

# Intuitionistic Hesitant Fuzzy Soft Set and Its Application in Decision Making

R.K. Mohanty and B.K. Tripathy

**Abstract** There are several models of uncertainty found in the literature like fuzzy set, rough set, intuitionistic fuzzy set, soft set, and hesitant fuzzy set. Also, several hybrid models have come up as a combination of these models and have been found to be more useful than the individual models. In everyday life we make many decisions. Making efficient decisions under uncertainty needs better techniques. Many such techniques have been developed in the recent past. These techniques involve soft sets and intuitionistic fuzzy sets. It is well known that intuitionistic hesitant fuzzy sets are more general than intuitionistic fuzzy sets. In this paper, we define intuitionistic hesitant fuzzy soft sets (IHFSS) and we also propose a decision making technique, which extends some of the recently developed algorithms. We also provide an application from real-life situations, which illustrates the working of the algorithm and its efficiency over the other algorithms.

**Keywords** Soft set · Fuzzy sets · Fuzzy soft sets · Intuitionistic fuzzy set · Hesitant sets · Intuitionistic fuzzy soft set · Decision making

## 1 Introduction

The notion of fuzzy sets introduced by Zadeh [1] in 1965 is one of the most fruitful models of uncertainty and has been extensively used in real-life applications. In order to bring topological flavor into the models of uncertainty and associate family of subsets of a universe to parameters, Molodtsov [2] introduced the concept of soft sets in 1999. A soft set is a parameterized family of subsets. Many operations on soft sets were introduced by Maji et al. [3, 4]. Hybrid models are obtained by suitably combining individual models of uncertainty have been found to be more

---

R.K. Mohanty (✉) · B.K. Tripathy  
School of Computer Science and Engineering, VIT University, Vellore, Tamilnadu, India  
e-mail: rknmohanty@gmail.com

B.K. Tripathy  
e-mail: tripathybk@vit.ac.in

efficient than their components. Several such hybrid models exist in literature. Maji et al. [5] put forward the concept of fuzzy soft set (FSS) by combining the notions of fuzzy set and soft set. Tripathy et al. [6] defined soft sets through their characteristic functions. This approach has been highly authentic and helpful in defining the basic operations like the union, intersection, and complement of soft sets. Similarly, defining membership function for FSSs will systematize many operations defined upon them as done in [7]. Many of soft set applications have been discussed by Molodtsov in [2]. An application of soft sets in decision making problems is discussed in [3]. Among several approaches, in [5], FSS and operations on it are defined. This study was further extended to the context of fuzzy soft sets by Tripathy et al. in [7], where they identified some drawbacks in [3] and took care of these drawbacks while introducing an algorithm for decision making. It has been widely known that the concept of intuitionistic fuzzy set (IFS) introduced by Atanassov [8] is a better model of uncertainty than the fuzzy set. The notion of non-membership function introduced, which does not happen to be one's complement of the membership function, introduces more generality and reality to IFS. The hesitation function generated as a consequence is what real-life situations demand. In case of fuzzy sets the hesitation component is zero. The intuitionistic fuzzy sets can only handle the incomplete information considering both the truth membership (or simply membership) and falsity membership (or non-membership) values. It does not handle the indeterminate and inconsistent information which exists in belief system.

Jiyang [9] introduced the concept of IVIFSS by combining the interval valued intuitionistic fuzzy sets (IVIFS) and soft set model. The concept of hesitant fuzzy soft sets was introduced by Sunil et al. They also discussed an application of decision making. In this paper, we introduce intuitionistic hesitant fuzzy soft sets. Here, we follow the definition of soft set due to Tripathy et al. [6] in defining IHFSS. Applications of various hybrid models are discussed in [7, 10–18]. The major contribution in this paper is introducing a decision making algorithm which uses IHFSS for decision making and we illustrate the suitability of this algorithm in real-life situations. Also, it generalizes the algorithm introduced in [5] while keeping the authenticity intact.

The concept of hesitant fuzzy sets was introduced by Torra [19]. This is an extension of fuzzy sets. It is sometimes difficult to determine the membership of an element into a set and in some circumstances this difficulty is caused by a doubt between a few different values. Some operations on hesitant fuzzy sets are defined in [20]. He also discussed an application of decision making. In this paper, we introduce the concept of IHFSS with the help of membership function.

## 2 Definitions and Notions

In this section, we introduce some of the definitions to be used in the paper. We assume that  $U$  is a universal set and  $E$  is a set of parameters defined over it.

**Definition 2.1** A fuzzy set  $A$  is defined through a function  $\mu_A$  called its membership function such that  $\mu_A: U \rightarrow [0, 1]$ .

**Definition 2.2** An intuitionistic fuzzy set over  $U$  is associated with a pair of functions  $\mu_A, \nu_A: U \rightarrow [0, 1]$  such that for any  $x \in U$ ,  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ .

The hesitation function  $\pi_A$  is defined as  $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ ,  $\forall x \in U$ .

**Definition 2.3** A soft set over the soft universe  $(U, E)$  is denoted by  $(F, E)$ , where

$$F: E \rightarrow P(U) \tag{1}$$

Here  $P(U)$  is the power set of  $U$ .

Let  $(F, E)$  be a soft set over  $(U, E)$ . Then in [7] it was defined as a parametric family of characteristic functions  $\chi_{(F,E)} = \{\chi_{(F,E)}^a | a \in E\}$  of  $(F, E)$  as defined below.

**Definition 2.4** For any  $a \in E$ , we define the characteristic function  $\chi_{(F,E)}^a: U \rightarrow \{0, 1\}$  such that

$$\chi_{(F,E)}^a(x) = \begin{cases} 1, & \text{if } x \in F(a); \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

**Definition 2.5** A hesitant fuzzy set on  $U$  is defined in terms of a function that returns a subset of  $[0, 1]$  when applied to  $U$ , i.e.,

$$T = \{\langle x, h(x) \rangle | x \in U\} \tag{3}$$

where  $h(x)$  is a set of values in  $[0, 1]$  that denote the possible membership degrees of the element  $x \in U$  to  $T$ .

**Definition 2.6** A pair  $(F, E)$  is called a hesitant fuzzy soft set if  $F: E \rightarrow \text{HF}(U)$ , where  $\text{HF}(U)$  denotes the set of all hesitant fuzzy subsets of  $U$ .

### 3 Intuitionistic Hesitant Fuzzy Sets

In this section, we introduce the notion of intuitionistic hesitant fuzzy soft sets.

**Definition 3.1** A pair  $(F, E)$  is called a intuitionistic hesitant fuzzy soft set if  $F: E \rightarrow \text{IHF}(U)$ , where  $\text{IHF}(U)$  denotes the set of all intuitionistic hesitant fuzzy subsets of  $U$ .

An IHFSS  $H$  on  $U$  is defined in terms of its membership function  $\mu_H : E \rightarrow P(\text{IHFS})$ ,  $\nu_H : E \rightarrow P(\text{IHFS})$  such that  $\forall a \in E$  and  $\forall x \in U$ ,  $\mu_H^a(x), \nu_H^a(x) \in P([0, 1])$  such that  $0 \leq \sup \mu_H^a(x) + \sup \nu_H^a(x) \leq 1$ .

Given three IHFEs in an IHFSS is represented by  $h, h_1$ , and  $h_2$ . Then we can define union and intersection operations as follows.

**Definition 3.2** For any two IHFSSs  $(F, E)$  and  $(G, E)$  over a common universe  $(U, E)$ , the union of  $(F, E)$  and  $(G, E)$  is the IFSS  $(H, E)$  and  $\forall a \in E$  and  $\forall x \in U$ , we have

$$\begin{aligned} h_1^a \cup h_2^a &= \{(\alpha_1, \alpha_2) | \alpha_1 \in h_1^a, \alpha_2 \in h_2^a\} \\ &= \left\{ \max \left\{ \mu_{\alpha_1}^a(x), \mu_{\alpha_2}^a(x) \right\}, \min \left\{ \nu_{\alpha_1}^a(x), \nu_{\alpha_2}^a(x) \right\} \right\} \end{aligned} \tag{4}$$

where  $\alpha_1 \in h_1^a, \alpha_2 \in h_2^a$ .  $h_1^a$  and  $h_2^a$  denote the hesitant fuzzy set.

**Definition 3.3** For any two IHFSSs  $(F, E)$  and  $(G, E)$  over a common universe  $(U, E)$ , the intersection of  $(F, E)$  and  $(G, E)$  is the IVIHFS  $(H, E)$  and  $\forall a \in E$  and  $\forall x \in U$ , we have

$$\begin{aligned} h_1^a \cap h_2^a &= \{ \min(\alpha_1, \alpha_2) | \alpha_1 \in h_1^a, \alpha_2 \in h_2^a \} \\ &= \left\{ \min \left\{ \mu_{\alpha_1}^a(x), \mu_{\alpha_2}^a(x) \right\}, \max \left\{ \nu_{\alpha_1}^a(x), \nu_{\alpha_2}^a(x) \right\} \right\}. \end{aligned} \tag{5}$$

**Definition 3.4** The complement of IHFSS  $(F, E)$ , represented as  $(F, E)^c$ , is defined as

$$h^c = \left\{ \left( \nu_{(F,E)}^a, \mu_{(F,E)}^a \right) \right\}. \tag{6}$$

**Definition 3.5** An IFHSS  $(F, E)$  is said to be a null IHFSS if and only if it satisfies

$$\mu_{(F,E)}^a(x) = 0 \quad \text{and} \quad \nu_{(F,E)}^a(x) = 1. \tag{7}$$

**Definition 3.6** An IFHSS  $(F, E)$  is said to be an absolute IHFSS if and only if it satisfies

$$\begin{aligned} \mu_{(F,E)}^e(x) &= 1 \\ \text{and} & \\ \nu_{(F,E)}^e(x) &= 1 \end{aligned} \tag{8}$$

### 4 Application of Intuitionistic Hesitant Fuzzy Set

In [2], Molodtsov has given several applications of soft set. In [10] the decision making example given depends on the decision of a single person. Here we discuss an application of DM in IHFSSs.

Many of researchers have tried to provide solutions for the decision making problems in lot many situations. Some of these approaches are preference ordering, utility values, preference values.

The parameters can be categorized as of two types [7].

We introduce the formula (9) to get a fuzzy value as score from an intuitionistic fuzzy value. It reduces the complexity and makes the comparison easier.

$$\text{Score} = \mu(1 + h) \tag{9}$$

The score will decrease with the increasing  $\nu$  value and score will increase when either  $\mu$  or  $h$  value increases. But, when  $\mu$  changes, the impact will be more in comparison to the  $h$  value as both the factors of the equation depends on  $\mu$ , whereas only one factor depends on  $h$ . Value of  $\mu h$  (2nd factor in equation) is inversely proportional to  $\nu$  value. So there is no need to consider  $\nu$  value again. The equation reduces to only  $\mu$  value in case of fuzzy soft set, i.e., if  $h = 0$ .

$$\text{Normalized Score} = \frac{\sum_{K=\{o,p,n\}} (C - R_K)^2}{|K| \times C^2} \tag{10}$$

where  $|K|$  is the number of approaches (e.g., optimistic, pessimistic, neutral, etc.),  $|C|$  is the number of objects to choose from.  $R_K$  is the rank with respect to approach  $x$ .

Consider the case of a company that wants to select a cloud service provider from the available service providers. Before the company selects the service provider, he needs to consider the parameters of the service provider. The parameters considered for comparison are efficiency, through put, security, delay, price, and feedback. Some parameters like price inversely affect the decisions. Those parameters are called negative parameters.

*Algorithm*

1. Input the parameter data table by ranking according to the absolute value of parameter priorities. If the priority for any parameter has not given, then take the value as 0 by default and that column can be opt out from further computations. The boundary condition for a positive parameter is  $[0, 1]$  and for a negative parameter is  $[-1, 0]$ .
2. Input the IHFSS table
3. Construct optimistic IFSS table by taking the maximum of membership values, minimum of non-membership values from IHFSS table and compute the hesitation values accordingly.
4. Construct pessimistic IFSS table by taking the minimum of membership values, maximum of non-membership values from IHFSS table, and compute the hesitation values accordingly.
5. Construct neutral IFSS table by taking the mean of the membership values and mean of the non-membership values from IHFSS table and compute the hesitation values accordingly.
6. Procedure Deci\_make(IFSS table)
  - 6.1. Multiply the priority values with the corresponding parameter values to get the priority table
  - 6.2. Construct the comparison table by finding the entries as differences of each row sum in priority table with those of all other rows taking membership and hesitation values separately.
  - 6.3. Construct the decision table by taking the sums of membership values and hesitation values separately for each row in the comparison table. Compute the score for each candidate using the formula (9).
  - 6.4. Assign rankings to each candidate based upon the score obtained.
    - 6.1 If there is more than one candidate having same score than who has more score in a higher ranked parameter will get higher rank and the process will continue until each entry has a distinct rank or they are equal with respect to all parameters. In later case, the whole group of candidates who are having that same score will get same rank.
  - 6.5. Return decision table.
7. Construct the decision tables for all approaches (optimistic, pessimistic, and neutral) using the procedure Deci\_make given in step 6.
8. Construct the rank matrix by taking the rank columns of all decision tables. Compute the normalized score using the formula (10).
9. Compute the final ranks for all candidates. The candidate having highest normalized score will get the highest rank and so on.

9.1 In case of more than one same normalized score, resolve the conflict by taking the scores given in the highest ranked parameter and continue the process until each entry has a distinct rank.

Let  $U$  be a set of cloud providers given by  $U = \{p1, p2, p3, p4, p5, p6\}$  and  $E$  be the parameter set given by  $E = \{e1, e2, e3, e4, e5, e6\}$ , where  $e1, e2, e3, e4, e5$  and  $e6$  represents efficiency, throughput, security, delay, price, and feedback, respectively.

The parameter data table is given in the Table 1. It contains all the details about the parameters. The parameter rank is decided by comparing the absolute priority value of a parameter.

Consider an IHFSS  $(F, E)$  which shows three opinions about the quality of service by the cloud service providers as shown in Table 2.

**Table 1** Parameter data table

Parameter	e1	e2	e3	e4	e5	e6
Priority	0.4	0.3	0.25	-0.3	0.1	0
Parameter rank	1	2	4	2	5	6

**Table 2** IHFSS table

$U$	e1		e2		e3		e4		e5		e6	
	$m$	$n$	$m$	$n$	$m$	$n$	$m$	$n$	$m$	$n$	$m$	$n$
p1	0.4	0.1	1	0	0.4	0.5	0.4	0.5	0.2	0.7	0.1	0.3
	0.8	0	0.4	0.4	0.6	0	0.2	0.6	1	0	0.5	0.3
	0.7	0.2	0.1	0.7	0.5	0.3	0.7	0.3	0.2	0.4	0.3	0.3
p2	0.1	0.7	0.2	0.6	0.2	0.7	0.6	0.1	0	0.1	0.1	0
	0.5	0	1	0	0.5	0.3	0.1	0.5	0.2	0.2	0.4	0.2
	0.8	0.1			0.8	0	0.4	0.1	0.9	0	0.1	0.6
p3	0.6	0.3	0.9	0	0	0.6	1	0	0.9	0	0.2	0.6
	1	0	0	0.1	0.4	0.2	0.1	0.5	0	0.6	0.2	0.7
	0.4	0.2	0.8	0	0.9	0	0.3	0.2	0.8	0	1	0
p4	0.4	0	0.9	0	0.2	0.1	0.1	0.1	0.3	0.3	0	0.1
	0	0.1	0.2	0.7	0.5	0.2	0.4	0.4	0.6	0.3	0.9	0
	0.5	0.2	0.5	0.1	0	0.6	0.1	0.8	0.3	0.1	0.8	0.1
p5	0.8	0.1	0.3	0.6	1	0	0.6	0.2	0.5	0.5		
	0	0.8	0.4	0.3	0.5	0.5	0.5	0.3	0.6	0.1	0.7	0.2
p6	0.1	0.7	0.8	0.1	0.4	0	0.6	0.4	0.4	0.2	1	0
	0.8	0.1	0.3	0.1	0.6	0.3	0.4	0.5	1	0	0.2	0.3
	0.4	0.6	0	0.3	0.4	0.2	0	0.6	0.7	0.1	0.6	0.4

**Table 3** Optimistic IFSS table

<i>U</i>	<i>e1</i>			<i>e2</i>			<i>e3</i>			<i>e4</i>			<i>e5</i>			<i>e6</i>		
	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>
<i>p1</i>	0.8	0.0	0.2	1.0	0.0	0.0	0.6	0.0	0.4	0.7	0.3	0.0	1.0	0.0	0.0	0.5	0.3	0.2
<i>p2</i>	0.8	0.0	0.2	1.0	0.0	0.0	0.8	0.0	0.2	0.6	0.1	0.3	0.9	0.0	0.1	0.4	0.0	0.6
<i>p3</i>	1.0	0.0	0.0	0.9	0.0	0.1	0.9	0.0	0.1	1.0	0.0	0.0	0.9	0.0	0.1	1.0	0.0	0.0
<i>p4</i>	0.5	0.0	0.5	0.9	0.0	0.1	0.5	0.1	0.4	0.4	0.1	0.5	0.6	0.1	0.3	0.9	0.0	0.1
<i>p5</i>	0.8	0.1	0.1	0.4	0.3	0.3	1.0	0.0	0.0	0.6	0.2	0.2	0.6	0.1	0.3	0.7	0.2	0.1
<i>p6</i>	0.8	0.1	0.1	0.8	0.1	0.1	0.6	0.0	0.4	0.6	0.4	0.0	1.0	0.0	0.0	1.0	0.0	0.0

**Table 4** Pessimistic IFSS table

<i>U</i>	<i>e1</i>			<i>e2</i>			<i>e3</i>			<i>e4</i>			<i>e5</i>			<i>e6</i>		
	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>
<i>p1</i>	0.4	0.2	0.4	0.1	0.7	0.2	0.4	0.5	0.1	0.2	0.6	0.2	0.2	0.7	0.1	0.1	0.3	0.6
<i>p2</i>	0.1	0.7	0.2	0.2	0.6	0.2	0.2	0.7	0.1	0.1	0.5	0.4	0.2	0.2	0.6	0.1	0.6	0.3
<i>p3</i>	0.4	0.3	0.3	0	0.1	0.9	0	0.6	0.4	0.1	0.5	0.4	0	0.6	0.4	0.2	0.7	0.1
<i>p4</i>	0	0.2	0.8	0.2	0.7	0.1	0	0.6	0.4	0.1	0.8	0.1	0.3	0.3	0.4	0	0.1	0.9
<i>p5</i>	0	0.8	0.2	0.3	0.6	0.1	0.5	0.5	0.0	0.2	0.6	0.2	0.4	0.5	0.1	0.2	0.6	0.2
<i>p6</i>	0.1	0.7	0.2	0	0.3	0.7	0.4	0.3	0.3	0	0.6	0.4	0.4	0.2	0.4	0.2	0.4	0.4

Optimistic IFSS table can be constructed by taking the maximum of membership values, minimum of non-membership values from IHFSS table and compute the hesitation values accordingly. Optimistic IFSS table is shown in Table 3.

Pessimistic IFSS table can be constructed by taking the minimum of membership values, maximum of non-membership values from IHFSS table and compute the hesitation values accordingly. Pessimistic IFSS table is shown in Table 4.

Neutral IFSS table can be constructed by taking the mean of the membership values and mean of the non-membership values from IHFSS table and compute the hesitation values accordingly. Neutral IFSS table is shown in Table 5.



**Table 5** Neutral IFSS table

<i>U</i>	<i>e1</i>			<i>e2</i>			<i>e3</i>			<i>e4</i>			<i>e5</i>			<i>e6</i>		
	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>	<i>m</i>	<i>n</i>	<i>h</i>
<i>p1</i>	0.633	0.100	0.267	0.500	0.367	0.133	0.500	0.267	0.233	0.433	0.467	0.100	0.467	0.367	0.167	0.300	0.300	0.400
<i>p2</i>	0.467	0.267	0.267	0.600	0.300	0.100	0.500	0.333	0.167	0.367	0.233	0.400	0.367	0.100	0.533	0.200	0.267	0.533
<i>p3</i>	0.667	0.167	0.167	0.567	0.033	0.400	0.433	0.267	0.300	0.467	0.200	0.300	0.567	0.200	0.233	0.467	0.433	0.100
<i>p4</i>	0.300	0.100	0.600	0.533	0.267	0.200	0.233	0.300	0.467	0.200	0.433	0.367	0.400	0.233	0.367	0.567	0.067	0.367
<i>p5</i>	0.400	0.450	0.150	0.333	0.467	0.200	0.800	0.167	0.033	0.433	0.367	0.200	0.500	0.267	0.233	0.450	0.400	0.150
<i>p6</i>	0.433	0.467	0.100	0.367	0.167	0.467	0.467	0.167	0.367	0.333	0.500	0.167	0.700	0.100	0.200	0.600	0.233	0.167

**Table 6** Optimistic priority table

<i>U</i>	<i>e1</i>		<i>e2</i>		<i>e3</i>		<i>e4</i>		<i>e5</i>	
	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>
<i>p1</i>	0.32	0.08	0.3	0	0.15	0.1	-0.21	0	0.1	0
<i>p2</i>	0.32	0.08	0.3	0	0.2	0.05	-0.18	-0.09	0.09	0.01
<i>p3</i>	0.4	0	0.27	0.03	0.225	0.025	-0.3	0	0.09	0.01
<i>p4</i>	0.2	0.2	0.27	0.03	0.125	0.1	-0.12	-0.15	0.06	0.03
<i>p5</i>	0.32	0.04	0.12	0.09	0.25	0	-0.18	-0.06	0.06	0.03
<i>p6</i>	0.32	0.04	0.24	0.03	0.15	0.1	-0.18	0	0.1	0

The priority tables can be constructed by multiplying the values in IFSS table with the respective priority values. The tables are not having non-membership value column further because the formula is dependent on membership and hesitation values only. Main idea behind this is, the change of non-membership value is always reflects in the values of either membership value or hesitation value or in both. The optimistic priority table is shown in Table 6.

In the same way, priority tables for pessimistic and neutral approach can be constructed.

Comparison tables can be constructed by taking the entries as differences of each row sum in priority table values. The optimistic comparison table is shown in Table 7.

In the same way, comparison of tables for pessimistic and neutral approach can be constructed.

Decision table can be constructed using formula (9). The optimistic decision table is shown in Table 8.

In the same way, the decision of tables for pessimistic and neutral approach can be constructed.

**Table 7** Optimistic comparison table

<i>U</i>	<i>P1</i>		<i>P2</i>		<i>P3</i>		<i>P4</i>		<i>P5</i>		<i>P6</i>	
	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>	<i>m</i>	<i>h</i>
<i>p1</i>	0.000	0.000	-0.070	0.130	-0.025	0.115	0.125	-0.030	0.090	0.080	0.030	0.010
<i>p2</i>	0.070	-0.130	0.000	0.000	0.045	-0.015	0.195	-0.160	0.160	-0.050	0.100	-0.120
<i>p3</i>	0.025	-0.115	-0.045	0.015	0.000	0.000	0.150	-0.145	0.115	-0.035	0.055	-0.105
<i>p4</i>	-0.125	0.030	-0.195	0.160	-0.150	0.145	0.000	0.000	-0.035	0.110	-0.095	0.040
<i>p5</i>	-0.090	-0.080	-0.160	0.050	-0.115	0.035	0.035	-0.110	0.000	0.000	-0.060	-0.070
<i>p6</i>	-0.030	-0.010	-0.100	0.120	-0.055	0.105	0.095	-0.040	0.060	0.070	0.000	0.000

**Table 8** Optimistic decision table

	sum $m$	sum $h$	Score	Rank
$p_1$	0.15	0.305	0.19575	2
$p_2$	0.57	-0.475	0.29925	1
$p_3$	0.3	-0.385	0.1845	3
$p_4$	-0.6	0.485	-0.891	6
$p_5$	-0.39	-0.175	-0.32175	5
$p_6$	-0.03	0.245	-0.03735	4

**Table 9** Rank matrix

	Optimistic rank	Pessimistic rank	Neutral rank	Normalized score	Final rank
$p_1$	2	1	2	0.76	1
$p_2$	1	3	3	0.573333	2
$p_3$	3	5	1	0.466667	3
$p_4$	6	6	6	0	6
$p_5$	5	4	4	0.12	5
$p_6$	4	2	5	0.28	4

From Table 9, it can be easily assess the quality of service provided by different service providers, which will help to take a suitable decision. Here,  $p_1$  is the best choice and so on.

## 5 Conclusion

In this article, we introduced a new definition of IHFSS, which uses the more authentic characteristic function approach for defining soft sets provided in [6]. This provides several authentic definitions of operations on IHFSS and made the proofs of properties very elegant. Earlier fuzzy soft sets were used for decision making in [4]. Their approach had many flaws. We pointed out those flaws and provided solutions to rectify them in [7]. This made decision making more efficient and realistic. Here, we proposed an algorithm for decision making using IHFSS, which uses the concept of negative parameters. Also, an application of this algorithm in solving a real-life problem is demonstrated.

## References

1. L.A. Zadeh, "Fuzzy sets", Information and Control, vol. 8, 1965, pp. 338–353.
2. D. Molodtsov, "Soft Set Theory - First Results", Computers and Mathematics with Applications, vol. 37, 1999, pp. 19–31.

3. P.K. Maji, R. Biswas, A.R. Roy, "An Application of Soft Sets in a Decision Making Problem", *Computers and Mathematics with Applications*, vol. 44, 2002, pp. 1007–1083.
4. P.K. Maji, R. Biswas, A.R. Roy, "Soft Set Theory", *Computers and Mathematics with Applications*, vol. 45, 2003, pp. 555–562.
5. P.K. Maji, R. Biswas, A.R. Roy, "Fuzzy Soft Sets", *Journal of Fuzzy Mathematics*, vol. 9(3), 2001, pp. 589–602.
6. B.K. Tripathy and K.R. Arun, "A New Approach to Soft Sets, Soft Multisets and Their Properties", *International Journal of Reasoning-based Intelligent Systems*, vol. 7, no. 3/4, 2015, pp. 244–253.
7. B.K. Tripathy, Sooraj, T.R., RK Mohanty, "A new approach to fuzzy soft set and its application in decision making", *Advances in intelligent systems and computing*, vol. 411, 2016, pp. 305–313.
8. K. Atanassov, "Intuitionistic Fuzzy Sets", *Fuzzy Set Systems*, vol. 20, 1986, pp. 87–96.
9. Y. Jiang, Y. Tang, Q. Chen, H. Liu, J. Tang, "Interval-valued intuitionistic fuzzy soft sets and their properties", *Computers and Mathematics with Applications*, vol. 60, 2010, pp. 906–918.
10. B.K. Tripathy, RK Mohanty, Sooraj T.R., K.R. Arun, "A New Approach to Intuitionistic Fuzzy Soft Sets and its Application in Decision-Making", *Advances in Intelligent Systems and Computing*, vol. 439, 2016, pp. 93–100.
11. Sooraj T.R., RK Mohanty, B.K. Tripathy, "Fuzzy Soft Set Theory and its Application in Group Decision Making", *Advances in Intelligent Systems and Computing*, vol. 452, 2016, pp. 171–178.
12. B.K. Tripathy, Sooraj T.R., RK Mohanty, "Advances Decision Making Usisng Hybrid Soft Set Models", *International Journal of Pharmacy and Technology*, vol. 8(3), 2016, pp. 17694–17721.
13. B.K. Tripathy, RK Mohanty, T.R. Sooraj, "On Intuitionistic Fuzzy Soft Sets and Their Application in Decision-Making", *Lecture Notes in Electrical Engineering*, vol. 396, 2016, pp. 67–73.
14. B.K. Tripathy, RK Mohanty, Sooraj T.R., "On Intuitionistic Fuzzy Soft Set and its Application in Group Decision Making", In: *Proceedings of the International Conference on Emerging Trends in Engineering, Technology and Science (ICETETS)*, Pudukkottai, 2016, pp. 1–5.
15. B.K. Tripathy, RK Mohanty, Sooraj T.R., A. Tripathy, "A Modified Representation of IFSS and Its Usage in GDM", *Smart Innovation, Systems and Technologies*, Vol. 50, 2016, pp. 365–375.
16. RK Mohanty, Sooraj T.R., B.K. Tripathy, "An application of IIVFSS in medical diagnosis decision making", *International Journal of Applied Engineering Research*, vol. 10(92), pp. 85–93.
17. B.K. Tripathy, Sooraj T.R., RK Mohanty, "A New Approach to Interval-valued Fuzzy Soft Sets and its Application in Decision-Making", *Advances in Intelligent Systems and Computing*, Vol. 509, 2017, pp. 3–10.
18. RK Mohanty, Sooraj T.R., B.K. Tripathy, "IVIFS and Decision-Making", *Advances in Intelligent Systems and Computing*, Vol. 468, 2017, pp. 319–330.
19. Torra V, Hesitant fuzzy sets and decision, *International Journal of Intelligent Systems* 25 (6), 2010, pp. 395–407.
20. Torra V, Narukawa Y, On hesitant fuzzy sets and decision, *The 18<sup>th</sup> IEEE international conference on Fuzzy Systems*, Jeju Island, Korea, 2009, pp. 1378–1382.