as follows:

Step1. Input corrupted image u_0, model parameters $\beta, \Delta t$ and iteration stopping tolerance ε;

Step2. Choose the most likely Gaussian mixing weights j_{max} for each patch $R_i u$;

Step3. Initially, we take the values of $\lambda_i > 0$ small enough so that

$$Q_{Oi}(u^\lambda) = \frac{1}{|O_i|} \left\| u^\lambda - u_0 \right\|^2 > \delta^2, \forall\, i = 1, \ldots, r$$

Step4. For each set of values $\lambda_i > 0$, we alternatively update (9) and (10), until we reach the asymptotic state u^λ.

Step5. For each $i \in \{1, \ldots, r\}$ recompute $\lambda_i = \max(\lambda_i + \rho(Q_{Oi}(u^\lambda - \delta^2), 0)$ (with $\rho > 0$ small enough)

Step6. Iterate steps 4–5 until the λ_i' satisfying stopping criterion.

3   Implementation and Experiment Results

In experiments, we compare our proposed method with the original EPLL in image denoising. The GMM with 200 mixture components is learned from 2×10^6 images patches which are sampled from the Berkeley Database. The experimental pictures are added Gaussian noise with zero mean and standard variance $\delta = 25$.

Figures 1 and 2 show the performance of the EPLL with Gaussian mixture priors and our method respectively on Test1 image (i.e. No. 37073) and Test2 image (i.e. No. 103070) in denoising. We can find that our proposed method outperforms the original EPLL in the denoised result. Because that the local constraints are equivalent
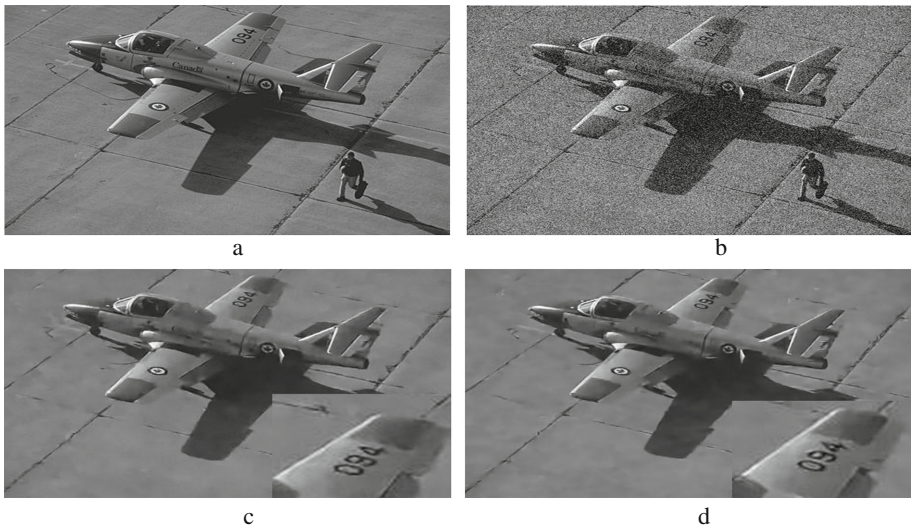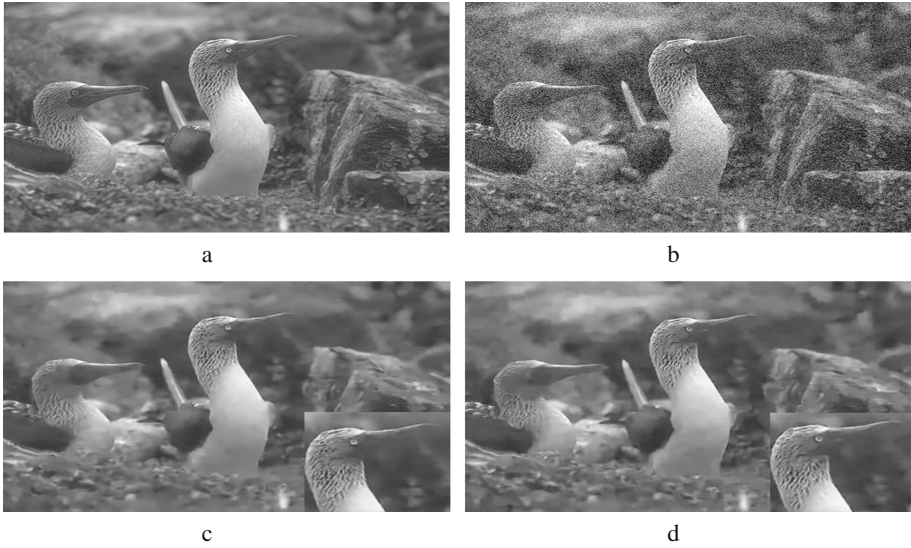

Fig. 1. Denoising results on Test1 image

Fig. 2. Denoising results on Test2 image

Table 1. The PSNR results of different denoising models

Image	EPLL	Our method
Test1	30.37	30.54
Test2	29.89	30.22

to the fidelity term with spatial adaptive parameters by Lagrange multiplier method. For the related quantitative comparison, as demonstrated in Table 1, the peak signal to noise ratio (PSNR) value of our method is also higher than the original EPLL.

4   Conclusions

Image priors play a vital role in image restoration tasks. In this paper, we devote to researching on Gaussian mixture model based on local constraints. We construct an adaptive regularization parameter coupling the local entropy of the image, which varies with different regions of the image and each λ corresponds to a region. The numerical results show our proposed method achieves a satisfying denoised result, compared with the original EPLL algorithm with fixed regularization parameters.

References

1. Vaksman, G., Zibulevsky, M., Elad, M.: Patch ordering as a regularization for inverse problems in image processing. Siam J. Image Sci. **9**(1), 287–319 (2016)
2. Dameravenkata, N., et al.: Image quality assessment based on a degradation model. IEEE Trans. Image Process. Publ. IEEE Signal Process. Soc. **9**(4), 636–650 (2000)
3. Boulanger Jr., C.K., Bouthemy, P.: Space-time adaptation for patch-based image sequence restoration. IEEE Trans. Pattern Anal. Mach. Intell. **29**(6), 1096–1102 (2007)
4. Yavuz, M., Fessler, J.A.: Maximum likelihood emission image reconstruction for randoms-precorrected PET scans. Nucl. Sci. Symp. Conf. Rec. IEEE **15**(2), 229–233 (2000)
5. Chantas, G., Galatsanos, N., Likas, A.: Maximum a posteriori image restoration based on a new directional continuous edge image prior. In: IEEE International Conference on Image Processing IEEE Xplore I-941-4 (2005)
6. Sigelle, M.: A cumulant expansion technique for simultaneous Markov random field image restoration and hyperparameter estimation. Int. J. Comput. Vis. **37**(3), 275–293 (2000)
7. Roth, S., Black, M.J.: Fields of experts. Int. J. Comput. Vis. **82**(2), 205–229 (2009)
8. Zoran, D., Weiss, Y.: From learning models of natural image patches to whole image restoration. In: 2011 IEEE International Conference on Computer Vision (ICCV), pp. 479–486. IEEE (2011)
9. Kurisu, K., et al.: Image segmentation using a spatially correlated mixture model with Gaussian process priors. In: Pattern Recognition, pp. 59–63. IEEE (2013)
10. Nguyen, T.M., Wu, Q.M.J.: Gaussian-mixture-model-based spatial neighborhood relationships for pixel labeling problem. IEEE Trans. Syst. Man Cybern. Part B Cybern. Publ. IEEE Syst. Man Cybern. Soc. **42**(1), 193–202 (2012)
11. Yu, G., Sapiro, G., Mallat, S.: Solving inverse problems with piecewise linear estimators: from Gaussian mixture models to structured sparsity. IEEE Trans. Image Process. **21**(5), 2481–2499 (2012)
12. Zheng, Y., et al.: Adaptively determining regularization parameters in non-local total variation regularisation for image denoising. Electron. Lett. **51**(2), 144–145 (2015)
13. Bertalmio, M., et al.: TV based image restoration with local constraints. J. Sci. Comput. **19**(1), 95–122 (2003)
14. Li, F., Zeng, T.: Image restoration via tight frame regularization and local constraints. J. Sci. Comput. **57**(2), 349–371 (2013)
15. Ciarlet, P.G.: Introduction to numerical linear algebra and optimization. Math. Comput. **55**(191), 159 (1990)