The Coordination and Dynamic Analysis of Industrial Clusters: A Multi-agent Simulation Study

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Abstract An agent based simulation model is presented to investigate the long-term behavior of firms in an industrial district. The firms are interconnected with each other through input-output relations, product markets, labor, and innovation spillover. The prices of the products depend on the supply-demand balance of the market as well as on the innovation levels of the firms. Dynamic strategies of the firms are examined and conditions for successful industrial cluster formation are developed.

Keywords Industrial clusters · Agent based simulation · Oligopoly theory · Innovation

1 Introduction

Industrial clusters are important examples of coordinated multi-agent systems in which the industrial firms are the agents that are interconnected to each other by their inputs and outputs as well as to the markets through inverse demand functions. The high complexity and the large sizes of industrial clusters make their analytical investigation impossible. In this paper agent-based simulation is used to examine the coordination and dynamic properties of industrial clusters. The interrelation of the firms is modeled as an extended oligopoly, when in addition to the competition of the firms on the product markets we are able to consider their competition for labor as well.

Traditional literature on industrial clusters has mainly focused on their identification, driving forces and policies. These studies try to answer the fundamental questions such as how the firms can benefit from belonging to a cluster. A very important problem is the evolution of industrial clusters. Investigations of the evolution mainly focused on the life cycle, entry, exit and growth of the clusters (Maskell 2001; Swann

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A. Matsumoto (ed.), *Optimization and Dynamics with Their Applications*,
DOI 10.1007/978-981-10-4214-0_7

et al. 1998). Results are drawn mostly from case studies or from empirical studies. Most of these studies analyzed the industrial clusters only after they became successful, and not during the transformation period. In addition, case studies can lead to special results from individual clusters, which cannot be generalized.

A new strand of study has recently emerged in which the evolution dynamics of industrial clusters are analyzed by using agent-based simulation. With two versions, spatial and non-spatial, these studies focus on the formation, development and coordination of artificial industrial clusters. In our paper, we will follow this strand, with the additional question: how the decisions and the behavior of industrial firms will promote the formation of a cluster when these firms are already in a given system structure of a district.

This question is very practical. It has been already discussed in literature that the initiation and support of public policies may be successful in the formation of clusters (Bresnahan et al. 2001). It is also known that clusters could grow through some types of network structure. However, how to ensure the formation, effectiveness and growth of clusters is a crucial question. Local government might help to build a structure or to introduce policies to promote the important local industries. For example, in the developing areas of some regions in China, the government plays a crucial role to initiate the development of certain industries. However, with similar policies and perhaps with similar environments, some districts were promoted to clusters while some others were not. There must be many other factors to explain this difference in the result. In this paper and in our future works, we are interested in the firm level influencing factors: what firms should do to help remove the barriers to the cluster formation and in exploring their own opportunities.

In this study the district structure means the topologies defined by Markusen (1996), who identified several types of system structures of industrial clusters. In this paper, we assume the particular structure, which is called 'Hub and Spoke' by Markusen. It consists of several large anchor companies and several small companies. (Hence we are not going to study industrial clusters with a large number of small and medium sized firms). In reality, some emerging clusters have similar system structure like this. Take again Chinese regional industrial clusters as an example, several foreign invested global companies were attracted into the developing district, and then many relatively small suppliers and accessorial firms moved in.

As mentioned earlier, we will propose an agent-based simulation model to show how an industrial cluster could emerge in a location which already includes several firms. Agent based simulation is a flexible tool to investigate emerged behaviors of complex systems from individuals. Researchers are already using this tool to examine industrial clusters. The reputation dynamics (Giardini et al. 2008) and the growth of clusters (Zhang 2003) are good examples. In the case of most studies, the individual-level decision rules are relatively simple, and the topologies of the district are never considered. In our study, by considering the environment of the designed system structure, we will adopt the Hub and Spoke topology and express it as a two-layer network. Firms will be modeled as bounded rational agents. Each agent has its own production input factors, labor, and production. During each period of time, each agent will make decisions based on its former behavior, the other firms former behavior and its own decision rules. For the decision making process of the agents, we will integrate oligopoly theory into the agent-based model. Hence our model will be an agent-based and game theory integrated model.

In the spatial version of the agent-based models of industrial clusters, moving and relocating agents are basic elements. However we will not consider these features, since firms cannot move easily like residents. Our primary model is a non-spatial one, and the distance of locations is not our concern. This is a reasonable assumption since it is more important to decide if a given firm is in the cluster or not. When we consider firms only in a specific location, spatial distance is not an important factor.

The methodology of this paper might have further applications. A potential study area is the examination of the change of behavior and decision patterns of firms that can transform declining clusters into new ones. In addition, with the relaxation of some assumptions, we may study more general situations. This paper is only a starting point of a long-term research project.

This paper develops as follows. Section 2 presents the related literature review. In Sect. 3, we will outline the fundamentals of agent-based models and oligopoly theory. Simulation methodology and numerical results will be reported in Sect. 4. Final conclusions will be drawn in Sect. 5.

2 Related Literature Review

In this section, we will briefly review the history of the two main tools that will be used in our study: oligopoly theory and agent-based social simulation.

2.1 Oligopoly Theory

The classical oligopoly theory dates back to the pioneering work of Cournot (1838). It examines an industry in which several firms produce identical product or offer identical service to a homogeneous market. Since then a significant number of researchers focused on the different extensions and generalizations of Cournots classical model. Comprehensive summaries of the earlier works and multi-product models are given in Okuguchi (1976), Okuguchi and Szidarovszky (1999). In the early stages, oligopolies were considered as noncooperative games in which the firms are the players, their output levels are the strategies, and the profit functions are the payoffs. The existence and uniqueness of the equilibrium was first the main issue, under certain monotonicity and convexity assumptions the existence and uniqueness of the equilibrium was proved. This important result was later extended to more realistic model variants including single product models with product differentiation, multi-product oligopolies, labor-managed and rent-seeking games among others.

The main focus of the studies in oligopoly theory has later turned into dynamic extensions. Models were developed with discrete and continuous time scales and the resulting difference and differential equation systems were investigated. The main issue was the asymptotical stability of the equilibrium; conditions were derived to guarantee that the output trajectories converge to the equilibrium in the long run. Most models were linear, where local and global stability are equivalent and very little attention was given to nonlinear dynamics until the late 80s. In developing dynamic models there are usually two alternative ways. In the case of best response dynamics it is assumed that each firm adjusts its output into the direction toward its best response. This approach requires the knowledge of the best response functions of the firms, which needs the solution of usually nonlinear optimization problems based on global information on the payoff functions. In the case of gradient adjustments it is assumed that the firms adjust their outputs in proportion to their marginal profits. This idea has a lot of sense, since in the case of positive (negative) gradient value the firms interest is to increase (decrease) its output level. This concept requires only local information about the payoff functions, so it is much more realistic than the use of best response dynamics. A comprehensive summary of the recent developments in this area can be found in Bischi et al. (2009).

Most studies in oligopoly theory considered only the market as a link between the firms; the unit price was always a function of the total output level of the industry due to the demand-supply balance. However in realistic economies the firms are linked together in much more complicated ways. First, they use common supply of energy, raw material, labor, capital etc., and therefore they also compete on this secondary market in addition to the market of their products. This idea was elaborated in the studies of oligopsonies (Szidarovszky and Okuguchi 2001). In multiproduct oligopolies on the other hand the firms might buy and use the products of other firms, so a network of firms develops. Network oligopolies were introduced and some results were reported in Szidarovszky (1997).

It has been also demonstrated that partial or complete cooperation of the firms in oligopolies will benefit the firms similarly to the well-known prisoners dilemma game (Chiarella and Szidarovszky 2005). Even by any increase in the cooperation level of the firms their benefit also increases.

In most models analytic results could be derived under only very special conditions, which are not the case in realistic economies. Instead of investigating very limited cases theoretically, it is much more important and practical to use computer simulation under realistic conditions and examine the evolution of more advanced production systems such as the industrial clusters.

2.2 The Agent-Based Industrial Cluster Model

In agent-based models, individuals are modeled as heterogeneous agents. Agents have goals and decision rules, and they interact with each other and with the environment. Agent-based model is a bottom up modeling method; it studies a system as an interaction evolving system. It can explicitly explain the decision process of the micro individuals, and the macro emergence from the individuals' interaction.

Agent-based models have been widely used in the analysis of complex economic and social systems (Tesfatsion and Judd 2006). Some initial attempts use agent-based simulation to study some special aspects of industrial clusters (Giardini et al. 2008; Zhang 2003; Albino et al. 2003, 2006a, b; Brenner 2001; Dawid and Wersching 2006; Fioretti 2005).

Fioretti (2005) explained what agent-based models are, the advantages of using agent-based model to study industrial clusters, and introduced some possible simulation tools. Fioretti also reviewed some connectionist models of industrial clusters that are related to agent-based modeling.

Brenner (2001) studied the spatial dynamics of entry, exit and growth of firms. Functions for productivity of firms, innovations, exit and entry of firms, public opinions etc. are modeled and then parameters' impact are analyzed by computer simulations.

Zhang considered a 100×100-lattice environment, on the lattice, agents are born and could choose whether to start a firm or not (Zhang 2003). Production functions and profit functions are adopted for firms. The emergence of a firm in a landscape could inspire its neighbors to choose to start firms; hence industrial clusters might emerge. Computer simulation was adopted to analyze dynamics of market price, firm size distribution, location of clusters, etc.

Giardini et al. (2008) modeled social evaluations as social links, and examined the effects of the reputation of the firms and the quality of the products in a cluster. Their simulation results show that higher reputation of the suppliers and information sharing will result in higher profit for the producers.

Albino et al. (2003) proposed a model to study the multiple forms of the cooperative and competitive relationships among agents and to prove the benefits of the selected type of interaction. In their model, firms and coordination mechanisms are agents; computer simulations were used to evaluate the benefit of cooperation. In the simulations, 3 buyers and 3 sellers were simulated and simple interaction rules were adopted. Albino et al. (2006a, b) introduced the concept of complex adaptive systems into agent-based model and the study mainly focused on innovation dynamics. Their simulation elements and efforts were very similar to their previous works.

3 The Agent-Based and Oligopoly Integrated Model

3.1 The Structure of the System

An industry of a region usually consists of several types of firms. In our model, we consider the situation in which there are several large firms and many smaller suppliers. The large firms produce final products that are sold directly to the market; their products could be substitutes or not. Small firms produce materials, parts, components that large firms buy and build in their final products; their products could be also substitutes or not. Therefore there is a complicated input-output relation between the large and small firms. For example, household appliances are manufactured in a certain location. Relatively large firms produce one or more of the following products: refrigerator, washing machine, television and air conditioner; and smaller firms provide resources to these larger firms.

Firms' interactions are in the form of networks. We establish the inter-firm network as a 2-layer network: one layer of all producers and one layer of all suppliers. The connections between firms in the producer layer and firms in the supplier layer are defined by input-output relations. Firms in the same layer compete for the resources and prices. Firms in the producer layer also compete with each other for new knowledge: the R and D investment of any firm spills over to others who can also benefit from the innovation. We assume that formal systematic R and D is performed only in large firms; this is based on the study of Santarelli and Sterlacchini (1990). All firms also compete in the secondary market. In the secondary market we consider only labor pool. The interactions of the firms in the system can be described therefore as the interaction among producers, the interaction among suppliers, the interaction between producers and suppliers (through supplies), and the interaction through the secondary market (the labor).

For the sake of simplicity, we assume that if a producer needs more supplies than the suppliers can produce, then it will buy them from outside the system with the same price; and when a supplier produces more than the producers need, it will sell the surplus outside the system for the same price. These assumptions will be relaxed in our next study.

3.2 Agents, Interactions and Environment

Individual firms in the system are modeled as agents. There are two types of agents: 'suppliers' who produce and offer their products to producers, 'producers' who produce final products to an open market. There are m supplier agents and n producer agents in the system. Producer agents have innovation ability, with relatively high technical advances, and they are linked together through the open market, the secondary market and by innovation spillovers. In this first model, only the size growth of the existing firms will be considered, the entry of new firms for the growth of the cluster will be studied in our future research. Hence the number of suppliers, m, and the number of producers, n, are considered fixed in the simulation model.

Agents have states and decisions. For any supplier i and any producer j, the main state variables are listed in Table 1, and notations related to the innovation of producer agents are given in Table 2. All variables of these tables vary with time according to state updating rules that will be introduced in the next subsection. In this simple model, we consider only the firms' productions and their innovation investments as factors influencing the formation of cluster. The decision variable of a supplier agent is its production level. The decision variables of a producer agent are its purchases from the suppliers, its output and innovation investment. The 4 types of interactions among agents and with environment are as follows: (1) interaction between suppliers and producers: supply demand balance; (2) interaction among suppliers: competition

Table 1 State variables of suppliers and producers

	Supplier i	Producer j
Number of firms	m	n
Productivity	s_i	z_j
Product price	p_i^s	p_j^p
Labor usage	L_i^s	L_j^p
Profit	φ_i^s	φ_j^p

Table 2 Notations related to the innovation of producers

Innovation development step	I_j
Total cumulative innovation level	$\widetilde{I}_j$
Impact of innovation level on sale price	$F(\widetilde{I}_j)$
Cost function of innovation	$D_j(I_j)$

without product interaction; (3) interaction among producers: competition, information transmission, possible relation in products; (4) interaction among all agents: competition for labor among all firms.

The environment supplies energy, raw material and labor, and it has its rule to change the labor price. The final products of the large firms are sold to the consumers in the environment. From the environment, all agent gathers information: the outputs of their competitors, market prices, spillover of innovation from its cooperators, and the price of the labor pool. Depending on the information from the environment and the agents own state, each agent will make its decision. The details of the decision rules are discussed in the next subsection.

3.3 Agents State Updating Rules Based on Oligopoly Theory

The agents decisions are based on their decision rules.

Let x_{ij} be the amount of the product that producer j purchased from supplier i, then the total physical product of a producer is represented by a production function which is assumed to be linear

$$z_j = \sum_{i=1}^{m} a_{ij} x_{ij} + a_{0j}, \tag{1}$$

where $a_{ij} \geq 0$, $a_{0j} \geq 0$. The marginal productivity of x_{ij} is denoted by a_{ij}. If $a_{ij} > 1$, then an increase in x_{ij} will result in a more than proportionate increase in the output of producer j; for $a_{ij} < 1$, the proportionate increase in the output of producer j is less than that of input x_{ij}; for $a_{ij} = 1$, the proportionate increases are equal.

The price of any supply is a decreasing function of the supplier's own output and the outputs of all other suppliers:

$$p_i^s(s_1, \ldots, s_m) = A_i - B_i s_i - \sum_{l \neq i} b_{il} s_l, \tag{2}$$

where $A_i > 0$, $1 \geq B_i > 0$ and $1 > b_{il} \geq 0$. Larger value of b_{il} represents higher level of similarity between the supplies, or higher level of competition among them. Similarly, the prices of the final products are also linear. It is also assumed that the final products are substitutes:

$$p_j^p(z_1, \ldots, z_n) = \overline{A}_j - \overline{B}_j z_j - \sum_{l \neq j} \overline{b}_{jl} z_l, \tag{3}$$

where $\overline{A}_j > 0$, $1 \geq \overline{B}_j > 0$ and $1 > \overline{b}_{jl} \geq 0$.

The revenue of a supplier is the product of its output and supply's price $s_i p_i^s$. For the revenue of a producer, we also have to consider the innovation effect. The innovation development and spillover of producer j are modeled as

$$\widetilde{I}_j(t+1) = \widetilde{I}_j(t) + I_j + \sum_{l \neq j} k_{jl} I_l, \tag{4}$$

that is, each producer invests in innovation development by increasing its technology level by a step I_j and can utilize the knowledge spillover from other producers. The spillover $k_{jl} I_l$ from agent l is proportional to agent l's innovation investment, where $1 > k_{jl} \geq 0$. The price of any final product is affected by the technology level dependent factor

$$F_j(\widetilde{I}_j) = 1 + (F_j^{max} - 1)(1 - e^{-\omega_j \widetilde{I}_j}). \tag{5}$$

In this function form we model the fact that with higher technological level, better and more expensive final products are produced. If $\widetilde{I}_j = 0$, then this factor equals 1, then it increases in $\widetilde{I}_j$ and converges to a maximum value F_j^{max} as $\widetilde{I}_j$ tends to infinity. The graph of function $F_j(\widetilde{I}_j)$ is shown in Fig. 1. With this innovation dependent factor, the revenue of producer j is given as $z_j p_j^p(z_1, \ldots, z_n) F_j(\widetilde{I}_j)$.

We assume that larger production level requires more labor, so the labor usage of supplier i is

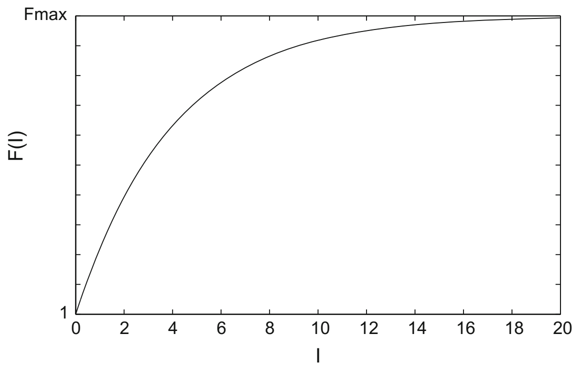

Fig. 1 The graph of function of $F_j(\widetilde{I}_j)$

$$L_i^s(s_i) = \gamma_i + \delta_i s_i. \tag{6}$$

The need of labor of producer j depends on its production and technical levels:

$$L_j^p(z_j, \widetilde{I}_j) = (\overline{\gamma}_j + \overline{\delta}_j z_j)e^{-\overline{\overline{\omega}}_j \widetilde{I}_j}, \tag{7}$$

that is, innovation decreases the labor need of the producers.

The price function of labor in the whole cluster is denoted by p^L, which depends on the total demand of labor. The price of labor is a linear function of the total labor usage:

$$p^L = c - d(\sum_i L_i^s + \sum_j L_j^p), \tag{8}$$

where $c > 0$ and $d > 0$. In this decreasing function form we model the fact that higher labor force usage decreases the ratio of skilled workers, so the average wage decreases.

The profit of a supplier is modeled as the difference of its revenue and labor cost:

$$\varphi_i^s = s_i p_i^s(s_1, \ldots, s_m) - L_i^s(s_i)p^L(\sum_{i=1}^m L_i^s(s_i) + \sum_{j=1}^n L_j^p(z_j, \widetilde{I}_j)), \tag{9}$$

For simplicity, we set all other costs to zero. The profit function of the producers is the following:

$$\varphi_j^p = z_j p_j^p(z_1, \ldots, z_n) F_j(\widetilde{I}_j) - L_j^p(z_j, \widetilde{I}_j) p^L(\sum_{i=1}^m L_i^s(s_i)$$
$$+ \sum_{j=1}^n L_j^p(z_j, \widetilde{I}_j)) - \sum_{i=1}^m x_{ij} p_i^s(s_1, \ldots, s_m) - D_j(I_j), \tag{10}$$

where the innovation development cost is also assumed to be linear:

$$D_j(I_j) = u_j + v_j I_j. \tag{11}$$

In this model the basic decision variables of the suppliers are their output levels s_i, those of the producers are the x_{ij} flows from the suppliers to the firms and the innovation investment I_j. We assume in our model that extra supplies can be sold outside the cluster for the same price, and in the case of supply shortages they can be purchased from sources outside the cluster.

3.4 The Decision Rules of Productions and the Innovation Step

In dynamic oligopoly models, there are two alternative ways to study the evolution of the system. In the case of best response dynamics it is assumed that each firm adjusts its output into the direction toward its best response. In the case of gradient adjustments it is assumed that the firms adjust their outputs in proportion to their marginal profits, which requires only local information about the payoff functions. So it is much more realistic than the use of best response dynamics.

In our earlier papers (Szidarovszky and Zhao 2009; Zhao and Szidarovszky 2008), we assumed gradient adjustment with constant speed of adjustment as updating rules. For producers, they adjust their inputs as

$$x_{ij}(t+1) = x_{ij}(t) + \frac{\varphi_j^p(x_{ij}(t) + \Delta^x) - \varphi_j^p(x_{ij}(t))}{\Delta^x} \varepsilon^x \tag{12}$$

and then the output of producer j at time period $t+1$ becomes $z_j(t+1) = \sum_{i=1}^m a_{ij} x_{ij}(t+1) + a_{0j}$.

The output of the suppliers is updated according to

$$s_i(t+1) = s_i(t) + \frac{\varphi_i^s(s_i(t) + \Delta^s) - \varphi_i^s(s_i(t))}{\Delta^s} \varepsilon^s. \tag{13}$$

In our earlier models (Szidarovszky and Zhao 2009; Zhao and Szidarovszky 2008), we selected $\Delta^x = 10$, $\Delta^s = 10$, $\varepsilon^x = 1$ and $\varepsilon^s = 0.1$. In this paper, we will also investigate the effects of these parameters on the behaviors of the agents and compare this linear decision updating rules to a special nonlinear rule, which is introduced next:

$$x_{ij}(t+1) = x_{ij}(t) + K_j^p \cdot \frac{2}{\pi} \arctan(\frac{\varphi_j^p(x_{ij}(t) + \Delta^x) - \varphi_j^p(x_{ij}(t)}{\Delta^x}) \tag{14}$$

$$s_i(t+1) = x_i(t) + K_i^s \cdot \frac{2}{\pi} \arctan(\frac{\varphi_i^s(s_i(t) + \Delta^s) - \varphi_i^s(s_i(t))}{\Delta^s}), \tag{15}$$

where $K_j^p = r^p x_{ij}(t)$, $r^p < 1$, $K_i^s = r^s s_i(t)$ and $r^s < 1$.

In the case of large marginal profits the adjustment schemes (12) and (13) might lead to large fluctuations of the output levels of the firms, which make the system unstable. By introducing the inverse tangent function into the adjustment rules we make all output changes bounded, so large fluctuations become impossible.

In our former papers (Szidarovszky and Zhao 2009; Zhao and Szidarovszky 2008), we assumed a constant step in innovation increase $I_j(t) = 0.001$. In this paper however, we will study the effect of innovation step on the behavior of the firms, so we selected a similar updating rule of innovation:

$$I_j(t+1) = K_j^I \cdot \frac{2}{\pi} \arctan(\frac{\varphi_j^p(I_j(t) + \Delta^I) - \varphi_j^p(I_j(t))}{\Delta^I}) \tag{16}$$

with $K_j^I = r^I \cdot \widetilde{I}_j(t)$, $r^I < 1$.

4 The Simulation Process

4.1 *Parameters of the Model*

We have a total population of 25 agents, including 20 suppliers and 5 producer ($m = 20$ and $n = 5$). For any supplier i, we have the maximum price of $A_i = 300$ and marginal price $B_i = 1$ in Eq. (2). To represent the relatively low level of interaction between the suppliers in their prices, b_{il} is selected as 0.1 for $l \neq i$. That is, the suppliers specialize in different supplies, so their prices do not interfere with each other much. The parameters of the labor function (6) of the suppliers are chosen as $\gamma_i = 10$ and $\delta_i = 0.4$. For any producer j, the parameters of its production function are chosen as $a_{0j} = 20$, $a_{ij} = 0.1$ in Eq. (1). We also assumed much higher prices for final products than those of the supplies, so we select the common maximum price of the producers as $\overline{A}_j = 1300$ and similarly to the situation of the supplier agents, we have $\overline{B}_j = 1$ and $\overline{b}_{jl} = 0.1$. As the knowledge spillover is concerned, we consider 10% of the innovation as spillover, hence $k_{jl} = 0.1$ for $l \neq j$. Besides, $F_j^{max} = 2$ and $\omega_j = 0.1$ for the innovation dependent factor of Eq. (5). The parameters of the innovation development cost in Eq. (11) are selected as $u_j = 50$ and $v_j = 0.1$. The sizes of the producers are assumed to be larger than those of the suppliers, hence $\overline{\gamma}_j = 50$, $\overline{\delta}_j = 0.3$ and $\overline{\overline{\omega}}_j = 0.05$. For the labor market, we have the maximum labor price $c = 300$ and d is selected as 0.2 in Eq. (8).

In this paper, we will analyze only the effect of the decision rules. At the beginning of the simulation process, the initial values $x_{ij}(0)$ were generated randomly by using uniform distribution from the interval [0, 20]. The initial value of s_i is $s_i(0) = \sum_j x_{ij}(0)$; the corresponding values of z_j are calculated according to Eq. (1). The same set of the initial $x_{ij}(0)$ values was used in the same simulation group for comparison purposes. The initial value of technology level of all producers was chosen as 1.

There might be situations when prices, labors might become negative in the process, therefore these variables will be bounded from below. It is reasonable to assume that final products are sold for higher prices than supplies. The prices of final products are bounded from below by 5, those of the suppliers are bounded from below by 0. Usually, government has minimum wage policy; hence, for the whole system the price of labor is bounded by 10 from below.

4.2 *Simulation Results*

4.2.1 The Effect of Parameters of Gradient Adjustment

First we fixed the values of Δ^x and Δ^s as 10, changed ε^s from 0.1 to 1.1 with the step size of 0.2, and with each value of ε^s, ε^x varies from 0.1 to 2 with varying step sizes depending on the pattern changes in the behavior of the agents.

1. $\epsilon^s = 0.1$

When ε^x varies gradually from 0.1 to 1.79, the patterns of the behavior of the agents remain the same: fast increase or decrease at the beginning (this can be interpreted as the primitive formation of clusters) then the patterns converge or increase (decrease) slowly (Fig. 2). If we consider a time cross section with increasing value of ε^x, the output, labor usage and profit of both the suppliers and the producers increase and the average price of all firms and the labor price decrease. In the long run, the output and profit of the producers always increase, but when ε^x is small (< 1 in the figure) the labor usage is actually decreasing slowly in time. In this situation, the producers will not increase their firm sizes. As $\varepsilon^x < 1$, the profits of the suppliers also decrease slowly and when t is large enough, the profit might drop down to a negative value. Hence the supplier firms might shut down or sell their products outside the

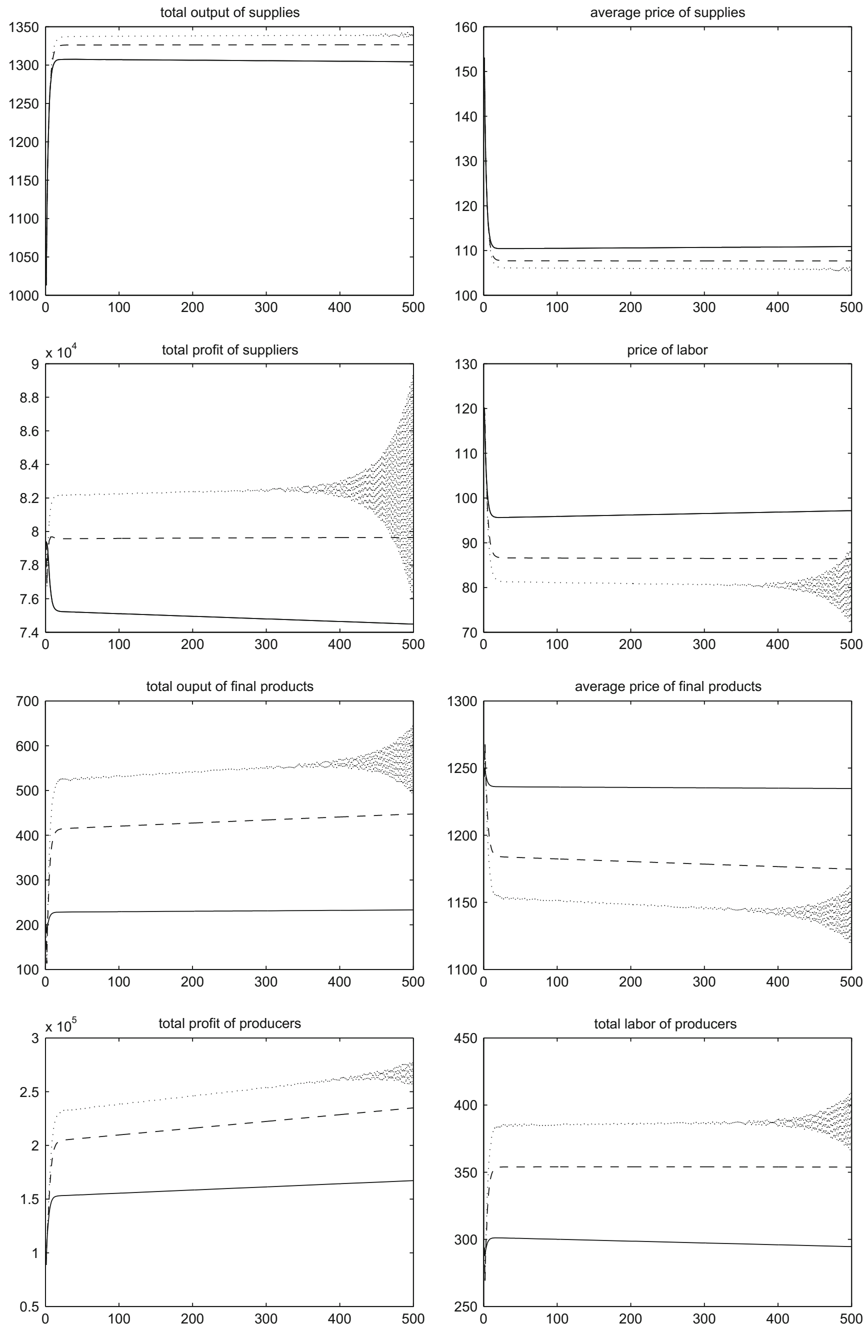

Fig. 2 Firms' behaviors when $\varepsilon^s = 0.1$, $\varepsilon^x = 0.1$ (*solid line*), 1 (*dashed line*), and 1.8 (*dotted line*)

cluster. In such situations, the cluster will never survive. Until ε^x is increased above 1, the labor of the producers and the profit of the suppliers keep a steady value after an increasing period. In this situation, it is hard to say that the exiting firms will expand.

From $\varepsilon^x = 1.795$, an oscillating behavior can be observed between two states (Fig. 2). That is, a two-period cycle emerges. The amplitudes of the oscillations become larger with time until they become stable. The oscillation starts after a linear pattern and its amplitude increases. Since we will have later other kinds of oscillation patterns, in order to distinguish between them, we call the oscillations just described as tail oscillations. Larger value of ε^x makes the oscillations start earlier in time and when ε^x becomes large enough, oscillations start almost at the beginning of the time scale, and the cycles will have more points. Hence for a stable system, when suppliers update their outputs as slowly as $\varepsilon^s = 0.1$, the producers should not choose large value of $\varepsilon^x(\varepsilon^x \geq 1.795)$. For a stable cluster that could stay, the range of ε^x should be $1 \leq \varepsilon^x < 1.795$.

2. $\varepsilon^s = 0.3$

Unlike higher values of ε^x which produce tail oscillations, higher values of ε^s induce behavior oscillation from the beginning of the time scale (we call this type oscillation as head oscillation) (Fig. 3). For $\varepsilon^x = 0.1$, there is a small oscillation at the beginning of time but the trajectories converge later. When ε^x increases, the amplitude and the length of the oscillating period become larger. If ε^x is larger than 1.4, then the trajectories do not converge anymore, and the shape of the time series looks like a dog bone as the result of the combination of the head oscillation and the tail oscillation. When $\varepsilon^s = 0.3$ and $\varepsilon^x < 1$, the long term behavior patterns are the same as those with $\varepsilon^s = 0.1$ and $\varepsilon^x < 1.795$.

Another impact of the higher value $\varepsilon^s = 0.3$ is that the tail oscillation patterns, which were induced by increasing values of ε^x, appear earlier than in the case of $\varepsilon^x = 0.1$. The two types of oscillations (head and tail) are combined again to the dog bone shape when ε^x is slightly larger than 1.4, and if ε^x becomes even larger, then the behavior oscillates between two stable states, forming a two-period cycle.

For $\varepsilon^s = 0.3$, smaller value of ε^x should be used to avoid the large amplitude oscillations. However, like in the case of $\varepsilon^s = 0.1$, with small value of ε^x, the small decreasing labor usage and decreasing profits imply that the cluster will not survive. Hence, for a surviving stable cluster, the range of ε^x should be $1 \leq \varepsilon^x \leq 1.3$.

3. $\varepsilon^s = 0.5$

The situation of $\varepsilon^s = 0.5$ is very similar to the case of $\varepsilon^s = 0.3$, however with larger amplitude of oscillation. The possible range of ε^x for a stable system is very narrow.

4. $\varepsilon^s \geq 0.7$

When $\varepsilon^s = 0.7$, the behaviors of both the suppliers and the producers oscillate irregularly, profits might drop down to negative values (Fig. 4). When ε^x increases, the producers production levels and profits also increase. When ε^x is increased to 1, the behaviors of the two types of agents converge, however the corresponding profits of suppliers become negative.

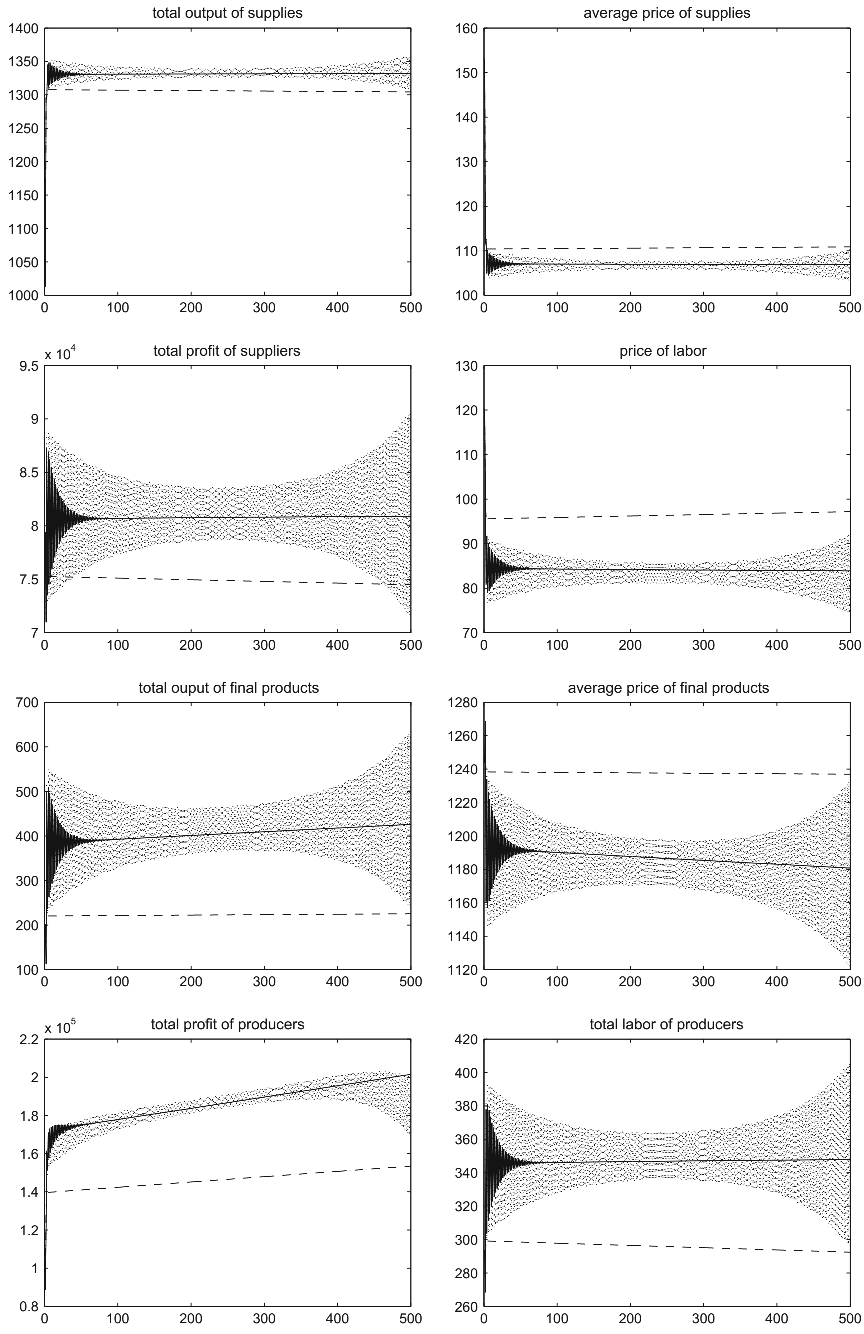

Fig. 3 Firms' behaviors when $\varepsilon^s = 0.3$, $\varepsilon^x = 0.1$ (*dashed line*), 1.3 (*solid line*), 1.4 (*gray dotted line*)

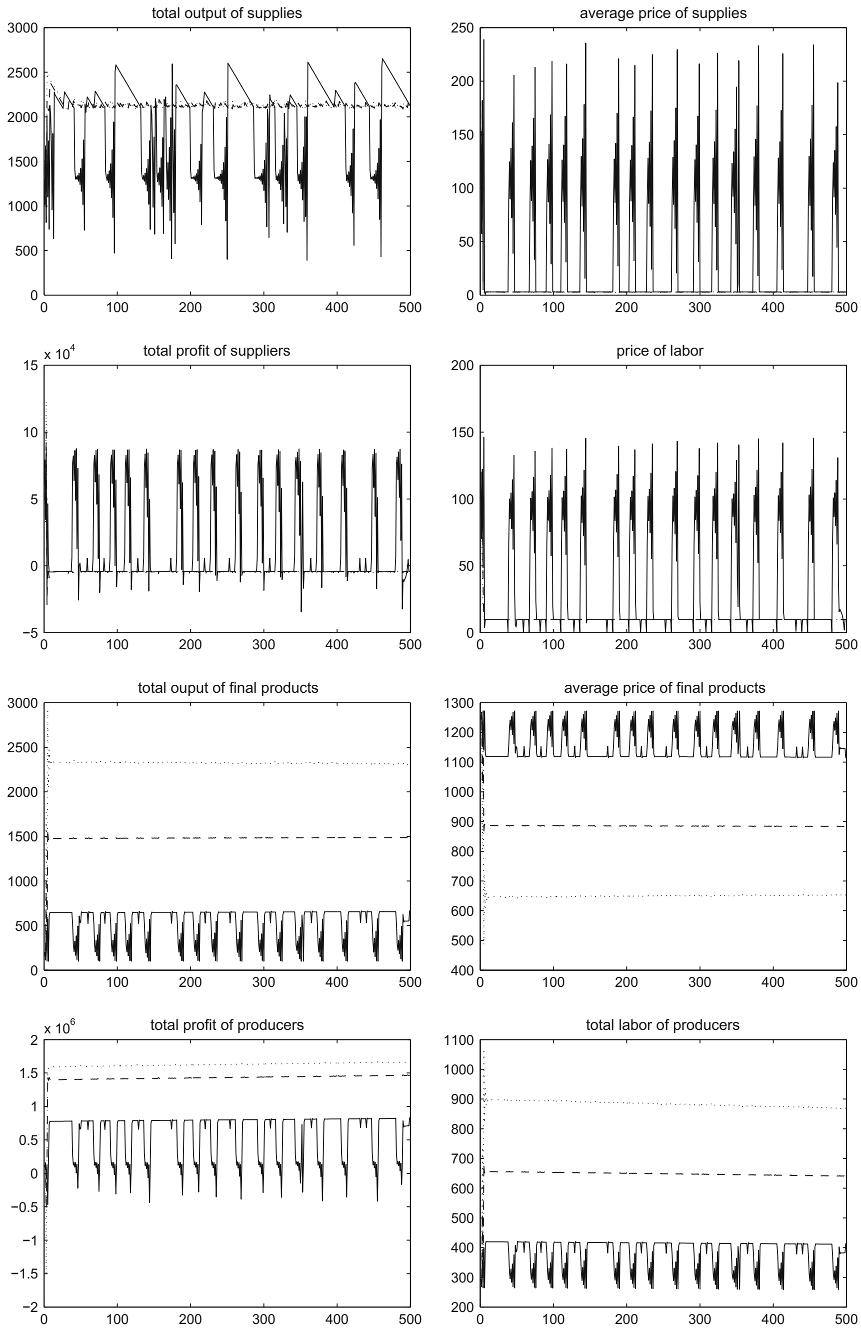

Fig. 4 Firm's behaviors when $\varepsilon^s = 0.7$, $\varepsilon^x = 0.3$ (*solid*), 1 (*dashed*), 2 (*dotted*)

When $\varepsilon^s = 0.9$, the patterns are similar to the previous case with the difference that when ε^x increased to 0.5, the behaviors of both types of agents converge, however the profits of the suppliers become negative.

When $\varepsilon^s = 1.1$, there are oscillations and sparks in the behaviors, they are never stable regardless of the value of ε^x (Fig. 5).

Overall, changes in the values of ε^x and ε^s have significant influence on the behaviors of the suppliers and the producers. The combination of the different values of ε^s and ε^x will generate many different patterns. Larger value of ε^s and larger value of ε^x induce unstable systems.

It is interesting to analyze the reason why oscillation is observed. When ε^s is very small, as 0.1, any increase of ε^x in a certain range will benefit the suppliers and the producers in the short-term, all make more profit even with decreased average prices. When ε^x is increased, then the behavior oscillates between two values. The amplitude of the oscillation increases in time through many iterations and then becomes stable. Our time scale is $0 \le t \le 500$. When $\varepsilon^x = 1.795$, oscillations emerge at the end of the time scale, and when $\varepsilon^x = 1.85$, oscillations emerge before time period 200. The reason is the following. When ε^x is increased, the outputs of the producers should also increase; this brings more labor to the cluster and decreases the labor price. This benefits the suppliers, increases their profits. Since the outputs and profits of the producers increase in time, until a certain time period, more and more outputs are produced, and when it accumulates to a certain value (when ε^x is large enough), the producers will over adjust their outputs, their profits decrease, then they adjust to the opposite direction. This drives the oscillations; the oscillation amplitude increases gradually until stable cycles occur.

If the updating step is too large, it will generate unstable behaviors, however if it is too small, then the firms development is also too slow. To form a stable cluster and also to keep the cluster for a longer time period, the values of ε^x and ε^s should be selected properly.

We also repeated the simulations for $\Delta^x = 1$. The pattern changes in the behavior of the firms were similar to those observed for $\Delta^x = 10$, only the critical values for pattern changes were slightly different.

4.2.2 Simulation Results with New Updating Rules

The combination of $\varepsilon^s = 0.1$ and $\varepsilon^x = 1$ is chosen as a benchmark for the comparison of the two different updating rules, the old rule (12)–(13), and the new rule (14)–(15). Figure 6 shows the results of the four different updating strategies: both types of agents use old updating rules and the innovation step is constant (dashed line); both types of agents use new updating rules and the innovation step is constant (dotted line); both types of agents use old updating rules but innovation increase uses new rule (dash-dot line); both types of agents use new updating rules and innovation increase uses new rule (solid line). In the new updating rules, $\Delta^x = 1$, $r^p = r^s = r^I = 0.1$. Even though the selected value of Δ^x is large, say 10, the behaviors of the firms become much smoother than before, only small oscillations within a small range can

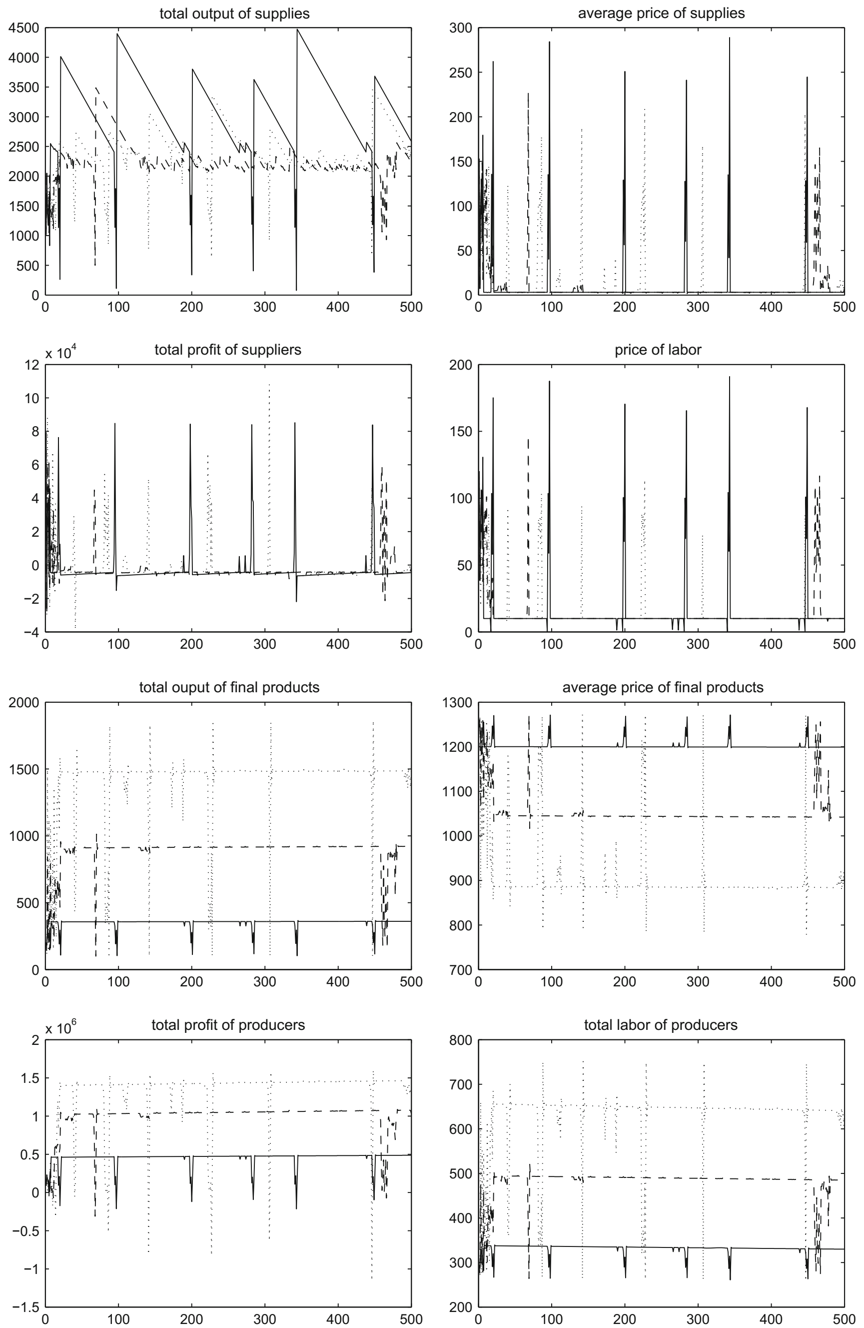

Fig. 5 Firms' behaviors when $\varepsilon^s = 1.1$, $\varepsilon^x = 0.1$ (*dashed line*), 0.5 (*solid line*), 1(*gray dotted line*)

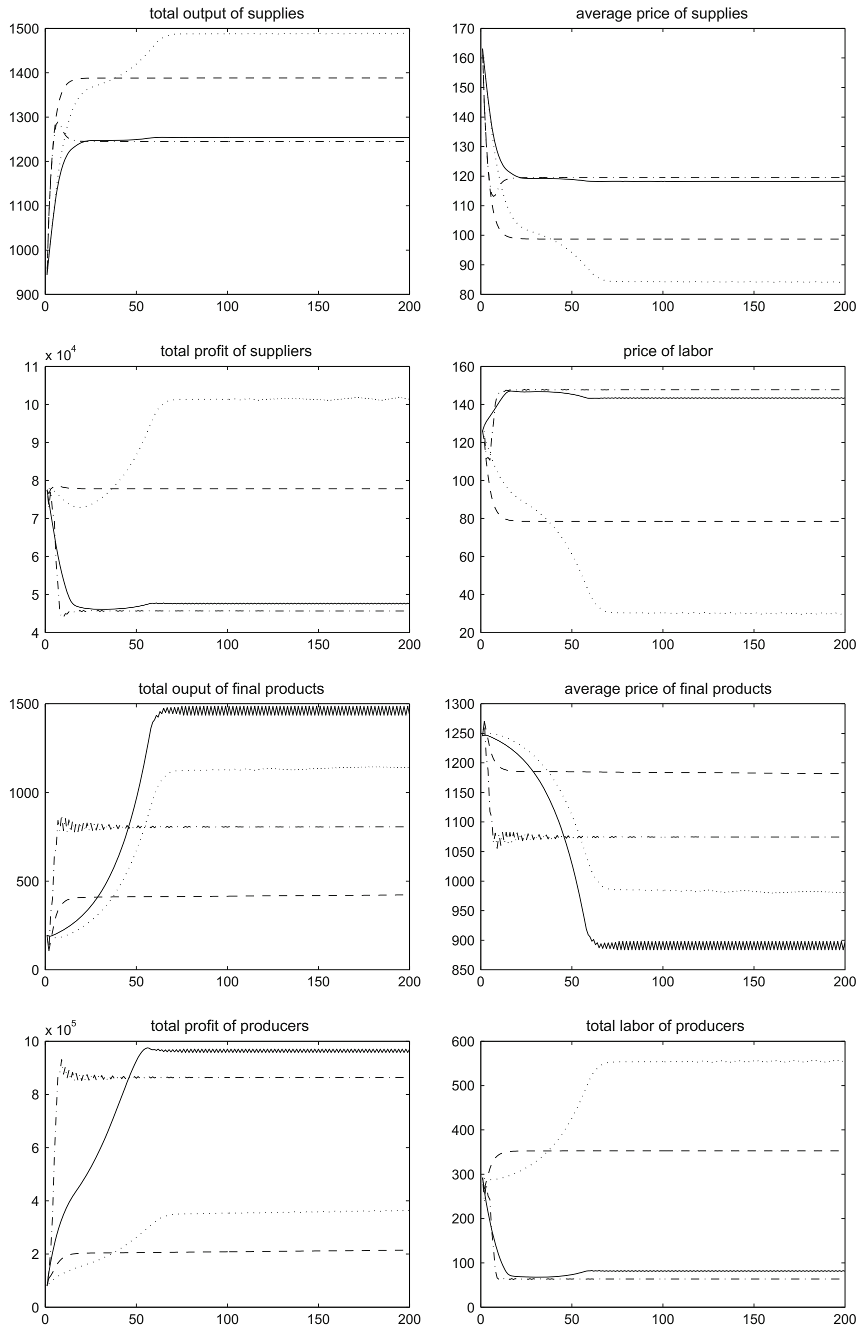

Fig. 6 Comparison of patterns of aggregated behavior by using different updating rules. *Dashed lines* old update rule; *dotted lines* x and z updated using new rule; *dash-dot lines* only I was updated using new rule; *solid lines* all use new rule

be observed. It is surprising to see that the new updating rule of the innovation step will harm the suppliers no matter the agents adopt new output strategy or not (since the suppliers' profits are decreased and the average prices are increased). Hence a stable innovation step is a relatively good choice. From the simulation results we have the main conclusion that to bring existing firms into a stable cluster, both types of agents should adopt new output strategies.

5 Conclusion

This paper presents an integrated model of agent-based simulation and network oligopoly to study the evolution of a group of local firms for the possibility of forming a long lasting industrial cluster. Agent-based simulation model is used to study the effect of firms' decisions on the formation of the cluster, and network oligopoly theory is used to model the decisions and interaction rules of the agents. We studied the very simple situation when the firms only concern is their marginal profits and their decisions are their productivity and innovation investments. Firms interact through the product market and the secondary market of labor. The structure of the system is similar to Markusen's 'Hub and Spoke' type of cluster. From the simulation results we demonstrated that under some production decision rules the group could have the potential to involve into a surviving cluster. This paper offers a starting point to study the cluster formation from exiting firms. More complicated situations will be considered in our future research. We considered only fixed network with existing firms. For investigating the growth of the cluster, innovations disperse and relationship establishment, dynamic spatial networks have to be used. This task will be the topic of our future work.

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