Parametric Modeling of Kinetic-Kinematic Polycentric Mechanical Knee

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Abstract— **The transfemoral amputation involves the loss of the knee joint, which is recognized as a common and complex case. The knee is replaced by a polycentric mechanism, which is exposed to high levels of structural stress. Therefore, mathematical models of the mechanics knees are commonly used to kinetic analysis and simulation and determine possible failures. This paper describes the procedure for determining a kinematic model of a four-bars polycentric knee using a geometric analysis and the Grashof Law for a double rocker. The kinetic model was found using parametric, linear and nonlinear identification techniques, for this were used knee force and angle data supplied by the free database Orthoload. The model couples the kinetics and kinematics ARX structure, these can represent bending angles 110 ° and the total force exerted on the instantaneous center of rotation of the knee.**

Keywords— **Biomechanics, kinematics, nonlinear dynamical systems, prosthetics, prosthetic limbs, system identification.**

I. INTRODUCTION

The amputation is a progressive problem associated with different causes like work or traffic accidents [1], diseases [2] and war [3]. Regardless of the cause, unilateral lower limb amputation is a common case [4], assorted according to amputation height [5].

The transfemoral prosthesis is a mechanical assistance device for above knee (AK) amputees. This prosthetic system consists of five parts: prosthetic foot, shank, knee unit, socket and suspension [6]. Each of these elements is subjected to different mechanical stresses decreasing the life cycle of elements, affecting mobility and patient confidence [7]. The mechanical knee facilitates patient mobility, improves gait performance and stability of the amputee during their activities of daily living [8]. The prosthetic knees are characterized by monocentric and polycentric mechanisms, providing greater or lesser flexion and extension of the joint [9].

The polycentric knees have different rotation axes, which converge in the instant center of rotation (ICR). These knees are used by it's biomechanics versatility, presenting good performance in the stance and swing phase gait, allowing greater confidence, stability, bending and factly.

A typical four bar knee is shown in Fig 1. This is characterized by four center cranks A, B, OB y OA linked by OA-A, A-B, B-OB, OA-OB, conform 4 links a, b, c, d respectively. Mechanical linkage configuration is based on Law of Grashof mechanisms, where the length of each link is determined to have a full joint revolution [10].

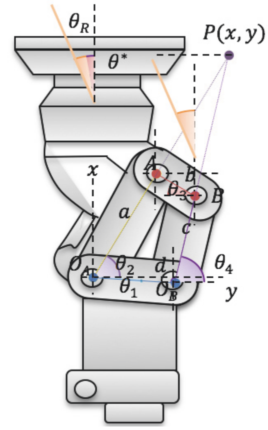

Fig 1. Extended knee geometry: links, articulation angles and instant center of rotation.

The knee should support the weight of the body and additional loads during gait cycles. The resulting forces stabilize the body and achieve the displacement during the phases of gait. If known the knee force magnitude and direction, is possible to choose a correct construction material, simulate the gait performance and estimate possible structural problems.

Two forms of analysis are discussed in this paper, kinematic and kinetic. To find kinetic model, the study is conducted with data from healthy patients, as suggested in [11]. Data is collected from 8 test subjects with an average weight of 100kg and different system identification techniques are evaluated. To find kinematic model, a geometric analysis of four- bar mechanism is done. The resulting model allows provides information about the ICR, marked with a *P(x,y)*.

Mohsen and colleagues [12] designed a method to parameterize a polycentric knee four-bars using the ICR and the floor reaction forces of (FRF) during gait (the initial contact, the support phase and voluntary bending before takeoff foot). To determine an appropriate measures and angles, a genetic algorithm uses FRF data of an amputee patient to parameterize the model of the knee.

The four-bars knee model and the optimal geometry of the joints are presented in [13]. They use the Grashof Law, the

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transmission angle criteria and the angle sequence. Has been used an optimization method known as musical harmony, which is based on better state of harmony.

A general model for the simulation of polycentric knee has not been studied considerably. Therefore, the paper also suggests applicate a system identification.

II. Methodology

A. Kinematic Analysis

The analysis of four-bar mechanism is performed following the geometry presented in Fig 1. The *a*, *b*, *c*, *d*, variables are lengths of the links, A, B, O_B, O_A depict joints.

The lower link d, corresponds of the mechanical knee distal view part (shank connection). The top link b connected to the proximal part (socket connection). The alignment angles are defined by θ_1 and θ^*. The variable θ_R represents the angle of the knee or the system input. It can be calculated as $\theta_3 = \theta_R + \theta^*$. By a geometric analysis, θ_2 and θ_4 angles can be describes in terms of the joints and some intermediate variables as (1) and (2) . In some cases, these equations may be indeterminate, so must include restrictions in evaluating simulation.

$$\theta_2 = 2tan^{-1}\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \quad (1)$$

$$\theta_4 = 2tan^{-1}\left[\frac{-B \pm \sqrt{E^2 - 4DF}}{2D}\right] \quad (2)$$

Where,

$$A = \frac{K_2}{2ab} - \frac{dK_1}{a} - cos(\theta_3) + \frac{dcos(\theta_1)}{b} \quad (3)$$

$$B = 2\left[sin(\theta_3) - \frac{dsin(\theta_1)}{b}\right] \quad (4)$$

$$C = \frac{K_2}{2ab} - \frac{dK_1}{a} - \frac{dcos(\theta_1)}{b} + cos(\theta_3) \quad (5)$$

$$E = 2\left[\frac{dsin(\theta_1)}{b} - sin(\theta_3)\right] \quad (6)$$

$$D = \frac{K_3}{2bc} - \frac{dK_1}{c} - \frac{dcos(\theta_1)}{b} + cos(\theta_3) \quad (7)$$

$$D = \frac{K_3}{2bc} - \frac{dK_1}{c} + \frac{dcos(\theta_1)}{b} - cos(\theta_3) \quad (8)$$

$$K_1 = cos(\theta_1)\,cos(\theta_3) + sin(\theta_1)\,sin(\theta_3) \quad (9)$$

$$K_2 = a^2 + b^2 - c^2 + d^2 \quad (10)$$

$$K_3 = b^2 - a^2 + c^2 + d^2 \quad (11)$$

Finally, joint points A, B, O_A and O_B can be calculated as,

$$O_A(X_{O_A}, Y_{O_A}) = [0,0] \quad (12)$$

$$O_B(X_{O_B}, Y_{O_B}) = [dcos(\theta_{!})dsin(\theta_{!})] \quad (13)$$

$$A(X_A, Y_A) = [acos(\theta_2) - asin(\theta_2)] \quad (14)$$

$$B(X_B, Y_B) = [X_{O_B} + ccos(\theta_4), Y_{O_B} + csin(\theta_4)] \quad (15)$$

The $P(x, y)$ point is located where the projections of the *a* and *c*, converge.

$$P_x = X_B + \frac{K_4}{K_5} - dcos(\theta_1) - ccos(\theta_4) \quad (16)$$

$$P_y = Y_B + \frac{K_4}{K_5}\tan(-\theta_2) - dsin(\theta_1) - csin(\theta_4) \quad (17)$$

A double swing is simulated on Matlab, making the shortest link "*b*" and the base "*d*" are fixed, fulfilling the condition of the second law of Grashof (*b+a* ≤ *c+d*). For simulation, *a=6cm*, *b=1cm*, *c=5.5cm* and *d=3cm* were chosen. An initial upper alignment angle (top of the knee) was considered by $\theta^* = -5°$ and lower alignment angle by $\theta_2 = -5°$. The model was tested with θ_1 = [-5º,-10º,-20º], simulating alignment changes on the socket and shank, then *P(x,y)* movements were evaluated.

B. Systems Identification

For model identification, the Orthoload data have been used without commercial use [14]. The data include information of 8 patients six men and two women, aged about 70 years, height 172cm, weight 91kg. Each patient flexed knee, the angle (°) and forces (N) were measured in each coordinate axis, then the joint total force was estimated. To eliminate weight patient variability, a proportional value of the force was found. The procedure of data processing used by Orthoload, includes a standardizes medial forces F_X, anterior F_Y, distal F_Z and resultant force F_{Res} about 100kg following the "dynamic time warping (DTW)" [15]. This procedure is a normalized time signal, a distorted of time signal, so that the summed squared errors between all of them become a minimum. Then, resulting signals are statistically treated obtaining different output signals: arithmetic mean, median, minimum, maximum, 25 percentiles, 75 percentiles. In this paper averages of one individual subject is used. Details of procedure are described in [16]. Data of resulting force and bending angle are seen in Fig 2.

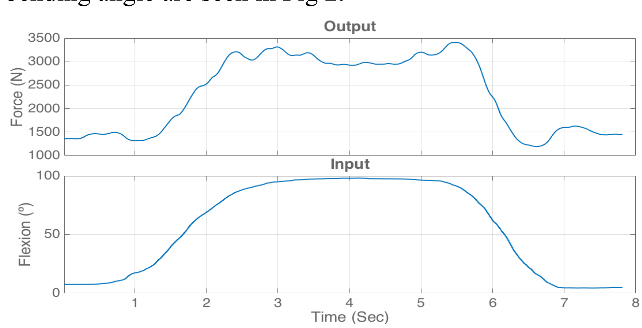


Fig 2. Resulting force is used as experimental input and flexion-extension used as experimental output.

Treatment of the data is performed before making the identification. First, an analysis is performed to determine frequency noise effects. In order to eliminate offset, the average values of the output signal and input were removed.

The linear polynomial structures tested were: ARX, ARMAX, OE and BJ. Then, a nonlinear Hammerstein-Wiener and nonlinear ARX. Nonlinear ARX model consists of a regressors block, a linear block and non-linear block. The Hammerstein-Wiener model consists of a series of linear block between two non-linear blocks. The best structure and order is performed from minimization of final prediction error criteria (18), the method of normalized Akaike criteria (19) and the adjustment percentage.

$$FPE = det\left(\frac{1}{N}\sum_{1}^{N} EE^{T}\right)\left(\frac{1+d/N}{1-d/N}\right) \quad (18)$$

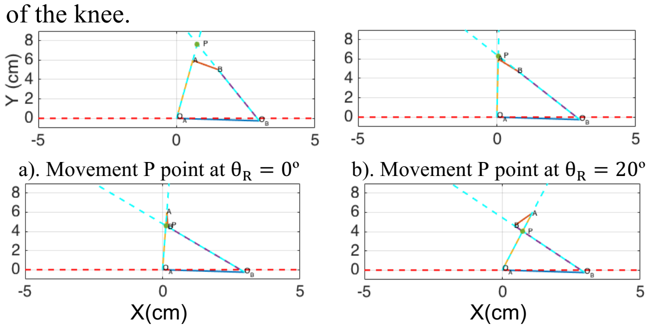
$$AICc = log\left(det(E^{T}E/N)\right) + 2\,d/N \quad (19)$$

Where E is the prediction error matrix, d is the number of estimated parameters, N is the number of estimation data samples.

III. RESULTS

A. Kinematic Model.

A knee flexion between $\theta_R = 10^o$ and $\theta_R = 140^o$ is applied. Six transitions of kinematic model simulation are showed in Fig 3. The P(*x,y*) describe an rotation angle between 10º and 140º is. This allowed to estimate the movement of the knee.



a). Movement P point at $\theta_R = 0^o$ b). Movement P point at $\theta_R = 20^o$

c). Movement P point at $\theta_R = 60^o$ d). Movement P point at $\theta_R = 140^o$

Fig. 3. Flexion knee on θ_3 joint. Displacement of instantaneous center of rotation.

B. System Identification Techniques

a) Linear parametric identification

Concerning linear structures, 1600 different ARX models varying parameters for $n_a = (1:10)$, $n_b = (1:10)$ and $n_k = (0:15)$ were tested. The models that minimized the two criteria (equations 18 and 19), FPE=0.69 and AIC=-0.29.

In the case of ARMAX structure, 2000 combinations were evaluated: $n_a = (1:5)$, $n_b = (1:5)$, $n_c = (1:5)$ and $n_k = (0:15)$. The minimizing of FPE = 0.74 and AIC = -0.30.

In the Output Error (OE) model, 1600 combinations of parameters $n_b = (1:10)$, $n_f = (1:10)$ and $n_k = (0:15)$. The FPE and AIC values were 1.49 y 9.61 respectively.

Concerning Box Jenquins (BJ) structure, 10.000 combinations were evaluated: $n_b = (1:5)$, $n_c = (1:5)$, $n_k = (1:5)$, $n_f = (1:5)$ and $n_k = (0:15)$, the minimization was achieved for FPE=0.74 and AIC=-0.31.

The minimizing criteria was achieved for each model (ARX, ARMAX, OE and BJ) with the same order for each one. Simulation of each model are presented in the figure 4.

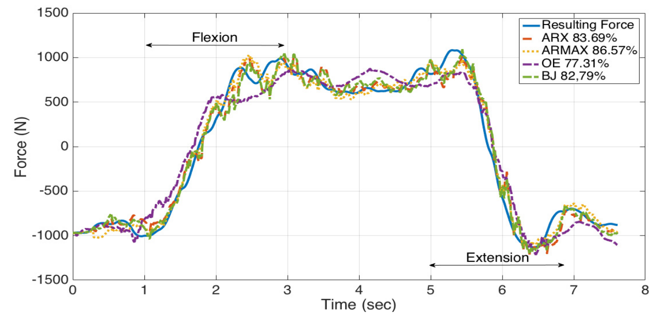


Fig 4. Linear models response. Minimizing values EFP and AIC.

b) Nonlinear identification

Model parameter of nonlinear ARX were varied in 3100 combinations for $n_a = (1:10)$, $n_b = (1:10)$ and $n_k = (0:30)$. The FPE=0.71 and AICc = -0.34. For the the Hammerstein-Wiener model was avaluated for 170 combinations of n_a=n_b=(1,3,5,7,9) and n_k=(0,5,10,15,20,25,30). The FPE=5.18 and AIC=8.55. The simulation models are present in the fig 5.

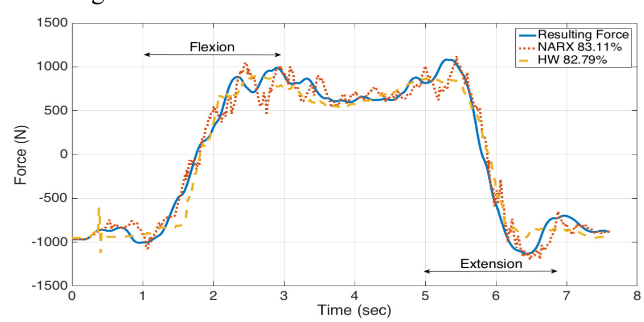


Fig 5. Non linear models response. Minimizing values EFP and AIC.

c) Model selection

The models were compared as shown in Table 1.

Table 1 Evaluation of models identified

MODELS	FPE	AIC	Mean error	Parameters*	FIT (%)
ARX	0,69	-0,29	1,72	(4,2,-,-,-,2)	83,69%
ARMAX	0,74	-0,30	-3,87	(5,3,1,-,-,2)	86,57%
OE	1,49	9,61	-2,05	(-,9,-,-,4,0)	77,31%
BJ	0,74	-0,31	-0,51	(-,1,1,5,3,2)	84,52%
NARX	0,71	-0,34	-2,65	(10,7,-,-,-,13)	83,11%
Ham.-Wiener	5,18	8,55	24,97	(-,5,-,-,5,30)	82,79%

The parameters row present the values of *na*, *nb*, *nc*, *nd*, *nf* and *nk*. The model that best adjusted all selection criteria in relation to minimizing information criterion, the average error of the order and maximizing the fit to the data was the ARX.

The continuous time model that describing the force exerted on the knee joint is presented in equation 20.

$$\frac{F_{Res}(s)}{\theta_R(s)} = e^{-0.013s}\frac{10.5s^3 + 2767s^2 + 232000s + 589600}{s^4 + 60.6s^3 + 1739s^2 + 9710s + 32100} \quad (20)$$

To found the ICR force (F_P), the P point angle is calculated by $\theta_P = cos(X_P, Y_P)$. This is the input to dynamic model, then ICR force is estimated.

IV. DISCUSSION

The resulting model simulates a prosthetic knee joint permitting a range of performance of 110º, with variations between 10º and 120°. The angle of knee $\theta_R(t)$ is the input of the model, generating the instantaneous center of rotation (ICR), which, added to the estimate of the value of the force exerted on the joint (found by identification), provides information of a *P(x,y)* angle and force.

The resulting overall model can simulate knee flexion of 110° and estimating parameters of strength in the joint. The results of this simple model can be used in the selection of building materials knees (in patients up to 120 kg), possible structural damage, in the simulation of amputee's gait deviations, and even in the simulation of humanoid robots, with an estimate of 83.69%. of the data.

Unlike other models found by Ortho Care, this allows to determine angle and strength, allowing be used for the design of knee prosthesis.

V. CONCLUSIONS

The resulting model allows simulations of possible movement made by the amputee and determine the forces that will undergo construction materials of mechanical knee.

The right balance between the different criteria for selecting a parametric model for system identification, to find a tight model representing multiple system states.

Should properly choosing the links lengths for proper knee simulation.

The use of two sub-models in series allows adjustments to each of these, facilitating the simulation work.

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this paper.

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