Performance Analysis of Compressed Sensing in Cognitive Radio Networks

N. Swetha, Panyam Narahari Sastry, Y. Rajasree Rao and G. Murali Divya Teja

Abstract In the recent research, compressive sampling (CS) has received attention in the area of signal processing and wireless communications for the reconstruction of signals. CS aids in reducing the sampling rate of received signals thereby decreasing the processing time of analog-to-digital converters (ADC). The energy minimization is the key feature of CS. In this work, CS has been applied to spectrum sensing in cognitive radio networks (CRN). The primary user (PU) signal is optimally detected using the sparse representation of received signals. The received PU signal is compressed in the time domain to extract the minimum energy coefficients and then applied to sensing. Further, the signal is detected using energy detection technique and recovered using l_1-minimization algorithm. The detection performance for various compression rates is analyzed.

Keywords Energy detection · Compression rate · l_1-minimization

1 Introduction

Dynamic spectrum access alleviates the problem of spectrum scarcity by introducing new wireless networks. CRN provides a promising solution for increasing the spectrum efficiency. Cognitive radio is a smart system that senses the electromag-

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netic spectrum for unused channels and then adapts them for communication without interfering with the licensed users. The channel is scanned continuously to identify the presence or absence of the signal using spectrum sensing algorithms. Wideband spectrum sensing needs efficient techniques to reduce the processing time. The wideband signal is filtered using several narrow band filters, and then the detection is performed. These signals need multi-channel ADCs sampling at a rate greater than Nyquist rate. Meanwhile, the traditional reconstruction techniques cannot provide necessary statistic using limited measurements. Therefore, a technique is required for sampling the signals at a rate lower than Nyquist frequency. CS accomplishes this task. CS theory refers to perform channel estimation using sparse samples compared to the conventional techniques [1]. A PU localization technique is developed using CS in order to improve the accuracy of sensing [2].

In [3], the signals are compressed using orthonormal basis functions and recovered using a l^p basis pursuit (BP) algorithm. However, the BP and orthogonal matching pursuit (OMP) algorithms use the preliminary information of the sparse nature for solving the problem. A tree-based OMP is proposed in [4] by exploiting the tree-like structure as additional information for reconstructing the signals. The advantage of the tree-like structure is that more elements are considered at a time thereby requiring less computation time than BS and OMP. In contrast with time domain CS, the multi-band detection technique is modeled in frequency domain [5]. Further, CS was applied to recognize the type of digital modulations in communication. The spectrum and moments of higher order are used as a measure to directly identify the different modulated signals without reconstructing the signals. The incoming radar pulses have been recovered using a photon-based CS system in [6].

This paper models a generalized framework for compressed spectrum sensing of a wideband signal and recovery using l_1-minimization algorithm. The wideband signal is first compressed using an orthogonal function in time domain for deriving the energy coefficients. Each narrow band signal is then detected using energy detection technique and recovered linearly. The rest of the paper is organized as follows. Section 2 introduces the compressed spectrum sensing model and Sect. 2 describes l_1-minimization algorithm. The results are discussed in Sect. 3 and finally, Sect. 4 summarizes the conclusions.

2   Compressed Spectrum Sensing Model

Consider a wideband signal x having L non-overlapping channels. The channel occupancy is not balanced at any particular location and time. The number of samples can be decimated using compression scheme.

2.1 Compression

Let x be a discrete time signal of size $N \times 1$. The sparse matrix y of size $K \times 1$ can be obtained as [7]

$$y = Ax \tag{1}$$

where A is an orthogonal matrix of size $K \times N$, in which N and K denote the total number of samples and number of observations. The condition for compression is $K \ll N$. Hence, y is called as a measurement vector that results in reduced sampling rate. Before applying this signal to sensing, the compressed signal needs to be filtered using a set of narrow band filters.

2.2 Energy Detection Technique

In general, spectrum sensing model can be formulated as a binary hypothesis problem having true and null hypothesis [8].

$$H_0 : s(n) = w(n) \tag{2}$$

$$H_1 : s(n) = y(n) + w(n) \tag{3}$$

where $s(n)$ is the received narrow band signal of each channel to the secondary user, $y(n)$ is the compressed signal, and $w(n)$ is the Gaussian noise having zero mean and variance (σ_ω^2). The energy detector test statistic is given as [9]

$$E(s) = \frac{1}{N} \sum_{n=1}^{N} |s(n)|^2 \tag{4}$$

The average energy $E(s)$ is compared with detection threshold T to evaluate the probability of detection. The performance factors that decide the ability of any sensing technique are probability of detection (p_d) (probability of successful detection under H_1), probability of false alarm (p_{fa}) (probability of incorrectly detecting under H_0), and signal-to-noise (SNR) wall of detection [10] (the minimum SNR value below which the detection is not possible). The detection threshold [8] is evaluated as

$$T = Q^{-1} \left(\frac{P_{fa}(n)}{\sqrt{N}} \right) + 1 \tag{5}$$

where $Q(x)$ is a complementary error function.

2.3 l_1-minimization algorithm

In recent papers, convex optimization methods are used for reconstructing the signals from insufficient data. Compressed sensing depends on least squares method referred as l_p-norm. In statistics, l_p-norm of x is computed as

$$\|x\| = \sqrt[p]{\sum_i |x_i|^p} \quad p \in \mathbb{R}$$

The mathematical properties of different norms ($p \geqslant 1$) vary dramatically. Compressed sensing scheme finds the sparsest solution for indeterminated systems. The vector with few nonzero entries is called as the sparsest solution. This problem is usually solved using linear programming or optimization methods [11].

The minimum energy x_0 can be evaluated using measurement vector y in Eq. (1) as

$$x_0 = A' y \tag{6}$$

The linear program used for recovery of the original signal is primary dual algorithm [12], stated as

$$min \, x_0 \quad subject \, to \quad Ax = b, \quad f_i(x_0) \leq 0 \tag{7}$$

where (search vector) $x \in \mathbb{R}^N$, $b \in \mathbb{R}^K$, and f_i is a linear function of x_0:

$$f_i(x_0) = <c_i x_0> + d_i \quad c_i \in \mathbb{R}^N \quad d_i \in \mathbb{R} \tag{8}$$

where c_i and d_i are constants. The optimized solution is picked, if Karush–Kuhn–Tucker (KKT) conditions are satisfied [12]. These are first-order conditions that can be used for linear as well as nonlinear programming. The Newton's iterative method arrives at an interior point (x_0, v, λ) provided $f_i(x_0) < 0$, $\lambda > 0$. The parameters v and λ are duals to x_0. The complementary slackness condition used in our problem is

$$\lambda_i^* f_i(x_0) = -\frac{1}{\tau} \tag{9}$$

The parameter τ is responsible for the iterations in the Newton's method. The increase of τ progresses the interior point toward the solution on the boundary. The proposed problem quantifies the residuals namely primal, dual, and central as the modified KKT conditions.

$$rprimal = Ax_0 - b$$
$$rcentral = \begin{bmatrix} -\lambda_1 f_1 \\ -\lambda_2 f_2 \end{bmatrix} - \frac{1}{\tau} \tag{10}$$
$$rdual = \begin{bmatrix} \lambda_1 - \lambda_2 + A' v \\ 1 - \lambda_1 - \lambda_2 \end{bmatrix}$$

Algorithm 1 Pseudo-code for primary dual algorithm

Set primary dual tolerance value (pdtol) and maximum number of iterations
(pdmaxiter) to 0.001 and 50, respectively.
 Step 1: Generate a minimum energy signal from equation (6)
Step 2: Define linear functions as $f1 = x_0 - u$ and $f2 = -x_0 - u$ (Eq. 8)
where $u = 0.95 \lfloor x_0 \rfloor + 0.1 \lfloor x_0 \rfloor_{max}$
Find dual variables $\lambda_i = -\frac{1}{f_i}$ $i \in [1, 2]$
Also find the third dual variable $v = -A(\lambda 1 - \lambda 2)$
Step 3: Derive the surrogate duality gap (sdg) and τ
$sdg = -(f_1' \lambda 1 + f_2' \lambda 2)$
$\tau = \frac{2\mu N}{sdg}$
Step 4: Compute rprimal, rdual, and rcentral from equation (10)
Step 5: Check for the stopping criterion variable *done*
$done = (sdg < pdtol) \lor (pditer \geq pdmaxiter)$
Step 6: Newton's method [12]

   while ($\sim done$) **do**
     Step 6.1:
     Calculate w_1, w_2 and w_3:
     $w_1 = -\frac{1}{\tau}(-\frac{1}{f_1} + \frac{1}{f_2}) - A'v$
     $w_2 = -1 - \frac{1}{\tau}(\frac{1}{f_1} + \frac{1}{f_2})$
     $w_3 = -rprimal$
     Step 6.2:
     Calculate $sigma_1$, $sigma_2$ and $sigma_x$:
     $sigma_1 = -\frac{\lambda_1}{f_1} - \frac{\lambda_2}{f_2}$
     $sigma_2 = \frac{\lambda_1}{f_1} - \frac{\lambda_2}{f_2}$
     $sigma_x = sigma_1 - \frac{sigma_2^2}{sigma_1}$
     Step 6.3:
     Calculate the deviation parameters *dx*, *du*, and *dv* using steps 6.1 and 6.2
     Step 6.4:
     Calculate the partial change in the dual variable λ_i
     Step 6.5:
     Calculate the suitable step size *s*
     Find the reconstructed signal x_p using back tracing search method
     $x_p = x_0 + sdx$
     $x_0 = x_p$
     repeat until *done* is true
   end while

The feasible step size *s* specifies the next move direction ($0 < s < 1$) to derive the
minimum energy coefficients of the signal. The complete process of recovering the
signal is given in Algorithm 1. This algorithm can be employed for face recognition
due to the sparse nature of human features [13].

3   Simulation Results

The wideband PU signal used in simulations is QPSK signal with five channels. The SNR assumption is 0 to -14 dB. The number of samples (N) is 512. The compression rate (K/N) varied from 10 to 100%. The SNR wall is evaluated at $p_d = 0.9$ and $p_{fa} = 0.1$ respectively. The detection performance of each narrow band-compressed signal is analyzed using ROC curves.

Figure 1 illustrates the original, minimum energy, and the recovered signals. The rate of compression assumed here is 50%, where $K = 256$ and $N = 512$. The original signal shown in Fig. 1a is the filtered narrowband signal of one channel. Figure 1b shows the compressed signal in time domain, evaluated using Eq. (6). It consists of minimum energy coefficients of the original signal. The recovery is made using Algorithm 1, shown in Fig. 1c. The figures clearly depict that most of the samples are successfully reconstructed even with 50% compression rate. The sparse nature of the compressed signal enables any spectrum sensing technique to identify the vacant spectrum bands in less time.
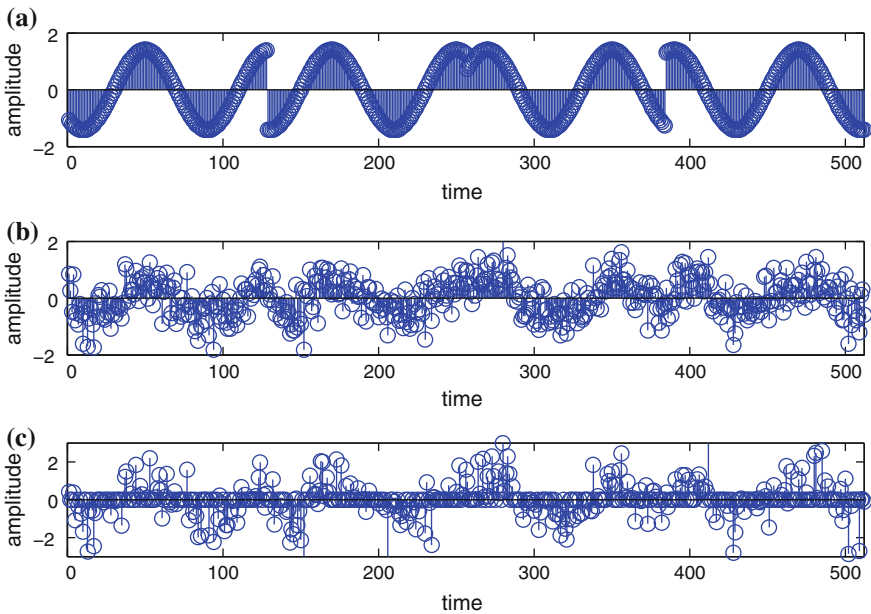

Fig. 1   **a** Received PU signal for single narrowband channel **b** Minimum energy signal **c** Recovery of compressed QPSK signal with $K/N = 50$%
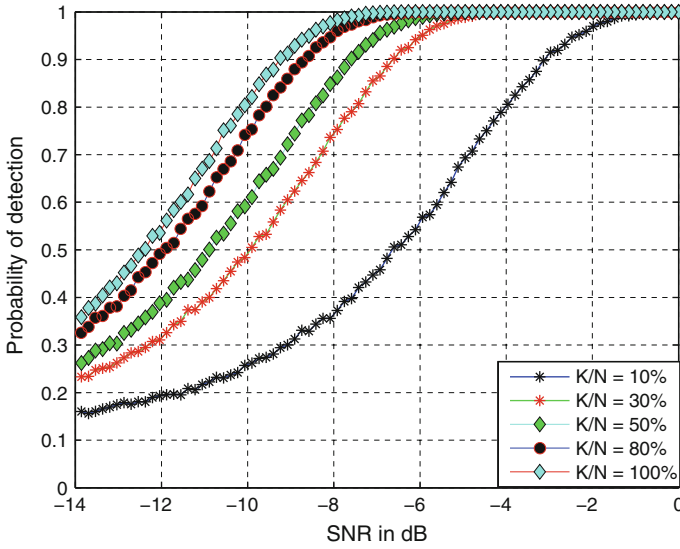
Fig. 2 SNR verses p_d of proposed CSS technique for different compression rates

Table 1 Comparison of SNR wall of the proposed method with variable compression rates

Compression rate in %	SNR wall (dB)
10	−3
30	−6.6
50	−7.6
80	−8.5
100	−9.2

Figure 2 shows the receiver operating characteristics (ROC) for observing the detection performance through p_d for each SNR value. The SNR wall of detection of the proposed technique is shown clearly for various compression rates. The 10% compressed signal is identified at −3 dB, 30% at −6.6 dB, 50% at −7.6 dB, 80% at −8.5 dB, and 100% at −9.2 dB SNR respectively. Table 1 shows these results. The detection strategy increases linearly with compression rate, but the increase in the compression rate increments the number of samples that add up the processing time. On an average, 50% compression can be chosen to detect the PU signals at −7.6 dB which achieves better performance than traditional energy detection technique. The same compression rates are considered for plotting p_d against p_{fa} in Fig. 3.
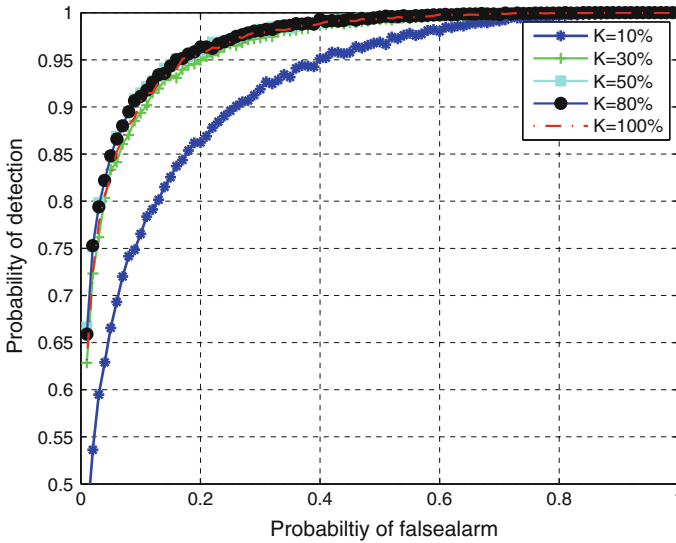
Fig. 3 p_d verses p_{fa} of proposed CSS technique for different compression rates

4 Conclusion

The computational complexity plays a significant role in wideband spectrum sensing. CS provides an optimal solution by compressing the samples at a rate less than 50%. The proposed work performs compression of a wideband signal and senses each narrowband channel using energy detection technique. In this work, a unique measurement matrix is produced to obtain minimum energy coefficients of the received signal. The spectrum sensing is initiated upon these condensed samples to analyze the performance of detection. The minimum energy signal is then reconstructed using l_1-minimization algorithm. The reconstruction can also be made feasible by adopting the optimization methods as a future direction of research. Furthermore, the simulation results are discussed for different compression rates making sparsest detection possible.

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