Chapter 10
Based on an Improved Sliding Mode Observer for Position Estimation of PMSM

Su Zhou, Daxiang Zhu and Zhe Hu

Abstract To reduce the chattering problem resulted from signum function in conventional sliding mode observer (SMO), a sensorless speed controller based on an improved SMO is proposed for permanent magnet synchronous motor (PMSM), where signum function is substituted for sigmoid function and the stability of the proposed SMO is verified using the Lyapunov method, the influence of different tilt parameters on position estimation is analyzed as well. To reduce phase delay in position estimation in conventional SMO, a new observer which has the structure of an extended Kalman filter is designed for back electromotive force estimation, which is expected to decrease estimation error of improved SMO. Based on improved SMO position sensorless control is modeled in MATLAB/SIMULINK, and result of simulation demonstrates correctness of improved SMO.

Keywords Position estimation · SMO · EKF · Chattering problem

10.1 Introduction

PMSM has been widely used in electric vehicle systems for its many advantages Compared with induction motor. Vector control is mainly used as a control method for PMSM [1], its core idea is that alternating stator current in stator reference frame is divided into exciting current component and torque current component in rotating coordinate system through coordinate transformation, so accurate rotor position is needed. The reliability of the position sensor is greatly reduced under poor and complex work environment of automobile motor, so sensorless control is expected. The position and speed of the rotor can be estimated using some electrical signals related to the motor windings through appropriate means. Several different

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sensorless techniques have been developed, these techniques can be divided into several categories: an injecting-signal method utilizing motor space saliency has been used to detect the rotor position of the PMSM in low speed and standstill [2, 3]. This injecting-signal method is not affected by the motor speed and load. By suitably choosing a high-frequency stator voltage signal or current signal, the rotor position and speed can be precisely obtained. This method, however, needs a high-frequency signal generator. In addition, a band-pass filter is required to process and detect the signal which is related to the rotor position. The second is state observer method, directly or indirectly extracted from the motor back-EMF position information [4, 5], which is mostly used in surface mount and buried permanent magnet synchronous motor for excellent dynamic performance.

The sliding mode controller for improving the robustness of the controllers has been proposed for some time. While in the sliding mode, these controllers are insensitive to parameter variations and disturbances. Therefore, the sliding mode observer has been presented as a robust estimation method. To reduce the chattering problem of sliding mode control resulted from signum function, reaching law has been presented, besides, filter and phase compensation element is reduced [6]. The cascade control method has been proposed for the achievement of an accurate tracking performance under an unknown motor and load parameter [7]. Signum function has been substituted for sigmoid function and the chattering problem has been effectively weakened by selecting a reasonable boundary layer thickness, but the speed estimation and close-loop control haven't been further studied [8]. Non-linear system state observation has been implemented by combining high-gain observers with sliding mode control under disturbance [9]. Sliding mode observer combining with software PLL has been proposed to achieve sensorless drive for a PMSM, estimated induced EMF is used as input of software PLL to estimate position and speed, but rotor in still position can't be estimated [10]. On the basis of the literature [10], the adaptive control algorithm has been proposed to estimate the rotor angle estimation error at different speeds through the implementation of adaptive coefficients [11].

Depending on the theory of variable structure, an improved sliding mode observer is designed based on sigmoid function and the Kalman filter. In a conventional sliding mode observer, a low-pass filter and an additional position compensation of the rotor used to reduce the chattering problem are commonly found in SMO using a signum function. Currently, a sigmoid function is used for the SMO as a switching function. Use of LPF in conventional SMO leads to phase delay during computing rotor position. To solve this problem, a Kalman observer is designed to extract the back EMF, so rotor position estimation error can be decreased. This paper proposes a new sensorless control algorithm based on the improved SMO for SPMSM and the superiority is proved by comparing with the conventional SMO through simulation. The Schematic of improved SMO is shown in Fig. 10.1.

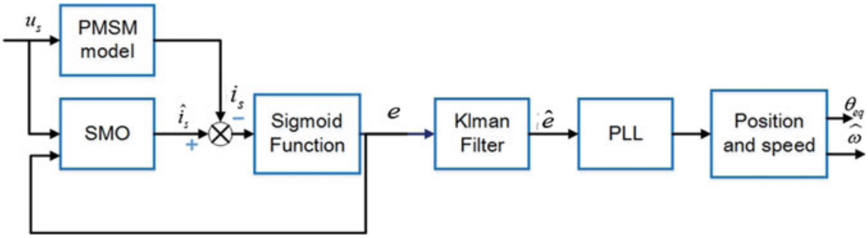


Fig. 10.1 Schematic of improved SMO

10.2 Sliding Mode Observer

10.2.1 Improved Sliding Mode Observer

For PMSM sensorless control, the rotor position is estimated so the stator equations can be used to model the system. The state equations, where the stator current is a state variable of the stationary frame voltage equation, can be represented as

$$\begin{aligned} \frac{di_\alpha}{dt} &= -\frac{R}{L}i_\alpha + \frac{1}{L}u_\alpha - \frac{1}{L}e_\alpha \\ \frac{di_\beta}{dt} &= -\frac{R}{L}i_\beta + \frac{1}{L}u_\beta - \frac{1}{L}e_\beta \end{aligned} \tag{10.1}$$

where i_α, i_β represent the stator current for each phase, u_α, u_β is the stator voltage of each phase, e_α, e_β is the electromotive force of each phase and R, L represent the stator resistance and inductance respectively. The electromotive force for each phase can be represented in the stationary frame as

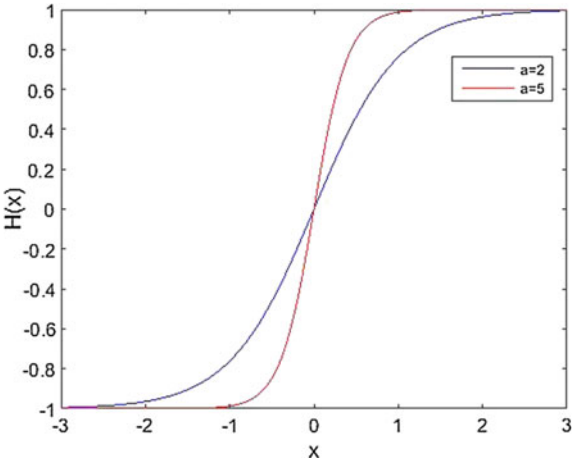
$$\begin{aligned} e_\alpha &= -\omega_e\psi_f \sin\theta \\ e_\beta &= \omega_e\psi_f \cos\theta \end{aligned} \tag{10.2}$$

where ψ_f, ω_e and θ represent the magnetic flux of the PM, the electric angular velocity, and the rotor angle, respectively. It is inferred from Eq. (10.2) that permanent magnet synchronous motor rotor position and speed can be extracted from back-EMF, thereby sensorless control can be realized.

To eliminate the undesirable chattering, sigmoid function is adopted in this research as the switching function. The improved sliding model observer equation in the stationary reference frame as follows:

$$\begin{aligned} \frac{d\hat{i}_\alpha}{dt} &= -\frac{\hat{R}}{\hat{L}}i_\alpha + \frac{1}{\hat{L}}u_\alpha - \frac{K_s}{\hat{L}}H(\bar{i}_\alpha) \\ \frac{d\hat{i}_\beta}{dt} &= -\frac{\hat{R}}{\hat{L}}i_\beta + \frac{1}{\hat{L}}u_\beta - \frac{K_s}{\hat{L}}H(\bar{i}_\beta) \end{aligned} \tag{10.3}$$

Fig. 10.2 Relationship between slope and tilt parameter



where $\hat{i}_\alpha$, $\hat{i}_\beta$ represent the estimated stator current for each phase, $\hat{R}$, $\hat{L}$ is the estimated value of R, L respectively. $\bar{i}_\alpha = i_\alpha - \hat{i}_\alpha, \bar{i}_\beta = i_\beta - \hat{i}_\beta$ denotes the estimation errors of the stator current. The new SMO resolves the problems of the conventional SMO by using a sigmoid function as the switching function. The sigmoid function is represented as

$$H(x) = \frac{1 - e^{-ax}}{1 + e^{-ax}} \tag{10.4}$$

where parameter a is used to change the slope of sigmoid function. Figure 10.2 shows how the slope is changed depending on the tilt parameter.

When $\hat{R} = R$, $\hat{L} = L$, estimation errors of the stator current equation can be configured

$$\begin{aligned} \frac{d\bar{i}_\alpha}{dt} &= -\frac{R}{L}i_\alpha + \frac{1}{L}e_\alpha - \frac{K_s}{L}H(\bar{i}_\alpha) \\ \frac{d\bar{i}_\beta}{dt} &= -\frac{R}{L}i_\beta + \frac{1}{L}e_\beta - \frac{K_s}{L}H(\bar{i}_\beta) \end{aligned} \tag{10.5}$$

10.2.2 Stability Analysis

The sliding surface $S(x)$ can be defined as functions of the errors between the actual current, i.e., i_α and i_β, and the estimated current, $\hat{i}_\alpha$ and $\hat{i}_\beta$, for each phase as follows:

$$S(x) = [\bar{i}_\alpha \quad \bar{i}_\beta]^{\mathrm{T}} = [\hat{i}_\alpha - i_\alpha \quad \hat{i}_\beta - i_\beta]^{\mathrm{T}} \tag{10.6}$$

When sliding mode is reached, i.e. estimation errors are on the sliding surface, estimation errors become zero. At that moment, the sliding surface becomes $S(x)$ and the observer becomes robust against the system parameters variation and disturbances. The Lyapunov function can be defined as

$$V = \frac{1}{2}S(x)^T S(x) = \frac{1}{2}(\bar{i}_\alpha^2 + \bar{i}_\beta^2) \tag{10.7}$$

Taking the time derivative of (10.14), and substitute (10.13) into

$$\dot{V} = -\frac{R}{L}(\bar{i}_\alpha^2 + \bar{i}_\beta^2) + \frac{1}{L}(e_\alpha \bar{i}_\alpha - K_s \bar{i}_\alpha H(\bar{i}_\alpha)) + \frac{1}{L}(e_\beta \bar{i}_\beta - K_s \bar{i}_\beta H(\bar{i}_\beta)) \tag{10.8}$$

where the observer gain can be derived to satisfy the inequality condition as:

$$K_s > max(|e_\alpha|, |e_\beta|) \tag{10.9}$$

10.2.3 Kalman Observer

The system behavior can be examined by apply equivalent control method. The back EMF can be expressed in the form derived from $S(x) = \frac{dS(x)}{dt} = 0$

$$e_\alpha = (K_s H(\bar{i}_\alpha))_{eq}, \quad e_\beta = (K_s H(\bar{i}_\beta))_{eq} \tag{10.10}$$

The back EMF is high frequency switch signal, a first order LPF is used to obtain smooth back EMF in conventional SMO, which leads to phase delay during computing rotor position. To solve this problem, a Kalman observer is designed to extract the back EMF, so rotor position estimation error can be decreased [12]. The state equation is as follows:

$$\begin{aligned} \frac{d\hat{e}_\alpha}{dt} &= -\hat{\omega}_e \hat{e}_\beta - K_l(\hat{e}_\alpha - z_\alpha) \\ \frac{d\hat{e}_\beta}{dt} &= -\hat{\omega}_e \hat{e}_\alpha - K_l(\hat{e}_\beta - z_\beta) \\ \frac{d\hat{\omega}_e}{dt} &= (\hat{e}_\alpha - z_\alpha)\hat{e}_\alpha - (\hat{e}_\beta - z_\beta)\hat{e}_\beta \end{aligned} \tag{10.11}$$

K_l is an observer gain.

The motor speed changes slowly comparing with electrical signals, since the motor mechanical time constant is much greater than the electrical time constant, a reasonable assumption that motor speed is constant can be made. The model of these induced back EMF is

$$\dot{e}_{\alpha} = -\omega_e e_{\beta}, \quad \dot{e}_{\beta} = -\omega_e e_{\alpha} \tag{10.12}$$

equations as follow derivate from (10.11), (10.12),

$$\begin{aligned} \frac{d\bar{e}_{\alpha}}{dt} &= \omega_e e_{\beta} - \hat{\omega}_e \hat{e}_{\beta} - K_l(\hat{e}_{\alpha} - e_{\alpha}) \\ \frac{d\bar{e}_{\beta}}{dt} &= \omega_e e_{\alpha} - \hat{\omega}_e \hat{e}_{\alpha} - K_l(\hat{e}_{\beta} - e_{\beta}) \\ \frac{d\bar{\omega}_e}{dt} &= (\hat{e}_{\beta} - e_{\beta}) e_{\alpha} - (\hat{e}_{\alpha} - e_{\alpha}) e_{\beta} \end{aligned} \tag{10.13}$$

where $\bar{e}_{\alpha} = \hat{e}_{\alpha} - e_{\alpha}$, $\bar{e}_{\beta} = \hat{e}_{\beta} - e_{\beta}$, $\bar{\omega}_e = \hat{\omega}_e - \omega_e$ are observer errors. The Lyapunov function is defined as

$$V = \bar{e}_{\alpha}^2 + \bar{e}_{\beta}^2 + \bar{\omega}_e^2 \tag{10.14}$$

Taking the time derivative of (10.14), and substitute (10.13) into

$$\dot{V} = -K_l(\hat{e}_{\alpha}^2 + \hat{e}_{\beta}^2) \leq 0 \tag{10.15}$$

It is proved that the estimates $\hat{e}_{\alpha}$ and $\hat{e}_{\beta}$ tend to e_{α} and e_{β} asymptotically by Lyapunov stability theory.

10.2.4 Position Estimation

Chattering problem arises when SMO enters sliding mode, the estimated back EMF is high frequency switch signal. Chattering problem can't be eliminated in rotor angle estimation based on arctangent function. Thus, the method of rotor angle estimation based on Phase Locked Loop (PLL) for PMSM is proposed in this paper, which has some advantages in application of rotor angle estimation. Method of rotor angle estimation based on PLL has a simple structure and is easy to implement, furthermore, it has higher speed astringency, which could satisfy the requirement of high real time characteristic for PMSM driving system.

From Eq. (10.2),

$$-e_{\alpha} \cos\theta = e_{\beta} \sin\theta \tag{10.16}$$

rotor angle can be derived from estimated back EMF and integration of speed estimated by PLL. Define the error as:

$$\varepsilon = -e'_{\alpha} \cos\theta' - e'_{\beta} \sin\theta' \tag{10.17}$$

Table 10.1 Parameters of the PMSM

Rated power/kW	3	Stator resistance/Ω	2.875
Rated line-to-line voltage/V	300	d-axis stator inductance/mH	8.5
PM flux/Wb	0.175	q-axis stator inductance/mH	8.5
Pole pairs	4	Rotor inertia/Kg*s	0.001

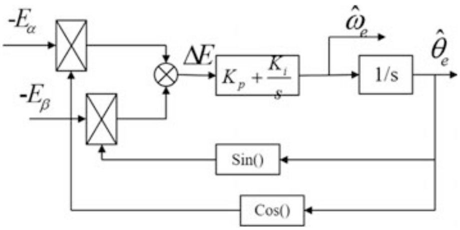


Fig. 10.3 Schematic of PLL

Estimated electrical rotor speed is obtained by PI regulator and estimated rotor angle is integration of estimated electrical rotor speed. Figure 10.3 demonstrates the principle of PLL.

10.3 Simulation Results

Based on field orientation control (FOC) and SVPWM technology, PMSM sensorless speed controller has been modeled in MATLAB/SIMULINK. To control the PMSM, the three-phase coordinates need to be transformed into rotational synchronous coordinates, which are a part of the vector control. As a result of the vector control, a reference current is generated and passed to the stator of the motor through the inverter. Using the error between the command and estimated velocities, the PID control law is implemented. The PID control is also used for the current control. To meet the current loop and speed loop dynamic and steady-state performance, set the current loop PI regulator parameters $K_p = 9.35, K_i = 140$ and the speed loop PI regulator parameter $K_p = 0.14, K_i = 7$. The SMO, using the sigmoid function as a switching function, estimates the rotor angle from the back EMF. The parameters of a 3 kW PMSM used are given in the Table 10.1.

10.4 Results Analysis

The steady-state performance of the SMO is important, since the reduction of the chattering is a critical factor for the SMO. Therefore, the steady-state performance of the two SMOs are compared at 209 and 628 rad/s with no load, which represents low speed and high speed. As shown in Fig. 10.4, the improved sliding model observer achieve good results in wide speed range that estimated rotor speed is

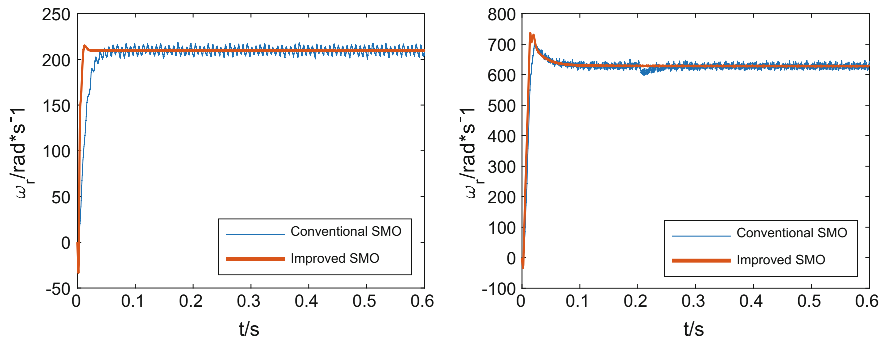


Fig. 10.4 Comparison of conventional and Improved SMO

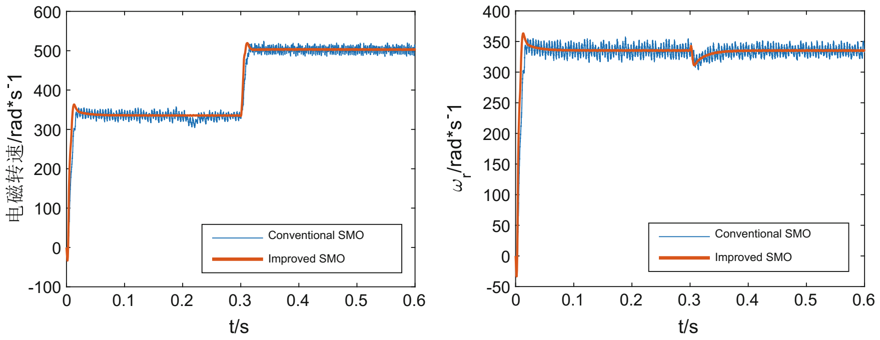


Fig. 10.5 Performance of improved SMO in change of operation conditions

quickly converge to the given speed in high and low speed where speed error is about ±5 rad/s, however, estimated speed of conventional SMO is ripple obviously and phase delay is inevitable. Improved SMO can well follow the actual speed and effectively reduce the chattering, while the speed ripple greatly in conventional SMO, it can be inferred that performance of improved SMO in speed tracking is superior to conventional SMO.

To study the performance of improved SMO in change of operation conditions, position estimation simulation result has been analyzed under the conditions that speed and torque changes abruptly. The first operation is when t equals to 0.3 s, rotor reference speed changes from 335 to 502 rad/s with no load, the second operation is that rotor reference speed is 335 rad/s and when t equals to 0.3 s, torque changes from 0 to 10 Nm. Figure 10.5 shows that the speed estimation error is substantially zero apart from zero speed nearby, which imply that improved SMO is still able to fast track the actual speed of the rotor under the given operation conditions. Traditional sliding mode observer also implements tracking speed, but the estimated speed still chatters obviously.

Slope of sigmoid function is changed depending on the tilt parameter. To analyze how tilt parameter effects improved sliding mode observer, set tilt parameter that a equals to 2 and 5 respectively. Figure 10.6 shows chattering problem is effectively reduced when the tilt parameter is small. On the contrary, estimated back EMF contains more high frequency information, increasing location estimation chattering.

To verify the improved SMO performance of resistance to parameter variations disturbance, motor parameters variation simulation is modeled. The motor stator resistance changes to $R = 2.5\,\Omega$ at the reference speed is 600 rad/s, the load is 5 Nm. As shown in Fig. 10.7, the rotor position estimated by improved SMO has no significant chattering, which indicates improved SMO is still able to accurately

Fig. 10.6 Sensorless speed control using the sigmoid function at a = 2, a = 5

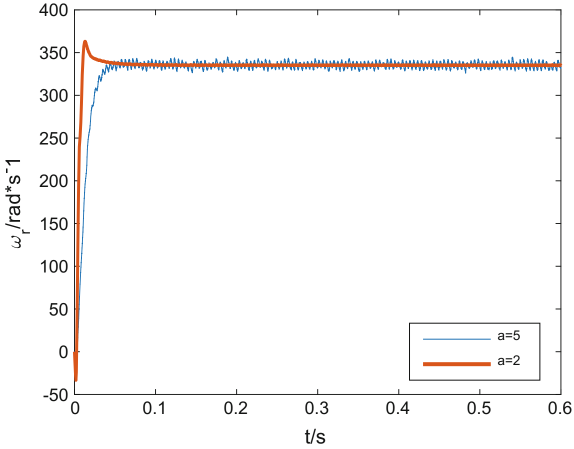
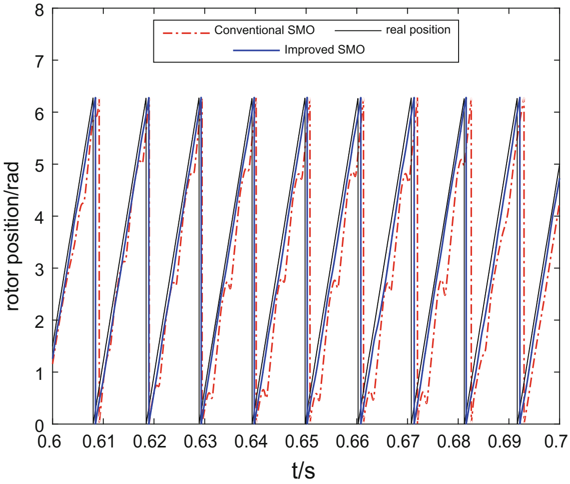


Fig. 10.7 Position estimation when stator resistance changes



estimate the rotor position after the parameters change. However, the rotor position estimated by traditional sliding mode observer fluctuates and is not accurate.

10.5 Conclusion

Depending on the theory of variable structure, an improved SMO is designed in which a sigmoid function is used as a switching function instead of signum function and a Kalman filter is used to extract back-EMF observer to eliminate phase delay. A PMSM position and speed estimation model is built in MATLAB/SIMULINK based on improved sliding mode observer. The corresponding simulation results show that:

(1) Tilt parameter has influences on improved SMO, chattering problem is effectively reduced when the tilt parameter is small;
(2) with respect to the conventional SMO, improved SMO can effectively reduce the chattering problem in position estimation;
(3) when the operation conditions changes, improved SMO can quickly and accurately track changes in speed;
(4) when the motor electromagnetic parameters change, improved SMO is still able to accurately estimate the rotor position.

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