

Chapter 6

Mixing Processes in Estuaries: Simplified Methods

The classical definition of an estuary establishes that it is a partially closed water body with openings to the adjacent ocean, where the seawater is diluted by fresh water of the fluvial drainage basin. The input of fresh water decreases the potential energy of the water column, which is supplied by tidal energy through the mixing process produced on the bottom and internal shear instabilities.

The presence of denser water at the estuary mouth generates a system which constantly pushes seawater into the estuary. The water mass in the mixing zone (MZ) is composed of fresh and salt water, which varies along the estuary. Due to the seaward salinity increase, the horizontal salinity gradient generates the baroclinic component of the gradient pressure force. The barotropic and the baroclinic components of the gradient pressure force, the fresh water input, and the wind shear cause agitation of the estuarine water mass and generate the mixing processes (advection and turbulent diffusion).

In this chapter, we take a semi-empirical approach to estuarine processes. The estuary will be considered as a *black box* and one-dimensional system, and the salinity and the fresh water will be used as tracers under steady-state conditions. The estuary geometry, river discharge (Q_f), tidal height (H_o), and the non-diluted salinity of the adjacent coastal ocean (S_o) will be considered as known. The conservation of salt and volume, with the assumption of a well-mixed estuary, will be used to calculate the longitudinal salinity distribution as well as the *flushing time*, which indicate the estuary capacity to flush out the salinity and concentrations of any conservative property.

The non-diluted salinity of the adjacent coastal ocean (S_o) is measured outside of the estuarine plume and, due to mixing processes on the continental shelf, may be subjected to slow seasonal variation. Then, it is advisable to take monthly time mean representative values for the seasonal period, if possible. For estuaries on the Southeastern Brazilian continental shelf (23°S–28°S), S_o values may be found in Castro and Miranda (1998).

6.1 Fundamental Concepts

The definition of an estuary may follow from the experimental evidence. In fact, the longitudinal salinity gradient in the mixing zone, shown in Fig. 4.4 (Chap. 4), indicates conclusively that, due to the measurable dilution of the seawater by the river fresh water, a parcel of this fresh water remains in the estuarine water body. According to Ketchum (1950), the *flushing time* (t_q) is the ratio of the fresh water volume retained in the MZ and the river discharge (Q_f), thus,

$$t_q = \frac{V_f}{Q_f}. \quad (6.1)$$

This property, with dimension $[t_q] = [T]$ has also been called *mean detention time* by Fischer et al. (1979), and is dependent on two main quantities intimately related: the fresh water volume retained in the MZ and the river discharge that dilutes the seawater entering the estuary. It should be noted that the tidal height, which determines the mixing intensity, and the direct and remote wind stress, which may dam or remove the estuarine water mass, may be important to determine this time interval.

In normal conditions, the river discharge (Q_f) may be considered constant during the tidal cycle; however, the fresh water volume (V_f) usually increase and decrease during the ebb and flood tide, respectively, but in the intratidal time scale it is very difficult to calculate this quantity. Therefore, it is more common to calculate a global *flushing time*, representative for a tidal period; it is an important quantity because it measures the time interval necessary for the fresh water volume retained in the mixing zone to be removed from the estuary, along with concentrations of other substances in the estuarine water mass.

The input of fresh water volume (R) into the estuary, during the time interval of a complete diurnal or semi-diurnal tidal cycle (T_p) is calculated by $R = T_p Q_f$. If the MZ of the estuary had already accumulated a fresh water volume, V_f , the flushing time for the tidal period is the ratio,

$$t_q = \frac{V_f + R}{R} T. \quad (6.2)$$

This result indicates that t_q is higher than or equal to the tidal period, and $t_q = T_p$ only when $V_f \rightarrow 0$. Small values of t_q indicate that the removal of all fresh water is due to macro or hyper tides, and the mean estuarine depth is similar to the tidal height. In these conditions, the estuarine water mass is completely renewed (the MZ is flushed out of the estuary) at each tidal cycle, because almost the whole water in the estuary is flushed out. Therefore, this environment is less susceptible to water pollution by pathogenic substances.

Let's now consider influences on the flushing time, which may occur due to the fortnightly tidal modulation, under the assumption that the river discharge and the salinity in the continental shelf don't vary and the wind forcing is negligible.

During neap tides, the estuarine circulation is less intense and the vertical stratification is high; however, during spring tides, the conditions are opposite (intense circulation and less vertical stratification). Thus, during the spring tidal cycle, less fresh water is retained than during in the neap tide, $(V_f)_S < (V_f)_N$. Applying the flushing time definition (Eq. 6.1) the corresponding values at the spring $(t_q)_S$ and neap $(t_q)_N$ tides are given by:

$$(t_q)_S = \frac{(V_f)_S}{Q_f}, \quad \text{and} \quad (t_q)_Q = \frac{(V_f)_Q}{Q_f}. \quad (6.3)$$

By combining these results it follows that:

$$(t_q)_S = \frac{(V_f)_S}{(V_f)_Q} (t_q)_Q. \quad (6.4)$$

Consequently, taking into account the inequalities of the fresh water volumes retained in the spring and neap tidal conditions, it follows from Eq. (6.4) that $(t_q)_S < (t_q)_N$. Then, according to the simplified conditions, the fresh water volume retained in the MZ is removed quickly during the spring tide than the neap tide.

The fresh water volume (V_f) necessary to calculate the flushing time may be obtained from the knowledge of the non-diluted salinity value at the continental shelf (S_0) . But, to do so, it is necessary to define the mean *fresh water fraction* or *concentration* indicated by \bar{f} . This quantity is defined as the ratio of the fresh water volume in the MZ (V_f) and the corresponding steady-state geometric volume (V) of the estuarine water mass,

$$\bar{f} = \frac{V_f}{V}. \quad (6.5)$$

The fresh water fraction (concentration) is a non-dimensional quantity which varies between the following extreme values: $\bar{f} = 1$, when the estuary is completely filled with fresh water, and $\bar{f} = 0$, when there is no fresh water in the MZ (meaning $Q_f \approx 0$). As this quantity is function of the fresh water volume (V_f) , which is the unknown required to calculate the flushing time (Eq. 6.1), then it is necessary to find an alternative way to calculate \bar{f} (Eq. 6.5), using another quantity. In this case we can use the salinity.

Considering a small control volume dV , it is possible to define fresh water fraction as $f = dV_f/dV$. Following its displacement from the estuary head (where $S = 0$ and $dV = dV_f$) to the estuary mouth, where the fresh water influence is small ($dV_f \approx 0$), the fresh water fraction of this volume varies from 1 to almost zero ($0 < f \leq 1$). This variation interval is the same as that presented above. Then, using the salinity definition and the conservation principle of the mass of salt in seawater, the fresh water fraction may be calculated as function of this physical-chemical

property. Consider a volume of salt water V_i with the total mass, M , a salinity S_i , and density ρ_i . Then, from the salinity definition,

$$S_i = \frac{m}{M} = \frac{m}{\rho_i V_i}, \quad (6.6)$$

where \underline{m} indicates the mass of the dissociated salts in the volume V_i . Adding to a the fresh water volume ΔV_f , to the initial water volume, the resulting salinity value S ($S < S_i$) due to this dilution is calculated by,

$$S = \frac{m}{\rho(V_i + \Delta V_f)}, \quad (6.7)$$

where ρ indicates the new density value of the solution. Solving these Eqs. (6.6) and (6.7) for the mass \underline{m} and equating the results gives,

$$\rho_i S_i V_i = \rho S (V_i + \Delta V_f). \quad (6.8)$$

Disregarding the density variation ($\rho \approx \rho_i$) gives the ratio:

$$\frac{\Delta V_f}{V_i} = \frac{S_i - S}{S}. \quad (6.9)$$

The first member of this equation is the fresh water fraction in relation to the initial volume. Then, this final result indicates that is possible to calculate the fresh water fraction if the initial and final salinity values are known. By analogy, and considering that in estuaries this fraction varies from 1 to 0, when the control volume of water is displaced along the estuary from its head down to the mouth, the mean value of this quantity may be calculated by (Ketchum 1950):

$$f = \frac{V_f}{V} = \frac{(S_0 - S)}{S_0}, \quad \text{and} \quad V_f = \left(1 - \frac{S}{S_0}\right)V. \quad (6.10)$$

As previously mentioned, the undiluted salinity value (S_0) or the salinity at the salt source, is known in this equation, and is a characteristic value of the water mass of the adjacent continental shelf without influence of the estuarine plume.

If the salinity distribution in the estuarine MZ is in steady-state condition, its spatial distribution depends only its spatial coordinates, $S = S(x, y, z)$. Then under this condition, if the salinity field is known, it follows from Eq. (6.10) that:

$$f(x,y,z) = \frac{S_0 - S(x,y,z)}{S_0} = 1 - \frac{S(x,y,z)}{S_0}, \quad (6.11)$$

and the fresh water concentration is also a function of the spatial coordinates. This equation may be solved for $S = S(x, y, z)$, if $f = f(x, y, z)$ is known, and we have $S(x, y, z) = S_0[1 - f(x, y, z)]$.

Applying Eq. (6.5) to a small differential volume dV , the corresponding fresh water volume is calculated by: $dV_f = f(x, y, z)dV$. Then, integrating in the finite geometric volume (V) of the estuarine MZ,

$$V_f = \int_V f dV = \iiint_V f(x,y,z) dx dy dz. \quad (6.12)$$

In this equation the fresh water fraction is calculated by the Eq. (6.11). Applying the *Mean Value Theorem* of calculus Eq. (6.12) may be rewritten as,

$$V_f = \bar{f}V = \left(1 - \frac{\bar{S}}{S_0}\right)V, \quad (6.13)$$

and the fresh water volume is obtained as a function of the mean salinity and the geometric volume of the MZ.

Combining the flushing time definition (Eq. 6.1) and Eq. (6.13), it follows that:

$$t_q = \frac{V_f}{Q_f} = \frac{\bar{f}V}{Q_f} = \frac{(S_0 - \bar{S})}{S_0} \frac{V}{Q_f}. \quad (6.14)$$

This equation indicates that the flushing time is directly proportional to the difference ($S_0 - \bar{S}$) and the fresh water volume V , and is inversely proportional to the fresh water discharge Q_f .

As an example of the flushing time determination, let us consider a laterally homogeneous estuary, with known values for its stationary salinity field $S = S(x, z)$ and non-diluted salinity (S_0) at the coastal region. Then, it is possible with Eq. (6.11) to convert the isohalines into the corresponding isolines of fresh water fraction $f(x, z) = \text{const.}$, presented in Fig. 6.1. With the assumption that the MZ width (B) may be considered constant, the fresh water volume (V_f) is calculated with an equation similar to (6.12):

$$V_f = B \iint_A f(x,z) dx dz, \quad (6.15a)$$

or if the estuary has a length, L , and \bar{A} is the mean cross section area, the fresh water volume is determined by,

$$V_f = \bar{A} \int_0^L f(x) dx. \quad (6.15b)$$

With the assumption that the estuary width is 500 m, and numerically calculating the area integral (Eq. 6.15a) with the data of Fig. 6.1, the computed fresh water volume (V_f) is approximately $61.0 \times 10^5 \text{ m}^3$. In the case of a river discharge

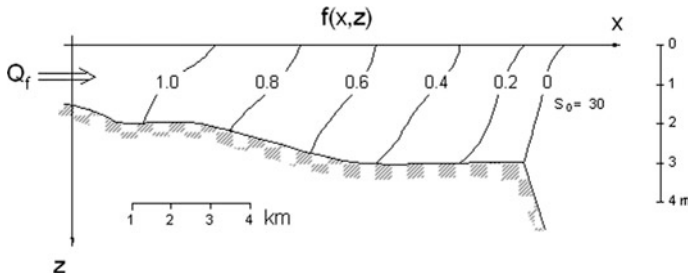


Fig. 6.1 Longitudinal steady-state distribution of the fresh water fraction $f = f(x, z)$ in a partially mixed and laterally homogeneous estuary

of $50 \text{ m}^3 \text{ s}^{-1}$, the flushing time of this estuary is approximately 34.0 h, which is almost three semi-diurnal tidal cycles.

As another example of flushing time and flushing rate calculation is the following from Fischer et al. (1979). “A well-mixed estuary has a constant cross-sectional area $A = 10^4 \text{ m}^2$, a length of $L = 30 \times 10^3 \text{ m}$, and a constant kinematic longitudinal diffusion coefficient $K_H = K = 10^2 \text{ m}^2 \text{ s}^{-1}$. The fresh water inflow is $30 \text{ m}^3 \text{ s}^{-1}$. Find the flushing time and the flushing rate according to Eqs. (6.1) and (6.15b), respectively”.

To simulate the longitudinal salinity distribution from its mouth, $S = S_0$ at $x = 0$, towards its head, $S = 0$ for $x \rightarrow -\infty$, a possible solution of the one-dimensional salinity variation may be expressed by,

$$S(x) = S_0 \exp\left[u_f \left(\frac{x}{K}\right)\right] = S_0 \exp(3.0 \times 10^{-5}),$$

and the corresponding longitudinal distribution of the fresh water fraction is

$$f(x) = 1 - \frac{S(x)}{S_0} = 1 - \exp(3 \times 10^{-5}).$$

The fresh water volume in the estuary volume (Eq. 6.15b) is calculated by the following integral:

$$V_f = A \int_0^L [1 - \exp(-3 \times 10^{-3})] dx = 1.02 \times 10^8 \text{ m}^3,$$

and the flushing time t_q , and the *flux rate* F (to be defined in Eq. (6.16), are calculated by

$$t_q = \frac{V_f}{Q_f} = \frac{1.02 \times 10^8}{30} = 3.41 \times 10^6 \text{ s} \approx 39.4 \text{ days}$$

and

$$F = \frac{V}{t_q} = \frac{V}{V_f} Q_f = 90 \text{ m}^3 \text{ s}^{-1}.$$

In the classical and recent literature there are several examples using the concepts and equations presented here to calculate the flushing time of estuarine systems: For example, Ketchum et al. (1951) calculated the flushing time near the Hudson river mouth in the New York Bight (USA) during high ($488 \text{ m}^3 \text{ s}^{-1}$) and low ($197 \text{ m}^3 \text{ s}^{-1}$) river discharge volumes, and the flushing time corresponding values were 6.0 and 10.6 days, respectively. Another classical result was published by Hughes (1958), who analyzed data collected during low river discharge ($25.7 \text{ m}^3 \text{ s}^{-1}$) in the Mersey Narrows estuary (Liverpool, England) and the calculated flushing time to be 5.3 days. The annual variations analysis by Pilson (1985), using various monthly time intervals of river discharge and salinity obtained in the Narragansett Bay (Rhode Island, USA) during 1951 and 1977, indicated large flushing time variations, from approximately 12 and 40 days, with the extreme values occurring during periods of high and low river discharges, respectively. Miranda and Castro (1993) investigated the flushing times associated with fortnightly tidal modulation; using data sampled during two spring and neap tide tidal cycles, in the Bertioga Channel (Chap. 1, Fig. 1.5), and obtained values of 2.5 and 3.2 days, respectively.

This methodology was applied by Geyer (1997) using moored measurements and along-estuary hydrographic stations in flushing time studies in two small and shallow (1–2 m depth) sub-estuaries (the Child and the Quashnet) in Waquoit Bay (Cape Cod, USA). These sub-estuaries were forced by different wind directions and intensities, had low average rivers discharges of 0.1 and $0.4 \text{ m}^3 \text{ s}^{-1}$, and had weak spring-neap tidal modulation a tidal height of approximately 0.5 m. This study demonstrates the strong influence of wind forcing on the salinity structure and flushing characteristics of these shallow estuaries. According to the article's conclusions, onshore winds inhibit estuarine circulation, increasing the along-estuary salinity gradient and reducing the flushing rate, due to the landward freshwater accumulation. Offshore winds enhanced the surface outflow, flushing out the freshwater and reducing the along-estuary salinity gradient. The flushing time of the Childs varied from less than one day, in offshore wind conditions, to 2.7 days during strong onshore winds (6.0 m s^{-1}), with a significant correlation at the 95% confidence level. Because onshore wind flushes water into the estuary increasing the MZ geometric volume, the flushing time may be explained by this volume increase, according to the Eq. (6.14), in which the mean salinity increase compensates for the decrease in the $S_0 - \bar{S}$. The flushing time of the Quashnet was

shorter, typically 15 and 17 h, with only one observational in which it was more than one day, and showed little wind-induced variability.

The flushing time may only be applied with rigor to a conservative pollutant that is adequately discharged into the estuary head; however, if the substance is discharged at another longitudinal position, its flushing time will be different (Bowden 1967a, b).

Another physical quantity related to the mixing process—combined effects of salt dilution due to the advection and diffusion, is the time rate exchange of the MZ volume (V) during the flushing time interval (t_q). This quantity (F), named the *flux rate*, is calculated by the ratio $F = V/t_q$, and, according to the Eq. (6.14), it may be calculated as (Officer 1976) and Officer and Kester (1991):

$$F = \frac{V}{t_q} = \frac{S_0}{(S_0 - \bar{S})} Q_f = \frac{Q_f}{\bar{f}}. \quad (6.16)$$

This equation indicates that the flushing rate is directly proportional to the river discharge and inversely proportional to the mean fresh water fraction, which is dependent on the mixing intensity (non-advective tidal processes and advective gravitational exchanges). The determination of the flushing rate with the data of the exercise of this topic (i.e. $50 \text{ m}^3 \text{ s}^{-1}$), and its definition or in function of the Q_f and \bar{f} (Eq. 6.16), then $F = 117 \text{ m}^3 \text{ s}^{-1}$, that is approximately to 2.4 times of the river discharge.

The mean values (\bar{S}) and (\bar{f}) are dependent on the diffusive up-estuary salt transport generated by the tidal forcing, the fresh water discharge and the gravitational circulation. When the estuary is dominated by the river discharge, and the mixing zone (MZ) is advected to the coastal region, the mean salinity and the fresh water fraction tend to zero and one ($\bar{S} \rightarrow 0$, $\bar{f} \rightarrow 1$), respectively. Then, from Eq. (6.16), the flushing rate (F) is equal to the river discharge, and, under this condition, the angular coefficient of this correlation, $F = f(Q_f)$ tends to one. Another limiting condition is: for $Q_f \rightarrow 0$ also $F \rightarrow 0$. When the turbulent diffusive process generated by the tide is predominant (as in a well-mixed estuary), the mean salinity, \bar{S} , may be considered independent on the river discharge. As F is inversely proportional to $(S_0 - \bar{S})$ it is possible, in first approximation, to also consider this rate as independent of the river discharge and $F = \text{const}$. The correlations under these limiting conditions ($F = Q_f$ and $F = \text{const}$.) are illustrated in Fig. 6.2.

To first-order effects, the tidal exchange flux should be independent of the freshwater input into the estuary, disregarding the dependence of the tidal exchange flux on the vertical shear stratification. If, for example, there were no the tidal diffusion exchanges, a plot F versus Q_f should be a curve with a zero intercept for F at $Q_f = 0$, and increasing values of F corresponding with increasing river discharge Q_f (Fig. 6.2, continuous line). Thus, for a more general situation, where both tidal and gravitational circulation processes are operative, the intercept value F_I , for F at $Q_f = 0$, will represent the tidal exchange flux, F values in excess of the intercept value will represent the various freshwater input conditions, and Q_f will represent

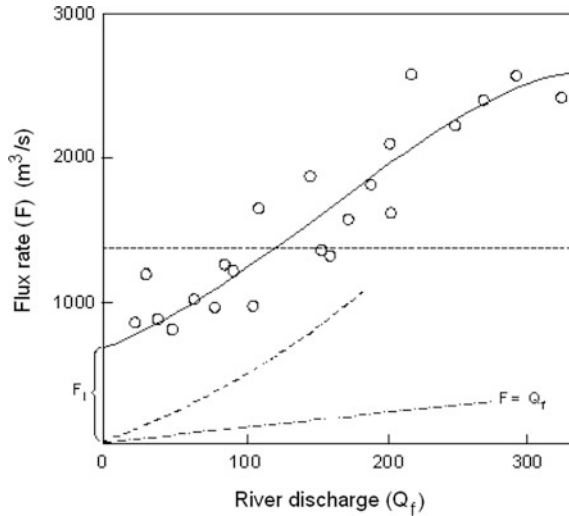


Fig. 6.2 Correlations of the *flushing rate* (F) and river discharge (Q_r) inferred under conditions $F = Q_r$ and $F = \text{const.}$ and with experimental data (\circ), and the mean correlation, for the estuarine system of the Narragansett bay (Rhode Island, USA). The intersection with the ordinate axis (F_I , for $Q_r \rightarrow 0$) represents the diffusive parcel of tidal mixing. The *dashed* and *dashed-point line curves* indicate the flushing rate due to the diffusive and advective processes, respectively (adapted from Officer and Kester 1991)

the gravitational circulation influences on salt flux (Officer and Kester 1991). Then, according to this result, the physical quantities F_I (for $Q_r \rightarrow 0$) and F are related to the mixing parameter, ν , of the classical Stratification-circulation Diagram of Hansen and Rattray (1966) (Chap. 3), which indicates the ratio of the up-estuary salt transport due to diffusion (Φ_{dif}) and the longitudinal total salt flux due to diffusion and advection (Φ_{adv}) expressed by:

$$\nu = \frac{F_I}{F} = \frac{\Phi_{\text{dif}}}{\Phi_{\text{dif}} + \Phi_{\text{adv}}} \tag{6.17}$$

From this ratio it follows that when $F = F_I$ or $F_I \rightarrow 0$ the parameter ν is equal to 1 or 0, respectively, and the up-estuary salt transport predominant to the mixing process is due to tidal diffusion or advection, respectively. This result is in close agreement with the physical interpretation of the ν parameter of the Hansen and Rattray (1966) (Eq. 11.96b, Chap. 11).

The Pilson (1985) data used to calculate flushing time variation, were revisited and the corresponding values of flushing rates (F) were calculated by Officer and Kester (1991). Figure 6.2 is a plot F versus Q_r , showing a well-defined dependence of these variables. An empirical curve has been drawn through the data points, with a zero intercept value for F , around $700 \text{ m}^3 \text{ s}^{-1}$; this value identifies F_I , which is the diffusive component of the tidal mixing. Values of the mixing parameter, ν , were

determined from Eq. (6.17) with tabulated values of F and the zero intercept value, F_1 , which were correlated with the river discharge. From this correlation, it was observed that for $Q_f \rightarrow 0$, the parameter $v \rightarrow 1$, showing that under this condition the estuarine bay, is dominated by tidal mixing. From the tabulated results of Officer and Kester (op. cit.), it is possible to observe that for $Q_f = 154 \text{ m}^3 \text{ s}^{-1}$, the parameter v is equal to 0.52. Then, for this observational period, the dynamic exchange processes due to tidal diffusion and gravitational circulation forced by the river discharge had almost the same magnitude.

Although being a simple procedure, estimation of the relative contribution of the tidal and gravitational circulation to the salt flux, using an alternative methodology to calculate the estuarine parameter v (Eq. 6.17), requires time series measurements of river discharge, and mean salinities of the estuary and in the coastal ocean.

The classical concepts we have introduced here are the basic concepts for the following topics related to the simplified mixing models, which, although semi-empirical, are important for providing the initial knowledge of the main estuarine characteristics influenced by the mixing processes (advection and turbulent diffusion). Their objectives are:

- To determine the salinity, fresh water fraction and the concentration of conservative substances in the estuary, in steady-state conditions.
- To calculate the time interval that the a small fresh water volume remains inside the estuary (flushing time).

6.2 Tidal Prism

The presentation of one-dimensional tidal prism mixing models must be initiated with the *tidal prism*, which is the simplest of the *box model*. The tidal prism will be applied to the salinity determination in an estuary with known tidal amplitude, estuarine surface area, fresh water discharge (Q_f) and coastal ocean salinity (S_0). It is applied to an ideal estuary (Fig. 6.3), with the assumption that the tidal prism ($T_{PR} = V_M$), defined in Chap. 2) with constant salinity (S_0) is completely mixed, with the fresh water from the river discharge introduced into the estuary during the flood tide. We also assume that the estuary is well-mixed (Type 1 or A). The fresh water volume at the disposal of the mixing at high tide is $(1/2)V_M Q_f = (1/2)R$ and, with the hypothesis that the low tidal water volume does not contribute to the mixture, it follow that the equality taking into account the salt mass conservation condition is:

$$\bar{\rho S} \left(V_M + \frac{1}{2} R \right) = \rho_0 S_0 V_M, \quad (6.18)$$

where $V_M = T_{PR}$. In this equation \bar{S} and S_0 are the mean salinity at high tide and that at the coastal sea, respectively.

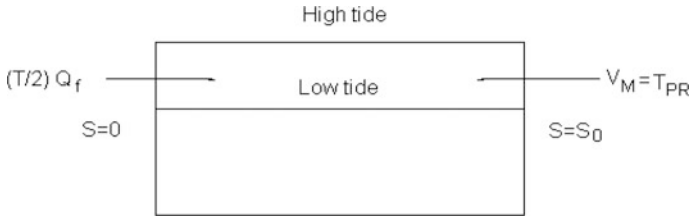


Fig. 6.3 Schematic diagram of the tidal prism model. Water exchanges and the salinities at the boundaries are indicated. For convenience the tidal prism (T_{PR}) will also be denoted by V_M

Solving Eq. (6.18) for the mean salinity value \bar{S} , it follows that the equation to calculate this property at high tide is:

$$\bar{S} = S_0 \frac{V_M}{V_M + \frac{1}{2}R}, \quad (6.19)$$

With this equation, it is possible to calculate the salinity mean value (\bar{S}) at high tide, knowing the tidal prism and the river water volume discharged into the estuary mixing zone (MZ) during half a tidal period, which are quantities that may be known for well-mixed estuaries. With this result, the mean fresh water fraction (\bar{f}) may be calculated by:

$$\bar{f} = \frac{S_0 - \bar{S}}{S_0} = 1 - \frac{V_M}{V_M + \frac{1}{2}R}, \quad (6.20)$$

or

$$\bar{f} = \frac{\frac{1}{2}R}{V_M + \frac{1}{2}R} = \frac{R}{2V_M + R}, \quad (6.21)$$

Taking into account Eqs. (6.1) and (6.5), the fresh water volume (V_f) and the flushing time (t_q) may be calculated by the following equations:

$$V_f = \bar{f}V = \frac{R}{2V_M + R}V, \quad (6.22)$$

and

$$t_q = \frac{V_f}{Q_f} = \frac{R}{(2V_M + R)Q_f}V = \frac{T_P}{2V_M + R}V. \quad (6.23)$$

This result indicates that the lowest flushing time interval T_P (tidal period) occurring when the geometric volume of the MZ (V) is equal to $2V_M + R$, which corresponds to an ideal condition when this volume is entirely removed to the

coastal zone during a complete tidal cycle. In this ideal condition, the mean salinity at the inner mixing zone tends to zero and the flushing rate is equal to the river discharge ($F = Q_f$). The flushing time may also be expressed in tidal period units (T_p). Then, it is suitable to divide Eq. (6.23) by T_p , and $t_q = \frac{V}{(2V_M + R)}$; when expressed in tidal period (T_p), the flushing time is directly proportional to the geometric volume of the MZ and inversely proportional to the tidal prism plus the fresh water discharged during the tidal cycle.

In Eqs. (6.19) and (6.20), the quantity $(1/2)R$ may be substituted by R , resulting in approximate mean values for the salinity and fresh water fraction, (\bar{S}) and (\bar{f}), during a complete tidal cycle. In this condition, it is possible to demonstrate that the fresh water volume and the flushing time are calculated by: $V_f = [R/(V_M + R)]$ and $t_q = [V/(V_M + R)]T_p$, respectively.

Finally, with the Eq. (6.23) and the flux rate (F) definition (Eq. 6.16) it follows that: $F = (2V_M + R)/T_p$.

The solution of this simplest box model (tidal prism) to calculate \bar{S} (Eq. 6.19), the flushing time t_q (Eq. 6.23) and the flux rate F may not give satisfactory results due to the following approximations:

- The fresh river water doesn't mix completely with the seawater during the flood tide or during a complete tidal cycle;
- The coastal region isn't a perfect sink and a water parcel flushed out to the near shore turbidity zone (NTZ) may return to the estuary in the next tidal cycle.

6.3 Segmented Tidal Prism Model

The tidal prism model hypothesis assumes a uniform steady-state salinity distribution at high tide, and can be applied to well-mixed estuaries. To eliminate this restriction, it has been re-worked by several researchers aiming to enable its application to stratified estuaries. In the pioneering article by Ketchum (1951), a one-dimensional model was presented, where the mixing zone (MZ) was partitioned in segments or cells.

The main hypothesis of the segmented tidal prism model is complete mixing of the river and sea water in each segment or cell at high tide. The conservation equation for this model is based on the principle of volume continuity of fresh water volume in the estuary. According to Ketchum (op. cit.), complete mixing occurs at high tide in each segment, while in the Dyer and Taylor (1973) model, complete mixing may occur in the segments at high and low water levels. Then, the segmented tidal prism may be applied to one-dimensional well-mixed estuaries ($v = 1$, according to the Hansen and Rattray 1966 classification method), which are dominated by vertical turbulent tidal diffusion.

Semi-empirical models of Ketchum and Dyer and Taylor will be presented and applied to an ideal estuary to an ideal estuary, and inter-comparisons will also be

made with observational data collected in the Winyah Bay estuarine system (South Carolina, USA).

The Ketchum paper presents a semi-empirical theory on the mixing of fresh river water with the seawater in selected segments distributed along the estuary (Fig. 6.4). The theory attempts to predict average conditions in successive volume segments for a constant river discharge and a mean tidal range. According to this theory, it is possible to calculate the one-dimensional mean salinity, the fresh water fraction distribution and the flushing time. This theory uses the fresh water as an indicator and may be easily adapted to include the one-dimensional variation of any conservative property concentration dissolved in the mixing zone. This theory assumes the following hypothesis:

- Steady-state river discharge and salinity field and a balance between inflow and outflow of sea water.
- Full mixing of fresh and salt water during flood tides.
- During a tidal cycle, a seaward volume of fresh water, equal to the input of fresh water discharged at the estuary head, must be moved.
- Salinity at the coastal sea has small temporal variation ($S_0 \approx \text{const.}$).

The inner end of the estuary (segment 0) is defined as the section above which the volume required to raise the water level from low to high water is equal to the river discharge input during the tidal cycle (Fig. 6.4); during the ebb tide, there will be a loss through this section of one river flow volume per tide. Then, by this definition, there is no seawater interchange at the boundary between *segments* 1 and 0, segment 0 being completely fresh water. During the flood tide and above the boundary of segments 0 and 1, the salinity value and the fresh water fraction are 0 and 1, respectively. It should be noted that this is a dynamic boundary, not a geometric definition, since the boundary location will move corresponding to changes in river discharge (Ketchum 1951).

The segment volumes along the estuarine channel are calculated by their water volume at low tide (V_n), with their corresponding tidal prism (P_n) volumes added at high tide, as indicated in Fig. 6.4. Then, at high tide, the segment volume are equal

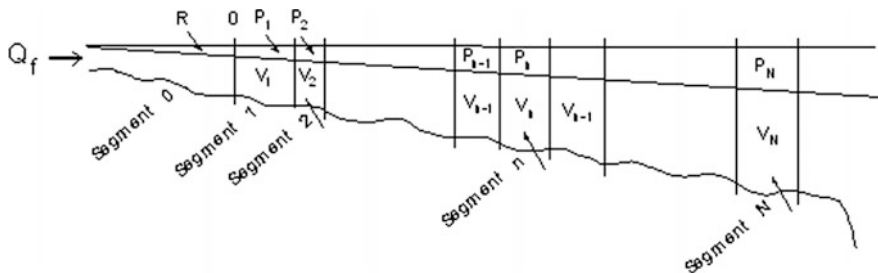


Fig. 6.4 Segmentation along an estuary model according to Ketchum (1951). P_n and V_n ($n = 0, 1, 2, \dots, N$) indicate tidal prism and low tidal volumes in a generic segment n . In the inner most segment ($n = 0$) there will be only fresh water given by $R = T_p Q_f$ (adapted from Dyer and Taylor 1973). The original notation of the tidal prism as P_i (for $i = 0$ to N) has been maintained

to $V_n + P_n$. It should be noted that, according to this notation, the index $n = 0, 1, 2, \dots, N$ indicates the segments located along the estuary, from the segment 0 (with $S = 0$), up to the segment N , located at the estuary mouth; by hypothesis, this last segment has the same salinity as the coastal region, $S(N) = S_N = S_0$, and along the estuary the total number of segments is $N + 1$.

From the definition of the inner most tidal prism segment (*segment 0*), its tidal prism is equal to the fresh water volume accumulated during a complete tidal period, that is, $P_0 = T_P Q_f = R$. As the fresh water volume is filled during the flood tide, Dyer and Taylor (1973) suggested that this volume should be equal to $(1/2)R$, because in the Ketchum's original theory the corresponding time interval $(1/2)T$ of the ebb tide has not been taken into account. However, the original theory will be presented, and later the changes which may be applied following the suggested correction. Then, the *segment 0* volumes are equal to V_0 and $V_0 + P_0$ at low and high tidal, respectively.

According to Ketchum (1951), consecutive volume segments are defined so that the distance between their inner and outer boundaries are equal to the average excursion of a water element on the flood tide. The average excursion is derived from the water volume entering each part of the estuary on the flood tide, as well as on the estuary topography. If the water volume entering with the flood tide was to act as a piston, displacing and pushing an equivalent volume of water upstream from the next landward segment, the distance moved would be the average excursion of a particle of water on the flood tide in that part of estuary. By definition, the high tide volume of any segment along the estuary length is equal to the low tide volume in the adjacent seaward segment. Consequently, along the estuary each segment is defined by the high tide volume in the landward segment, which is equal to the low tide volume in the adjacent seaward segment. Beginning with the *segment 0*, defined above, the entire estuary can thus be subdivided into a series of volume segments composed of low tide volume (V_n) for a generic segment, and the local intertidal or tidal prism volume (P_n), as shown in Fig. 6.4. Salinities and volumes in the segments are distributed as follows: the *segment 0* at high tide (P_0 , with salinity zero), and the corresponding landward segments, *segment n*, for $n = 1, 2, \dots, N$, with volumes equal to $V_n + P_n$ and salinities $S(1), S(2), \dots, S(N)$. By definition, the segment located at the estuary mouth has salinity equal to the adjacent coastal sea S_0 , $S(N) = S_0$.

The tidal excursion, defined in Chap. 2 (Eq. 2.26) is directly proportional to the amplitude of the velocity generated by the tide, or to the tidal amplitude at the estuary mouth; high values of these quantities generate high salinity intrusion lengths, increasing the number of volume *segments* along the estuary.

With $R = T_P Q_f$ indicating the volume of the river water introduced during the tidal cycle, and from the above definitions of P_n and V_n as segment volumes, the following equations summarize these fundamental definitions (Ketchum op. cit.):

$$P_0 = R, \quad (6.24)$$

$$V_1 = V_0 + P_0, \quad (6.25)$$

$$V_2 = V_1 + P_1 = V_0 + P_0 + P_1, \quad (6.26)$$

$$V_3 = V_2 + P_2 = V_0 + P_0 + P_1 + P_2, \quad (6.27)$$

...

$$V_N = V_{N-1} + P_{N-1} = V_0 + \sum_i P_i, \quad (i = 1, 2, \dots, N - 1), \quad (6.28)$$

or generically $V_n = V_{n-1} + P_{n-1}$, with $n = 1, 2, \dots, N$.

The next step in performing the practical segmentation of the estuary is the geometric determination of the volume segments at low tide. A bathymetric nautical chart can be used to subdivide the estuarine channel into auxiliary cross-sections, and from this determine the segment areas and volumes. If there is no such nautical chart for the region being investigated, echo-sounding measurements of the estuarine channel must be made.

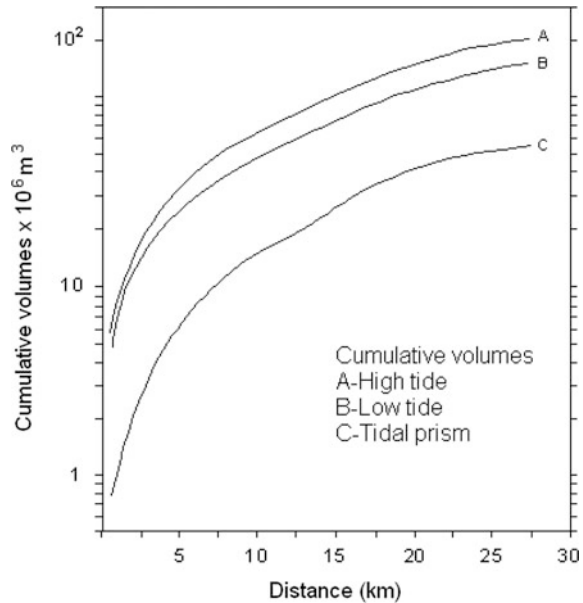
The volume of each segment is calculated as the product of the distance between the areas and the mean cross-section of its limiting areas. The chosen distance between the cross section areas along the estuarine channel and the depths variations must be as uniform as possible, to minimize errors in the calculations due to the non-uniformity of the limiting areas. With the data generated from this procedure, it will be possible to calculate the sum of the cumulative low tide volumes along the estuary, and these cumulative values may be plotted as function of the longitudinal distance from the head to the estuary mouth.

The tidal prism volumes from each auxiliary partition are calculated from the product of the surface area between limiting cross-sections and the tidal height; details of these volume calculations may be obtained in Anderson (1979). These volumes may also be cumulatively plotted as functions of the longitudinal distance between the head and estuary mouth. Finally, the sum of the low tide volumes (V_n) and their corresponding tidal prism volumes (P_n) are equal to the high water volumes ($V_n + P_n$), which may also be plotted as function of the estuary longitudinal distance.

The longitudinal variation of the cumulative sum of the high water volumes ($\sum_n (P_n + V_n)$), low ($\sum_n V_n$), and tidal prism volumes ($\sum_n P_n$), were calculated for the Winyah Bay estuarine system (South Caroline, USA), and indicated by curves A, B and C, respectively (Fig. 6.5).

If the fresh water volume discharged by the river during the tidal period (R) is known, it is possible to perform the estuary segmentation according to the equations system (6.24–6.28), and determinate its geometric limits along the estuary. In fact, by plotting the ordinate $P_0 = R$ in Fig. 6.5, the interception with curve C (tidal prism) determines the landward limit of the *segment 0*. With this abscissa value, it is possible to determine the ordinates corresponding to the volumes V_0 and $V_0 + P_0$

Fig. 6.5 Cumulative volumes at high (A) and low (B) tides and the tidal prism (C), as function of the longitudinal distance in the Winyah Bay (South Carolina, USA) estuarine channel



from curves B (low tide) and A (high tide), respectively. In the segmentation system of equations, the volume *segment 1* at low tide is equal to $V_0 + P_0$ (Eq. 6.25); then, on the cumulative curve B, this ordinate determines the landward limit of this segment in the abscissa axis. In turn, this abscissa value on curve A corresponds to the volume $V_1 + P_1$, which is equal to the volume V_2 , according to the segmentation equation (Eq. 6.26). Using this V_2 value, the process may be repeated considering curve B.

This procedure will then be repeated until the last estuary segment is found in the abscissa axis of Fig. 6.5. Then, the volume of each segment at low tide is equal to the adjacent segment at high tide (Fig. 6.4). As the salt intrusion length limits the upper MZ position, the segmentation process is very important to this theory application, enabling the geometric limits of the segments their volumes at low (V_n) and high ($V_n + P_n$) tides, from the estuary head down to the mouth, to be obtained.

The estuary divided into volume segments as described above, indicates the limits of each segment and the average excursion of a water element with the flood tide. With the assumption that the water within such a volume segment is completely mixed at high tide, the proportion of water removed on the ebb tide will be given by the ratio of the local intertidal volume and the high tide volume of the segment. This proportion of river water will be removed by the ebb tide taking with it any particles dissolved or suspended in it. Thus, an exchange ratio (r) was defined by Ketchum (1951), for a generic segment n as the ratio of the tidal prism (P_n) by the high tide element volume ($V_n + P_n$),

$$r_n = \frac{P_n}{V_n + P_n} \quad (6.29)$$

This ratio quantifies the fraction of fresh water renewed of the total fresh water discharged into the estuary in a complete tidal cycle. Its extreme values are $r_n = 0$ and $r_n = 1$ due to the following conditions:

- $r_n = 1$, when the tidal height is equal to the estuary depth and all water is removed at the low tide ($V_n \rightarrow 0$ or $V_n \ll P_n$);
- $r_n < 1$ or $r_n \ll 1$, in estuaries forced by regular or micro tidal forcings ($P_n < V_n$ or $P_n \ll V_n$);
- $r_n = 0$, when tidal prism is too low ($P_n \rightarrow 0$).

In the first condition ($r_n = 1$), the estuary is an ideal system to renewing the salinity or concentration of any substance in its water body, because the water volume introduced during a complete tidal cycle is completely removed during the ebbing tide, acting as a perfect sink. In the third condition ($r_n = 0$) there is no water renewal in the estuary and it may accumulate salt (or other substances). The intermediate condition is the most common in partially or highly stratified estuaries.

The river water present in the estuary is a mixture of fresh and salt water, accumulated during many tidal cycles. In the condition of a constant input of fresh water, during each tidal cycle, each segment receives an influx of river water (R) equal to the total volume introduced into the estuary by the river during the tidal period. Taking R_1 as the volume of river water entering a segment during the current tidal cycle (*age of one tidal cycle*), then the amount removed on the ebb tide will be $r_n R_1$, and the amount behind will be $(1 - r_n)R$. Considering one step forward in time, the portion of river water that arrived in the previous time-step, R_2 , was not fully removed during ebb currents. Therefore, the remaining portion of fresh water from previous time-step is required to be taken on the following time-step (e.g. *age of two tidal cycles*). For two time steps (or two tidal cycles), fresh water removed will be $r_n(1 - r_n)R_2$ and the remaining fresh water for two successive ebb tides, will be $(1 - r_n)^2 R_2$. The proportion of water of various tidal ages which is removed (1, 2, 3, ... m), or remaining behind within the segment, as a result of the exchanges on any given ebb tide, may be summarized as follow in Table 6.1 (Ketchum 1951).

Table 6.1 Water mass volumes removed and accumulated into the estuary as function of the tidal age (according to Ketchum 1951)

Tidal age	Removed fresh water (volumes)	Accumulated fresh water (volumes)
1	$r_n R$	$(1 - r_n)R$
2	$r_n(1 - r_n)R$	$(1 - r_n)^2 R$
3	$r_n(1 - r_n)^2 R$	$(1 - r_n)^3 R$
...
m	$r_n(1 - r_n)^{m-1} R$	$(1 - r_n)^m R$
Σ	R	R/r_n

The parcels summation of the first line of Table 6.1 is the fresh water balance at the tidal age 1 and, as this result is equal to R , the principle of volume conservation in this first tidal cycle (*tidal age 1*) is satisfied. The second column corresponds to the total water volume removed during the successive tidal cycles. As the ratio of terms of this row is constant and equal to $(1 - r_n)$, its summation may be easily calculated by the formula of the geometric progression series. Considering a series with a great number of elements ($m \rightarrow \infty$), it is convergent to R which confirms the fresh water volume conservation.

As, by hypothesis, the fresh water input is constant, all values R are equal and the steady-state condition can be assumed. The total volume (V_f) of river water accumulated within any volume segment (n) of the estuary at high tide, is calculated by the sum of the remaining volumes given in the final column of Table 6.1. Since the equation is written for the high tide condition, one volume of the river flow (R) which has not been depleted is also present, and the fresh water volume, $(V_f)_n$, accumulated in high tide is:

$$(V_f)_n = R \left[1 + (1 - r_n) + (1 - r_n)^2 + \dots + (1 - r_n)^m \right]. \quad (6.30)$$

As the expression between the square bracket is the sum of a geometric progression with a ratio equal to $(1 - r_n)$, the fresh water volume accumulated in the segment, n , is calculated by:

$$(V_f)_n = \frac{R[1 - (1 - r_n)^m(1 - r_n)]}{[1 - (1 - r_n)]}. \quad (6.31)$$

As $(1 - r_n) \leq 1$ and the number (m) of tidal cycles (m) is great, the final result for the fresh water volume is:

$$(V_f)_n = \frac{R}{r_n}, \quad (6.32)$$

and is determinate by the volume of the fresh water discharged by the river during the tidal cycle (R) divided by the exchanged ratio r_n (Eq. 6.29). This relationship states that the volume of fresh water discharge is flowing seaward during the tidal cycle, and is the product of the exchange ratio (r_n) and the accumulated volume of river water (rQ_f), satisfying the hypothesis of the steady-state condition.

The exchange ratio was defined on the assumption of complete mixing of the water mass in each segment at high tide. The average excursion of seawater during the flood tide is presumed to set the upper limit of the saline intrusion length, over which complete mixing was assumed.

Before using this method to calculate the fresh water fraction (f), the average longitudinal salinity, $S = S(x)$, and the flushing time (t_f), it is opportune to observe that Eq. (6.32) allows immediate determination of the flushing time in a generic segment, using its definition (Eq. 6.1):

$$(t_q)_n = \frac{(V_f)_n}{Q_f} = \frac{R}{r_n Q_f} = \frac{T_P}{r_n}. \quad (6.33)$$

This equation agrees with the result already obtained with the Eq. (6.2); the lower flushing time ($t_q = T_P$) occurs when the exchange ratio is equal to one, $r_n = 1$; also, $t_q \rightarrow \infty$ when $r_n \rightarrow 0$.

Combining the previously calculated values for the fresh water volume discharge ($R = T_P Q_f$), the tidal prism (C) and the low tidal volumes (B) obtained from Fig. 6.5, with Eqs. (6.29) and (6.32), it is possible to calculate the accumulated fresh water volumes in the generic segment (n) applying the fresh water fraction definition (6.5), and the result is:

$$f_n = \frac{V_{fn}}{V} = \frac{V_{fn}}{V_n + P_n} = \frac{R}{r_n(V_n + P_n)} = \frac{R}{P_n}. \quad (6.34)$$

Using Eq. (6.10), which defines the fresh water fraction as a function of the salinity, it follows that:

$$f_n = \frac{S_0 - S_n}{S_0}, \quad (6.35)$$

and combining this definition with Eq. (6.34), the mean salinity at the segment, n , may be calculated from the known undiluted salinity at the coastal ocean (S_0),

$$S_n = S_0(1 - f_n), \quad (6.36)$$

and, combining with Eq. (6.34),

$$S_n = S_0 \left[1 - \frac{V_{fn}}{(P_n + V)_n} \right] = S_0 \left(1 - \frac{R}{P_n} \right). \quad (6.37)$$

From this result, it is possible to calculate the mean salinity (S_n) at each segment, for $n = 0, 1, 2, 3, \dots, N$. It then follows that for the segment 0 the salinity is zero, because at this segment $P_0 = R$ (Eq. 6.24).

The flushing time for the *segment n* may be calculated by Eq. (6.33) and its sum for each segment is equal to the estuary flushing time,

$$t_q = T_P \sum_n \frac{1}{r_n}, \quad (6.38)$$

or in tidal period (T_P) units,

$$t_q = \sum_n \frac{1}{r_n}. \quad (6.39)$$

To satisfy the assumption of the complete mixing of the fresh river water discharge and the seawater at high tide, this method must be applied to well-mixed estuaries or to low stratified partially mixed estuaries. Thus, the incomplete mixing in the segments implies difficulty in application of this. However, in such cases, the exchange ratio (r_n^*) will be dependent on the segment depth (h) and on the well-mixed thickness (D) (Ketchum 1951):

$$r_n^* = \frac{P_n}{V_n + P_n} \left(\frac{D}{H} \right). \quad (6.40)$$

In this equation, D is the height of the *segment n*, and H is the mixed layer thickness (or also its height). When the exchange ratio is larger ($r_n^* > r_n$), then the resulting accumulation of river water ($(V_f)_n = R/r_n^*$) will be small. In cases, the segmentation of the estuary is also made using volumes computed to the mixed layer depth. The entire treatment is therefore developed, with the assumption that the water below the mixed layer takes no part in the tidal mixing.

The Ketchum's method has been applied for three different estuaries in almost all characteristics, the Raritan river and Bay (New Jersey, USA), the Alberni Inlet (Columbia, Canada) and Great Pond (Massachusetts, USA); however, the method was only described in detail for the Raritan river, and the theoretical results corresponded closely to the observed distributions of salinity and fresh water.

For simplicity, the method was applied for a model estuary with rectangular cross sections and constant depth, and with equal low tide and tidal prism volumes ($V_n = P_n$); then, the seaward variations of these cumulative volumes are equal, its longitudinal distributions are coincident ($B = C$), and the volumes at high tide ($V_n + P_n$) are indicated by (A), as shown in Fig. 6.6. For further simplification, for the tidal cycle, a fresh water discharge equal to one ($R = 1$) implying $P_0 = 1$, was adopted. This ordinate, plotted in the figure, starts the segmentation process, enabling the determination of the geometric limits of the estuary segments. As $V_n = P_n$ and $R = 1$, it is possible, using Eqs. (6.29) and (6.32), to calculate the exchange ratio (r_n) and the fresh water volume (V_f) retained in the segments, which are constants equal to $1/2$ and 2, respectively.

The calculate values of the exchange rate, fresh water volume, relative salinity (S/S_0) and the flushing time (t_q) in tidal period units are presented in Table 6.2. The relative salinity in the *segment 0* is zero, and converges to one (1) at *segment 10*; this convergence is accentuated in the first segments and tends asymptotically to one (1) from *segment 4*. The flushing time of this model estuary, determined by the sum of the corresponding value of each segment (2), is 20 tidal periods.

The semi-empirical segmented tidal prism was applied to the estuarine system of Winyah Bay (South Caroline, USA) (Fig. 6.7). As this estuary is partially mixed, but with low vertical stratification, the exchange ratio was calculated with the assumption that it is well-mixed, and its segmentation was performed with the longitudinal variation of the cumulative tidal prism at low and high tide, as presented in Fig. 6.5. The results in Table 6.3 were calculated using the following hydrologic and hydrographic data: input of the average discharge of fresh water by the river

Fig. 6.6 Schematic diagram of the longitudinal variation of the cumulative volumes of high (A) and low (B) tide, and the tidal prism (C) of an estuary model with $V_n = P_n$, and $R = 1$, according to Miranda (1984)

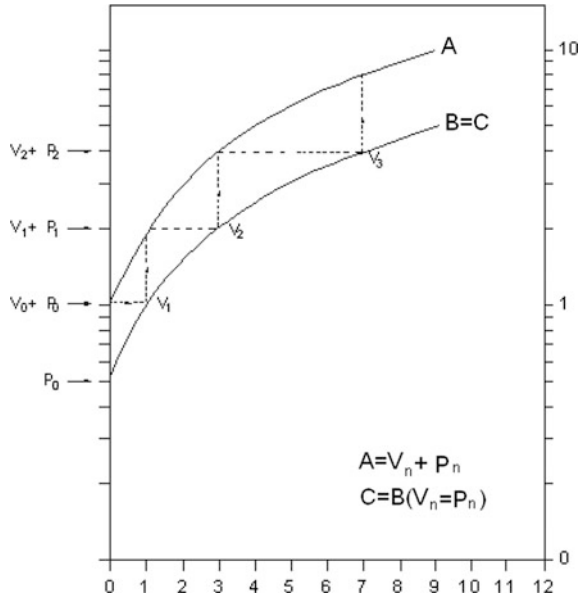


Table 6.2 Results of the estuary model ($V_n = P_n$ and $R = 1$): exchange ratio (r_n), fresh water volume ($(V_f)_n$, relative salinity (S/S_0) and flushing time $(t_q)_n/T$

n	V_n	$V_n + P_n$	r_n	$(V_f)_n$	f_n	S/S_0	$(t_q)_n/T$
0	1	2	1/2	2	1	0	—
1	2	4	1/2	2	0.5	0.5	2
2	4	8	1/2	2	0.25	0.75	2
3	8	16	1/2	2	0.125	0.875	2
4	16	32	1/2	2	0.062	0.938	2
5	32	64	1/2	2	0.032	0.968	2
6	64	128	1/2	2	0.016	0.984	2
7	128	256	1/2	2	0.008	0.992	2
8	256	512	1/2	2	0.004	0.996	2
9	512	1024	1/2	2	0.002	0.998	2
10	1024	2048	1/2	2	0.000	1.000	2

during a tidal cycle, $R = 8.6 \times 10^6 \text{ m}^3$, and non-diluted salinity at the coastal sea, $S_0 = 34.0\text{‰}$.

Table indicates that, using this method, only a few segments were determined along the estuary, and the flushing time was calculated to be 14.4 semi-diurnal tidal cycles (7.2 days). Of course, the steady-state longitudinal variation of salinity, forced by river discharge and tide, must be validated with observational data.

To calculate the results presented in Tables 6.2 and 6.3, the estuary segmentation process from the longitudinal variation of segment volumes A, B and C (Fig. 6.5), associated with Eqs. (6.24–6.28), were used, along the following equations:

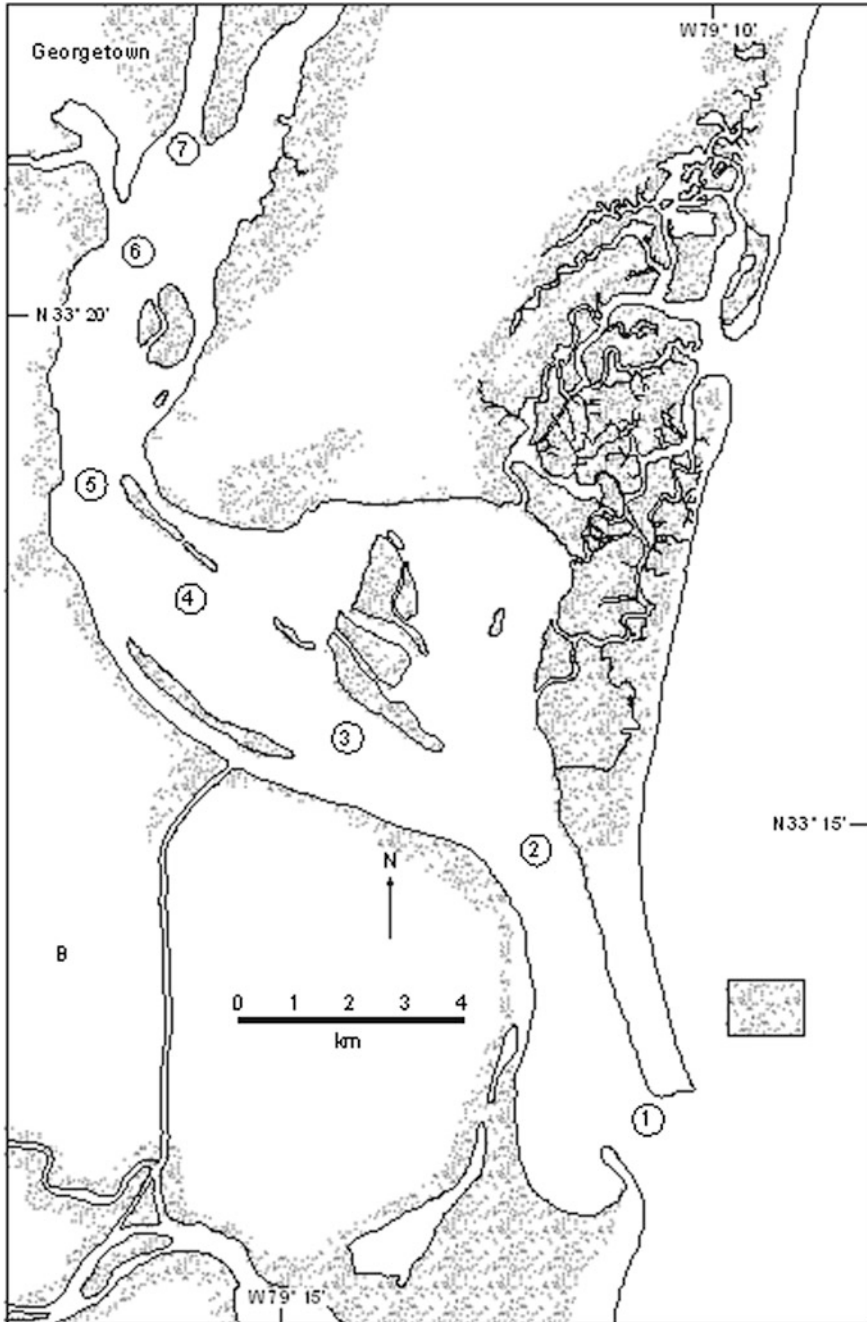


Fig. 6.7 Winyah Bay estuarine system located SE of South Carolina (USA). The along channel numbers (1–5) indicate the segments boundaries

Table 6.3 Results of the Ketchum (1951) method applied to the Winyah Bay estuary

n	$V_n + P_n$	r_n	$(V_f)_n$	f_n	S		$(t_q)_n/T$
0	25.3	0.17	25.3	1.0	0.0	–	–
1	60.5	0.28	30.7	0.87	4.4		3.6
2	111.7	0.31	27.7	0.54	15.6		3.2
3	182.7	0.28	30.8	0.43	19.4		3.6
4	–	0.25	34.4	0.36	21.8		4.0
5	–	–	–	0.00	34.0		–

Salinity (‰), flushing time $[(t_q)_n/T]$. With: $R = 8.6 \times 10^6 \text{ m}^3$, $S_0 = 34.0\text{‰}$ and V_n and P_n (in unities of 10^6 m^3)

$$r_n = \frac{P_n}{V_n + P_n}; \quad (V_f)_n = \frac{R}{r_n}; \quad f_n = \frac{(V_f)_n}{V_n + P_n}. \tag{6.41}$$

and

$$S_n = S_0(1 - f_n), \quad \text{and} \quad (t_q)_n = \frac{T_P}{r_n}. \tag{6.42}$$

In the Ketchum’s theory, the tidal prism volume of the *segment 0* was taken as R ($P_0 = T_P Q_f = R$). However, as previously mentioned, Dyer and Taylor (1973) suggested a correction for this volume as half of the value in the original paper, $P_0 = (1/2)R$. Although this correction is applied at the very beginning of the segmentation procedure, and therefore alters the volume of segments along the MZ, all semi-empirical equations from Ketchum’s original paper remain the same.

This method has been applied in several investigations, and, in some cases, the longitudinal mean salinity distribution values were acceptable, however, in others they were not. These inconsistent results indicated that further investigations should be sought, and a modified version of the original segmented tidal prism model was developed by Dyer and Taylor (1973). The model presented by Dyer and Taylor was based partly on a combination of Ketchum’s method and Maximon and Morgan’s (1955) concepts, allowing for additional inflows into the estuary from tributaries and outfalls, while keeping the method simple, with more consistent physical interpretations of the mixing processes and fresh water continuity.

In order to make this second method more comprehensible, the fundamental differences between the methods of Ketchum (1951) and the Dyer and Taylor (1973), will be described, including the terminology and notation of variables. According to Maximon & Morgan’s (op. cit.) theory, the seaward mean salinity is calculated at high and low tide, allowing for time dependence of various quantities involved and the introduction of solutes (or salinity) into the estuary. Secondly, in Dyer and Taylor’s segmentation equations, a non-dimensional *mixing parameter* (a) was included, enabling adjustments and validation based on experimental data. Concerning terminology and notation, the term *fresh water concentration* (C) in Dyer and Taylor’s, was used instead *fresh water fraction* (f); thus $C = f$, and,

according to Dyer and Taylor's original paper, the conditions at high and low tides will be identified by upper letters H and L, respectively, and the following equalities will exist: $C_n^H = f_n^H = f_n$. In this case, the *fresh water concentration* at high tide in the segment, n , is numerically computed following similar approach to the fresh water fraction of Ketchum's paper.

The *segment 0* contains only fresh water, and its fresh water concentration will be denoted as C_0 , which by definition is equal to one ($C_0 = 1$). For the segment located at the estuary mouth ($n = N$), the fresh water content is practically equal to zero, following the equality $C_N^H = C_{N+1}^L = 0$; then, the $N + 1$ index for the low tide concentration indicates the segment adjacent to the estuary mouth, located coastal region.

The segmentation of the Ketchum's model prescribe that the low tide volume of a generic segment ($n + 1$) is equal to the high tide volume of the adjacent *segment* n , located landward ($V_{n+1} = V_n + P_n$). This process implies that during the flood a volume equal to V_{n+1} crosses this segment boundary. Also, as R is the fresh water volume accumulated during the tidal cycle, the water volume transported through the segment boundary in the ebb tide is equal to $V_{n+1} + R$. Then, taking into account the fresh water concentration (and hence fresh water fraction) for a complete tidal cycle, the following identity to satisfies the principle of fresh water conservation (Dyer and Taylor 1973):

$$(V_{n+1} + R)C_n^H - V_{n+1} \cdot C_{n+1}^L = RC_0, \quad n \geq 0. \quad (6.43)$$

This identity is satisfied only when $C_n^H = C_{n+1}^L = C_0 = 1$. As previously indicated, the fresh water concentration and fresh water fraction are equal numeric quantities at high tide, that is:

$$C_n^H = f_n^H = \frac{(V_f)_n}{(V_n + P_n)}. \quad (6.44)$$

However, according to Eqs. (6.29) and (6.32),

$$(V_f)_n = \frac{R}{r_n} = R \frac{(V_n + P_n)}{P_n}. \quad (6.45)$$

Combining Eqs. (6.44) and (6.45), it follows that:

$$C_n^H = f_n^H = \frac{R}{P_n} = C_0 \frac{R}{P_n}. \quad (6.46)$$

With the fresh water balance expressed by Eq. (6.43), we have already concluded that $C_n^H = C_{n+1}^L = C_0 = 1$; this result is incompatible with Eq. (6.46), because it is true when $R = P_n$. Then, it was shown that the Ketchum's model doesn't agree completely with the principle of volume conservation, because the

fresh water concentration is not taken into account during the transition from low to high tide.

Dyer and Taylor’s model retains the simplicity of the Ketchum’s method and assures a more consistent fresh water balance, applying the same simplifying hypothesis: stationary conditions of the mean salinity field and complete mixing at low and high tide.

The geometric limits of the estuary segments are also determined using prior knowledge of the cumulative volumes from the head and estuary mouth at low and high tides, exemplified for the estuary system of Winyah Bay (Fig. 6.5, curves B and C, respectively). Using the same notation for the identification of the volume segments at low tide (V_n) and the tidal prism (P_n), the estuary segmentation is schematically shown in Fig. 6.8.

Volumes $(1 - a)V_n$ at low tide ($n = 2, 3, \dots N$) between the segment (sections A and B in Fig. 6.8), limited by the dashed line in this figure are accounted in the mixing at high tide. Then, the total volume of this segment at high tide is equal to $(V_n + P_n)$ because:

$$(1 - a)V_n + aV_n + P_n = V_n + P_n, \tag{6.47}$$

with $n = 1, 2, 3 \dots N$. The parameter associated with the mixing process (a) may vary from zero to one. It could, in principle, be determined from observational tidal excursion data, and potentially to vary from one segment to another.

Similar to Ketchum’s method, the upstream end of the model is defined by the section across which there is no flow during the flood tide. If R is the river flow per tidal cycle, the tidal prism volume above the segment 0 will be $R/2$ (not R as stated by Ketchum). This definition is unaffected if the tidal limit is determined by a weir. The segmentation equations of this model are (Dyer and Taylor 1973):

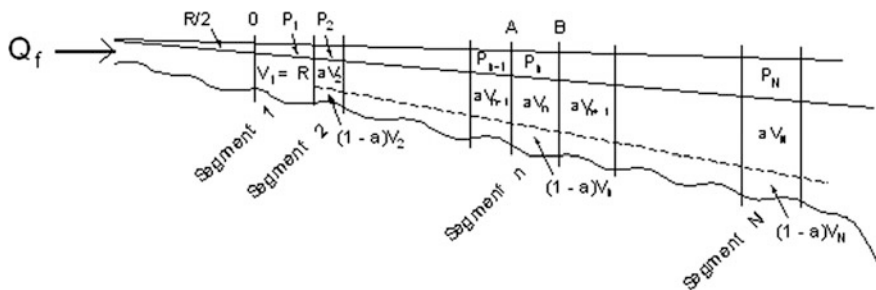


Fig. 6.8 Dyer and Taylor’s estuary segmentation. P_n and V_n are the volumes of the tidal prism at low and high tide in the generic element n (a is the mixing parameter), $(1 - a)V_n$ is the low tide volume to be used for mixing at high tide. A and B are control boundaries [after Dyer and Taylor (1973)]

$$V_1 = R, \quad (6.48)$$

$$aV_2 = P_1, \quad (6.49)$$

$$aV_3 = aV_2 + P_2 = P_1 + P_2, \quad (6.50)$$

$$aV_N = aV_{N-1} + P_{N-1} = \sum_i P_i (i = 1, 2, 3, \dots, N - 1), \quad (6.51)$$

or generally, $aV_n = aV_{n-1} + P_{n-1}$, for $n = 3, 4, \dots, N$. If the mixing parameter is equal to one ($a = 1$), these equations are equal to the Ketchum's segmentation (Eqs. 6.26–6.28). In Dyer and Taylor's analysis, this parameter was assumed to be constant ($a = \text{const.}$), giving reasonable agreement between computed and observed high and low tide mean salinity distributions.

Equations (6.48–6.51) indicate that the segments are defined as follows: on the flood tide, the water volume occupying aV_{n+1} of the segment $n + 1$ is moved up-estuary to occupy the volume $aV_n + P_n$ at high tide, or just the volume P_1 in the segment 1. During this process, it is assumed that the volume at high tide is mixed with the portion of water remaining in the segment, n , from the low, i.e., with $(1 - a)V_n$, or with the volume V_1 , when $n = 1$, because at low tide this volume is entirely supplied by the river discharge. Then, the high tide volume at any segment is equal to $V_n + P_n$, for $n = 1, 2, \dots, N$ (Fig. 6.8).

This model may be applied for different river discharge volumes, and also taking into account additional fresh water contributions in the MZ boundaries. The volume of segments increases seaward, and if a volume aV_{n+1} crosses the segment boundary (B in Fig. 6.8) during the flood, due to volume continuity a water volume equal to $aV_{n+1} + R$ will cross this boundary during the ebb tide.

After defining the estuary segmentation, the following step is to find the equations to calculate the concentrations C_n^H and C_n^L for each segment. This may be established by applying the volume continuity, to assure that during each tidal cycle the fresh water volume transport out of the estuary is equal to R .

6.3.1 High Tide Fresh Water Balance

Consider a generic control segment n th, which occupies a volume aV_{n+1} ($aV_{n+1} = aV_n + P_n$, according to the segmentation Eqs. 6.48–6.51) at high tide, and is completely mixed with a water volume $(1 - a)V_n$ disposable at low tide (Fig. 6.8). Then, according to the volume conservation principle, during the flood tide the following fresh water balance will occur through boundary B of this segment:

$$(V_n + P_n)C_n^H - (1 - a)V_n C_n^L = aV_{n+1} C_{n+1}^L, \quad n \geq 2, \quad (6.52)$$

or

$$(V_n + P_n)C_n^H = aV_{n+1}C_{n+1}^L + (1 - a)V_nC_n^L, \quad n \geq 2. \quad (6.53)$$

For the segment $n = 1$, the fresh water balance, equivalent to the Eq. (6.52) is given by the following expression:

$$(V_1 + P_1)C_1^H - V_1C_1^L = aV_2C_2^L. \quad (6.54)$$

6.3.2 Low Tide Fresh Water Balance

Again considering the n th control segment, in the ebb tide the flow travels from the adjacent segment $(n - 1)$ to the segment, n , through the control boundary A (Fig. 6.8). Then, a water volume $(aV_n + R)$, with a concentration C_{n-1}^H , will enter the segment, n , and mix with the water volume $(1 - a)V_n - R$ that remained in this segment at low tide, with a concentration C_n^H , in the segment n . With this procedure, we are making the assumption that an additional volume, equal to R , is coming from the water volume $(1 - a)V_{n-1}$. Then, to establish the volume conservation it is necessary that:

$$V_nC_n^L = (aV_n + R)C_{n-1}^H + [(1 - a)V_n - R]C_n^H, \quad n \geq 2. \quad (6.55)$$

For *segment 1* holds the following conservation equation:

$$V_1C_1^L = RC_0. \quad (6.56)$$

As according to the segmentation process, $V_1 = R$ (Eq. 6.48), it follows from this equation that $C_1^L = C_0 = 1$.

6.3.3 Fresh Water Balance During the Tidal Cycle

An additional relationship may be obtained with the assumption, according to the volume conservation principle, that after a complete tidal cycle the net water volume flow across any cross section boundary is equal to $R = RC_0$, then:

$$(R + aV_{n+1})C_n^H - aV_{n+1}C_{n+1}^L = RC_0, \quad n \geq 1. \quad (6.57)$$

Equations (6.53), (6.55) and (6.57) are not independent, and the unique relationship with the two unknowns C_n^L and C_n^H may be obtained by combining Eqs. (6.53) and (6.57). In effect, Eq. (6.57) may have its parcels rearranged as:

$$R(C_0 - C_n^H) = aV_{n+1}C_n^H - aV_{n+1}C_{n+1}^L, n \geq 1. \quad (6.58)$$

Now, the terms of Eq. (6.53) will be rearranged in order to isolate the last parcel of Eq. (6.58), and the result is:

$$aV_{n+1}C_{n+1}^L = (V_n + P_n)C_n^H - (1 - a)V_nC_n^L, \quad (6.59)$$

and Eq. (6.58) is rewritten as:

$$(1 - a)V_nC_n^L + [aV_{n+1} - (V_n + P_n)]C_n^H = R(C_0 - C_n^H). \quad (6.60)$$

The expression in brackets of the first member of this equation may be rewritten taking into account the following identity for the high tide volume of the n th segment (Eq. 6.47):

$$(V_n + P_n) = (1 - a)V_n + aV_n + P_n, \quad (6.61)$$

hence, from the segmentation equations system,

$$(V_n + P_n) = (1 - a)V_n + aV_{n+1}. \quad (6.62)$$

Then,

$$aV_{n+1} - (V_n + P_n) = -(1 - a)V_n, \quad (6.63)$$

and substituting Eq. (6.63) in Eq. (6.60), it follows that,

$$(1 - a)V_n(C_n^L - C_n^H) = R(C_0 - C_n^H), \quad (6.64)$$

or, rearranging its terms,

$$C_n^L = C_n^H + \frac{R(C_0 - C_n^H)}{(1 - a)V_n}, \quad \text{with } n \geq 2. \quad (6.65)$$

As the quantities R , a and the volumes V_n are known, Eq. (6.65) has two unknowns C_n^L and C_n^H . However, the fresh water concentration, C_n^H , may be calculated by Eq. (6.58),

$$C_n^H = \frac{RC_0 + aV_{n+1}C_{n+1}^L}{aV_{n+1} + R}. \quad (6.66)$$

Now, calculating the above equation for $n = N$ and assuming that pure water enters the estuary mouth on the flood tide ($C_{N+1}^L = 0$), it is possible to calculate the fresh water concentration at high water in the last segment (C_N^H),

$$C_N^H = \frac{RC_0}{(aV_{N+1} + R)}, \quad (6.67)$$

and from the segmentation equations $aV_{N+1} = V_N + P_N$, the final value of C_N^H is:

$$C_N^H = \frac{R}{(aV_N + P_N + R)}. \quad (6.68)$$

As all variables are known to calculate C_N^H with this equation, the value of C_N^L may be determined by Eq. (6.65) for $n = N$. Repeating this procedure, sequentially for $n = N - 1$, $n = N - 2$, ..., $n = 2$, $n = 1$, Eqs. (6.66) and (6.65) correspond to a system with two equations and two unknowns (C_N^H and C_N^L). This equation system may be solved to obtain high and low volumes of fresh water concentration (or salinity) starting from the segment $n = N$, located at the estuary mouth. With these results, the equations to calculate the salinity values are:

$$S_n^H = S_0(1 - C_n^H), \quad (6.69)$$

and

$$S_n^L = S_0(1 - C_n^L). \quad (6.70)$$

The volumes of fresh water retained in the estuary at high and low tide during the flood (V_{fn}^H) and ebb (V_{fn}^L), respectively, may be calculated with known corresponding geometric volumes,

$$V_{fn}^H = C_n^H(V_n + P_n), \quad (6.71)$$

and

$$V_{fn}^L = C_n^L(V_n). \quad (6.72)$$

The flushing time (t_q) at high and low tide are functions of the fresh water volumes and may be calculated by:

$$t_{qn}^H = \frac{V_{fn}^H}{Q_f}, \text{ or in tidal cycles } t_{qn}^H = \frac{1}{T_P} \frac{V_{fn}^H}{Q_f}, \quad (6.73)$$

and

$$t_{qn}^L = \frac{V_{fn}^L}{Q_f}, \text{ or in tidal cycles } t_{qn}^L = \frac{1}{T_P} \frac{V_{fn}^L}{Q_f}. \quad (6.74)$$

In practical applications of the model, negative values of the fresh water volume concentration in the low tide segment (C_n^L) located near the seaward end of the estuary may be found when $(1 - a)V_n < R$. If this occurs, the appropriated interpretation is that any salt water passing upstream into the segment (n), on the flood tide is entirely removed on the ebb tide so that $C_n^L = C_0$, and in consequence $C_{n-1}^H = C_0$, $(C_{n-1}^L) = C_0 = 1$, which should only occur near the head of the estuary (Dyer and Taylor 1973).

In order to exemplify the application of this method, it was applied to the same ideal estuary previously used for Ketchum's tidal prism segment model with $V_n = P_n$, the mixing parameter (a) equal to $a = 0.8$ and $R = 1$. The calculated volumes V_n , P_n , aV_n and $(1 - a)V_n$, using Eqs. (6.48–6.51) are presented in Table 6.4.

With the results of Table 6.4, Eqs. (6.65) and (6.66) may be calculated successively for $n = 10, 9, \dots, 2, 1$, and fresh water concentrations at low (C_n^L) and high tide (C_n^H) for all segments will be obtained. With Eqs. (6.69) and (6.70), the relative salinity values for high, $(S_n/S_0)^H$, and low tide, $(S_n/S_0)^L$, can be easily calculated and are presented in Table 6.5. As fresh water volumes are calculated by the product of the fresh water concentration to the segments at low and high tides, respectively, the flushing times may also be calculated by Eqs. (6.73) and (6.74). The results of this table also indicate the convergence of the low (C_n^L) and high (C_n^H) fresh water concentrations to the value 1 (one), indicating the absence of salt water in segments 1 and 2. Therefore, these segments correspond to segment 0 in the Ketchum's model. Also, as may be observed, $(C_n^L) > (C_n^H)$ and the relative salinity values are higher at high tide than at low tide. In comparing the flushing times there is a great difference between results from the two methods. In tidal periods, these values are 20 and 52 for the Ketchum's and Dyer and Taylor's model, respectively (Tables 6.2 and 6.5).

Results of the longitudinal mean relative salinity variation for the model estuary (Tables 6.2 and 6.5) calculated with the Ketchum's (K) and Dyer and Taylor's

Table 6.4 Partial volumes of a simple estuary model ($V_n = P_n$, $R = 1$ and $a = 0.8$), according to Dyer and Taylor (1973)

n	V_n	P_n	aV_n	$(1 - a)V_n$
1	1.0	1.0	0.8	0.2
2	1.25	1.25	1.0	0.25 ^a
3	2.81	2.81	2.25	0.56 ^a
4	6.33	6.33	5.06	1.26
5	14.24	14.24	11.39	2.85
6	32.04	32.04	25.63	6.41
7	72.08	72.08	56.67	14.42
8	162.20	162.20	129.70	32.44
9	364.90	364.90	292.00	73.00
10	821.12	821.12	656.90	164.22

^aNote that $(1 - a)V_n < R$ and so $C_n^L = 1.0$

Table 6.5 Results of a simple estuary model ($V_n = P_n$, $R = 1$ and $a = 0.8$) of fresh water concentrations (C_n^L), (C_n^H), relative salinities $(S_n/S_0)^L$, $(S_n/S_0)^H$ and flushing times $(t_q)_n^H/T$, according to Dyer and Taylor (1973)

n	(C_n^L)	(C_n^H)	$(S_n/S_0)^L$	$(S_n/S_0)^H$	$(t_q)_n^H/T$
1	–	–	–	–	–
2	–	–	–	–	–
3	1.00	0.92	0.00	0.08	5.17
4	0.91	0.58	0.09	0.42	7.34
5	0.54	0.30	0.46	0.70	8.50
6	0.27	0.13	0.73	0.87	8.33
7	0.12	0.06	0.88	0.94	8.65
8	0.05	0.02	0.95	0.98	6.49
9	0.02	0.01	0.98	0.99	7.43
10	0.01	0.00	0.99	1.00	0.00

(D&T) methods are comparatively presented in Fig. 6.9a, b. Salinity variations indicates some differences, as should be expected. However, the longitudinal salinity distributions are very close (Fig. 6.9b).

Dyer and Taylor’s method was also applied to the Raritan river estuary and bay, using the volumetric data given by Ketchum (1951), with different values of the mixing parameter (\underline{a}). A mixing parameter $\underline{a} = 0.5$ gave reasonable comparison with the salinity distribution in high tide observed by Ketchum in the Raritan river. For further details on these comparisons, as well as for the Thames river estuary, may be found in the Dyer and Taylor’s original paper.

Dyer and Taylor’s method has also been applied to Winyah Bay (Fig. 6.7), using the previous volumetric data (Fig. 6.5). The results are in Table 6.6, calculated with hydrologic measurements and salinity at the coastal sea conditions, as previously indicated ($R = 8.6 \times 10^6 \text{ m}^3$, and $S_0 = 34.0\text{‰}$), and used in the application of the first method, and the mixing parameter used $\underline{a} = 0.8$. The comparative analysis of the mean theoretical salinity distribution along the bay, obtained with these

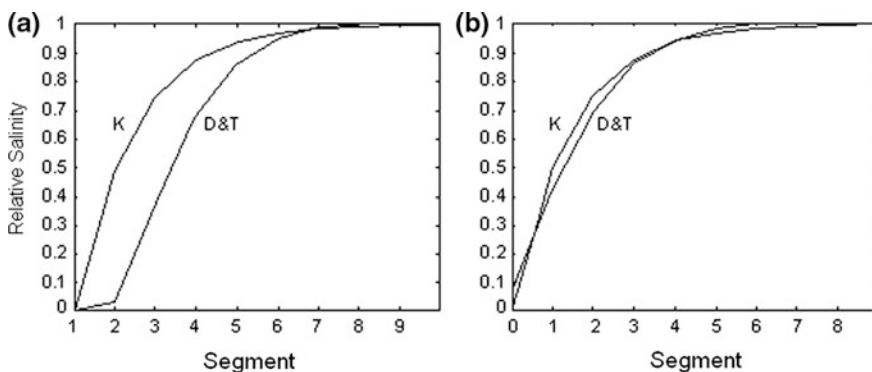


Fig. 6.9 Mean longitudinal salinity variation in the estuary model with $V_n = P_n$. **a** Values calculated with Ketchum’s (K) and Dyer and Taylor’s (D&T) methods, with the mixing parameter $a = 0.8$. **b** The best agreement was obtained displacing the second method to the left

methods, is shown in Fig. 6.10. Similar to the model estuary (Fig. 6.9), differences were observed in the salinity values, which were also minimized by displacing the first segment of the Dyer and Taylor's to the left. The results of this figure indicate variations which have some dependence on the used method, and, due to the higher number of segments in it, the salinity varies smoothly from the head down to the estuary mouth.

The results in Fig. 6.10 indicate the dependence of the longitudinal salinity variation on the chosen method. The confidence in the Ketchum's and Dyer and Taylor's methods may only be validated by comparing both results with experimental data. However, Dyer and Taylor's method satisfies the volume conservation of the fresh water input, and should presents longitudinal salinity distributions and flushing time more consistently. It may also be observe that the longitudinal salinity variation is nearly linear in the central MZ (Fig. 6.10), and this result has also been confirmed with observational data. Another observation of the results of these methods is that the flushing time from Dyer and Taylor's (28.9) is twice the duration calculated by Ketchum's method (14.4), in tidal periods.

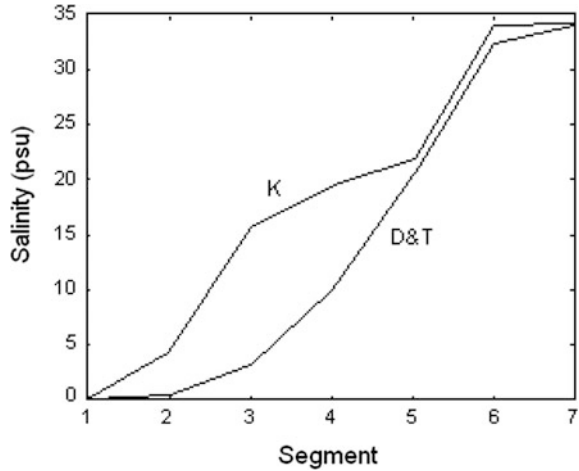
Dyer and Taylor's model was adapted by Brown and Arellano (1980) for a branching estuary in order to study the mixing of salt within the Great Bay estuary (New Hampshire, USA). This estuarine system has two main branches with their own river discharge, and it was necessary to take into account this particular morphology. This estuary is classified as vertically well-mixed (type 1) most of the year, with a few exceptions of highest river discharge periods, when this estuary has been classified as partially mixed with low stratification (type 2a). In the application of this model, the mixing parameter (\underline{a}) was allowed to vary and was chosen on the basis of a calibration procedure using observational data. The predicted mean salinity distribution over a range of river discharges volumes were in agreement with observational data when the flux ratio was higher than 1 (one) (tidal prism much less than the river discharge per tidal cycle). As another result of the Brown &

Table 6.6 Results of the Dyer and Taylor (1973) method applied to the Winyah Bay estuarine system

n	V_n	P_n	aV_n	C_n^L	C_n^H	S_n^L	S_n^H	t_{qn}^H/T
1	8.6	1.5	6.9	–	–	–	–	–
2	21.2	4.2	17.0	–	–	–	–	–
3	26.5	6.5	21.2	1.0	1.0	0.00	0.00	3.8
4	34.6	9.7	27.7	1.0	0.99	0.00	0.34	4.6
5	46.7	14.3	37.4	0.99	0.91	0.34	3.10	6.4
6	64.6	21.4	51.7	0.90	0.71	0.34	9.86	7.1
7	91.4	34.8	73.1	0.68	0.40	10.88	20.40	5.9
8	134.8	50.0	107.8	0.35	0.05	22.10	32.30	1.1
9	197.4	–	157.9	0.00	0.00	34.00	34.00	–

Salinities S_n^L and S_n^H in ‰. With $R = 8.6 \times 10^6 \text{ m}^3$, $S_0 = 34.0\text{‰}$ and mixing parameter $\underline{a} = 0.8$. Volumes in units of 10^6 m^3

Fig. 6.10 Theoretical mean salinity variation in the MZ in the Winyah Bay estuary. Ketchum's (K) and Dyer and Taylor's (D&T) methods



Arellano (op. cit.) investigation was that related to the flushing times calculations; for water parcels entering at the estuary head during periods of low and high river flow the flushing times were 54.5 and 45.9 tidal cycles, respectively.

The Dyer and Taylor (1973) one-dimensional tidal prism model was also been used by Bradley et al. (1990) to simulate the changes in the longitudinal mean salinity distribution, which occurred in the Cooper River (South Carolina, USA), because a diversion in 1985 caused a reduction in the mean river discharge from 442 to 130 m³ s⁻¹. The model simulation indicated that a salinity increased of 10–14‰, has occurred in the region of the river where the marsh plant community shifts from a virtual monoculture of *Spartina alterniflora* to a more diverse brackish community. The flow reduction, due to the river diversion, and the associated salinity increase are expected to result in the dominance of the halophyte, *S. alterniflora*, and a progressive exclusion of the less halotolerant species that currently inhabit the region.

A segmented tidal prism model has also been developed by Wood (1979) and presented comparatively with the previously described methods by Miranda (1984), and we encourage the reader to follow the analysis of the Wood's model.

The one-dimensional segmented tidal prism models gives better results to estimate the fresh water, salinity, flushing times in well-mixed estuaries, and could be also applied to other conservative properties, as long as their input rates are known. These models are convenient because it is only necessary to know the basic estuarine data, such as tidal height, river discharge, geometric characteristics of the estuary and the salinity in the coastal sea. Of course, to achieve validation, observational data for the steady-state salinity distribution must also be known.

6.4 Concentration Estimates of a Conservative Pollutant

The concepts and semi-empirical models related to the steady-state mean salinity and fresh water estimates in an estuary, may be applied to other conservative chemical constituents or pollutants introduced into estuaries, provided their flux or transport inputs are known. Consider a one-dimensional estuary partially mixed (type 2 or B), forced by fresh water discharge, with tidal mixing due to horizontal flow associated with the flood and ebb tidal currents. Its mixing zone (MZ) may also be schematically segmented according to Fig. 6.11, and $R = T_P Q_f$ is the fresh water volume disposable to mixing during a complete tidal cycle with period (T_P). In this type of estuary, salinity increases with depth, as well as progressively increasing seaward due to the mixing process related to advection and turbulent diffusion. To maintain the volume (mass) conservation, this seaward transport must be compensated by an equal up-estuary fresh water volume (Q) in the sub-surface layer; for steady-state volume conservation $Q = R$.

Due to the tidal forcing attenuation towards the estuary head, in the uppermost segment the advective influence of the river discharge predominates, and the entrainment is the main process transporting water into the surface layer. The landward mass transport of salt in the bottom layer is equivalent to $6Q^{(1)}$, and $1Q^{(2)}$ is the compensating upward transport due to the entrainment between the bottom and the surface layers (Fig. 6.11). Due to dilution of the upward subsurface water by the fresh water volume, R , the salinity of this layer increase from 0 at the estuary head to 3‰ in the upper layer of the adjacent segment; in fact, due to this salt balance transport it follows that: $6Q = S(2R)$ and $S = 3‰$. We must also observe that in this segment the longitudinal volume and salt balance are equal to $2R - Q = R$ and $3(2R) - 6Q = 0$, respectively.

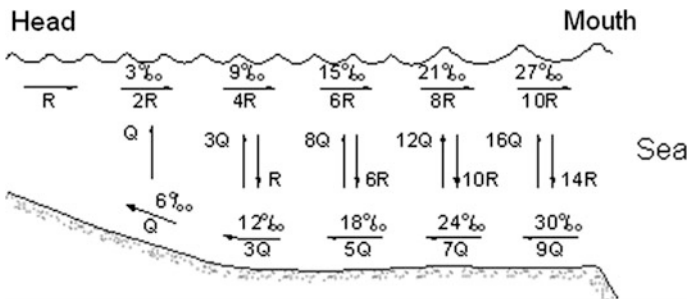


Fig. 6.11 Schematic changes in the mean salinity and in the volume of fresh water transported by advection in order to maintain the steady-state balance in an estuary during a complete tidal cycle. Salinity in ‰. River (nR , $n = 1, 2, 4, \dots 10$) and fresh water (mQ , $m = 1, 3, 5, \dots 9$) indicate its contents in seawater volumes, respectively. Landward and seaward, up and downward arrows indicate interchanges of water volumes (adapted from Ketchum 1953)

- (1) In the salt balance the mass transport is calculated by ρSQ . Adopting $\rho = 1.0 \times 10^3 \text{ kg m}^{-3}$, S in ‰ (g kg^{-1}) and Q in m^3 , and the mass of salt is calculated by $1.0 \times 10^3 \times S \times 10^{-3} \times Q = S \times Q$.
- (2) The compensating upward transport (upwelling) of the seaward surface flow has been observed for the first time by F. Ekman, in 1876, during his studies on the circulation and salt observations at the mouth of the Gotaelv river flowing into the Elfsborgsfjord (Sweden).

The tidal mixing increase in the adjacent seaward segment due to turbulent diffusion surpasses the entrainment, and the net volume transport between the bottom and surface layers becomes $2Q + (3Q - R)$; consequently, the seaward volume transport on the surface layer increase to $4R$ (Fig. 6.11). By the mass of salt conservation principle, the seaward salinity in the surface layer increases to 9‰ due to the salt balance: $12 \times 3Q = S \times 4R$ and $S = 9\text{‰}$ on the upper layer. By analogy, as in the previous segment, the longitudinal volume and salt balance are equal to: $4R - 3Q = R$ and $9(4R) - 12(3Q) = 0$, respectively, and the seaward transport of water increases in proportion to its salt content.

This process is repeated in all segments located seaward and, according to the continuity principle, the net volume and salt mass across any cross section are equal to R and zero, respectively. Also, as illustrated in Fig. 6.11, there is an increase in seaward transport of mixed water and the compensating landward transport of salt. A direct consequence of this simple relationship is that the total circulation in the estuary increases enormously in volume as the water moves from the river towards the sea. This volume increase associated with the mixing process is called the *equivalent down-estuary transport*, which is a fictitious quantity and would only be measured under unusual conditions (Officer 1978).

The process just described is related to the volume and salt mass conservation principle under steady-state conditions or near steady-state conditions, within the time frame of the tidal period. In the cross section located at the estuary head, where $\bar{S} = 0$ and $f = 1$, the equivalent down-estuary transport (Q_d) is equal to the river discharge, and $Q_d = Q_f$, according to Eq. (6.75), and shown in Fig. 6.12. In any other section located seaward, the net volume transport is equal to Q_f . However, if $f = 0.5$ in this section, the equivalent transport is equal to $2Q_f$, to compensate for the water parcel retained in the system due to the mixing process. Thus, the ratio $Q_d/R = 1/f$, is a measure of the total process of removing a pollutant from an estuary compared with the advection effect due to the river discharge R (Officer 1978).

$$Q_d = \frac{R}{f} = \frac{S_0}{S_0 - \bar{S}} Q_f = \frac{Q_f}{f}. \quad (6.75)$$

As Eq. (6.75) is equal to the Eq. (6.16), which defines the *flushing rate* F , it has been proved that this *flushing rate* and the *equivalent down-estuary transport* are the same physical quantity. Hence, the mixing zone (MZ) volume is exchanged in the time interval equal to the flushing time (t_q). Another interpretation is that the

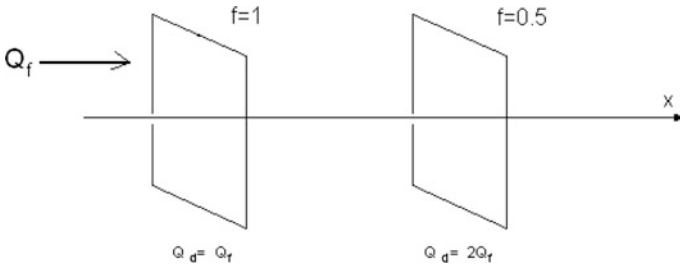


Fig. 6.12 Diagram showing the relationship of the volume transport, Q_f , and its *equivalent* (Q_d), and the fresh water fraction (f) (according to Officer 1978)

ratio, Q_d/R , is a measure of the total process for removing a conservative pollutant from an estuary compared to the simple advection effect of the river discharge, Q_f .

Let's consider now a mass transport, W , $[W] = [MT^{-1}]$, of a conservative effluent that is discharged into a river cross-section (Fig. 6.13). By hypothesis, this discharge is made through a multiport diffuser system to increase the effectiveness of the dilution of the less dense ascending plume located at the bottom (not indicated in the figure), extended across the estuarine channel. Then, the initial cross sectional average concentration per unit volume of sea water (c_0^*) is calculated by:

$$c_0^* = \frac{W}{Q_f}, \tag{6.76}$$

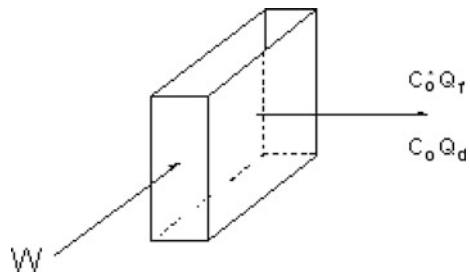
where Q_f is the steady-state river discharge, and $[c_0^*] = [ML^{-3}]$, and $kg\ m^{-3}$ in units of the SI.

For an estuary, the river advection must be replaced by the equivalent downstream transport (Q_d) at the outfall, and the effluent concentration (c_0) is determined by,

$$c_0 = \frac{W}{Q_d} = \frac{W}{F}, \tag{6.77}$$

or, taking into account Eq. (6.75):

Fig. 6.13 Schematic diagram of the input of a discharge, W , of a conservative effluent into a cross-section of a non-tidal river, or estuary (according to Officer 1978)



$$c_0 = \frac{W}{Q_f} f_w, \quad (6.78)$$

where c_0 has dimension of $[ML^{-3}]$, and f_w is the average fresh water fraction at the outfall. This result of Ketchum (1955), obtained with the implementation of the segmented tidal prism model, indicates that under steady-state conditions the initial concentration is directly proportional to the fresh water fraction, and inversely proportional to the river discharge which is assumed to be constant.

From Eqs. (6.76) and (6.78) it follows that the relationship between river (c_0^*) and estuary (c_0) concentrations is,

$$c_0 = (c_0^*) f_w. \quad (6.79)$$

As $0 \leq f_w \leq 1$, this implies that $c_0 < c_0^*$. Also, it should be noted that $f_w \rightarrow 0$, and also $c_0 \rightarrow 0$ at the estuary mouth.

Downstream of the outfall, when steady-state conditions are achieved, the pollutant must pass a cross section at the same rate it is discharged from the source, and its concentration is (Officer 1978):

$$(c_x)_d = \frac{W}{Q_f} f_x, \quad (6.80)$$

where $(c_x)_d$ and f_x are the average concentration of pollutant and the fresh water fraction at the cross-section located at the longitudinal position x , respectively.

Combining Eqs. (6.80) and (6.78) gives the following expression to calculate the pollutant concentration at the position (x) downstream of the pollutant introduction:

$$(c_x)_d = c_0 \frac{f_x}{f_w}. \quad (6.81)$$

This result indicates that the average concentration in the transversal section is directly proportional to the initial concentration (c_0) and the fresh water fraction at position x , and inversely proportional to the fresh water concentration at the position of the pollutant discharge. Using the expressions to calculate the fresh water fraction as a function of salinity (Eq. 6.10), Eq. (6.81) may be written as:

$$(c_x)_d = c_0 \frac{(S_0 - S_x)}{(S_0 - S_w)}. \quad (6.82)$$

In this equation, S_0 and S_x are the salinities at the adjacent coastal sea and in the cross section downward of position x , respectively.

The pollutant will also be carried upstream from the outfall by the diffusion and advection of tidal currents during tidal flood and ebb, and above the outfall there will be no net exchange across any boundary when the steady-state condition is reached. The pollutant quantity carried up-estuary will be exactly balanced by the

quantity carried down-estuary. This is the same criterion that applies to the salt distribution up-estuary from the outfall. Thus, the up-estuary distribution of a conservative pollutant will be directly proportional to the salinity distribution, as given by:

$$(c_x)_u = c_0 \frac{S_x}{S_0}. \quad (6.83)$$

It is clear from these relationships that the knowledge of the distribution of salinity is essential in order to predict the expected steady-state distribution of conservative pollutants. These derivations were originally given by Ketchum (1955) and are a simple and direct method for estimating the distribution of a conservative pollutant or other index quantity in an estuary, with the knowledge of the salinity distribution alone. The pollutant distribution is calculated directly in terms of the salinity distribution without recourse. However, as stated in Officer (1978), it is important to emphasize that only the longitudinal effects have been considered and the definition contains the implicit assumption that the ocean at the mouth of the estuary is a perfect sink.

An observed fresh water concentration and the expected distribution of a conservative pollutant has been derived from the salinity distribution and fresh water fraction in the Raritan river and bay (Fig. 6.14). Four locations (A, B, C and D) have been arbitrarily selected for its position at an outfall. The horizontal distribution of pollutant concentration in percentage is obtained, assuming that the

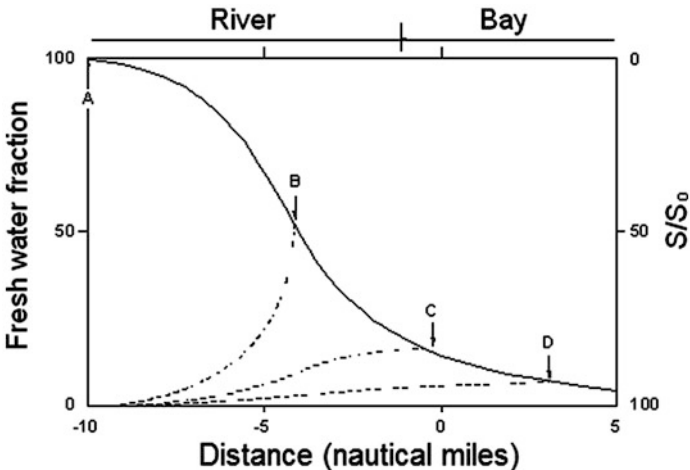


Fig. 6.14 The observed average distribution of fresh water fraction (in %) and the calculated distribution of a conservative pollutant (*continuous and dashed lines*) in Raritan river and bay for four possible outfall locations (A, B, C and D) (according to Ketchum 1955)

pollutant is uniformly mixed in the estuarine water through a multiport diffuser system.

From Fig. (6.14) and Eq. (6.81), at the pollutant releasing location $f_x = f_w$, the pollutant concentration decreases towards the estuary mouth from the maximum concentration (point A), as shown by the solid line of the fresh water concentration. The concentration distribution is directly dependent on its initial value. Therefore, different release location of pollutant may be less effective to the environment (e.g. points B,C and D). Up-estuary, from each outfall location, the pollutant distribution will follow the salinity distribution curve, or its inverse (the fresh water fraction curve), indicated by the dashed lines in Fig. 6.14). Two important consequences of moving the outfall downstream or upstream, respectively are (Ketchum 1955; Officer 1978):

1. As the outfall is progressively moved further down the estuary, the pollutant concentration up-estuary from the outfall is decreased while the concentration down-estuary remains the same.
2. The concentration of pollutants upstream from the outfall is decreased.

This method estimates the longitudinal variation of the concentration of a conservative pollutant discharged into an estuarine channel, under steady state conditions, and uses the salinity and/or the fresh water concentration as indicators. Thus, the method sensitivity is dependent on the river discharge and the salinity distribution, with the assumption that the coastal sea is a perfect sink, which is a very simplified boundary condition.

Pollutants that decay or decrease with time will be less concentrated throughout the estuary than the predicted concentrations of the conservative pollutants. This degradation of pollutants over time, which is superimposed on the circulation and diffusion has also been presented in the Bowden (1955) classical article.

To exemplify some concepts of this topic a practical example will be given based on Fischer et al. (1979). "During one complete tidal cycle, the multiport outfall diffuser of an industry discharges $200 \text{ m}^3 \text{ s}^{-1}$ of effluent, containing fifteen parts per thousand of a toxic material, into an estuary that is less dense than seawater. The mean river fresh water discharge is $500 \text{ m}^3 \text{ s}^{-1}$, the salinity at the coastal sea and the mean salinity value, at the outfall point are 34.0 and 20.0‰, respectively". Estimate the initial concentration of this substance at the transverse section, as well as the flushing time and rate, with the assumption that the estuarine MZ has a volume of 10^8 m^3 .

With the Eq. (6.10) the fresh water concentration at the outfall (f_w) is determined by:

$$f_w = 1 - \frac{20}{34} = 0.41. \quad (6.84)$$

Knowing the exiting transport, and the pollutant concentration within this transport, then $W = 0.015 \times 200.0 = 3.0 \text{ m}^3 \text{ s}^{-1}$, and the initial pollutant concentration (c_0) is calculated by:

$$c_0 = \frac{W}{Q_f} f_w = 2.46 \text{ g/kg} = 2.46 \times 10^{-3} \text{ kg/kg}, \quad (6.85)$$

The concentration $c_0 = 2.46 \times 10^{-3}$ is an average for the transverse section and representative if the outfall diffuser was adequately projected. The flushing time (t_q) is calculated by,

$$t_q = \frac{V_f}{Q_f} = \frac{(S_0 - \bar{S}) V}{S_0 Q_f} = 23 \text{ h} \ (\approx \text{two semi-diurnal tidal cycles}).$$

As an exercise, the reader may solve the following expression for t_q ,

$$t_q = \frac{V_f}{Q_f} = \frac{V_f c_0}{W f_w} = \frac{V c_0}{W}. \quad (6.86)$$

The *flushing rate* (F) may also be calculate, combining its definition (Eq. 6.16) with Eqs. (6.1) and (6.85):

$$F = \frac{V}{t_q} = \frac{V Q_f}{V_f}, \quad \text{or} \quad F = \frac{V Q_f}{V_f} = \frac{V f_w W}{V_f c_0}. \quad (6.87)$$

Introducing the known numerical values $F \approx 1214 \text{ m}^3 \text{ s}^{-1}$.

6.5 Water Mass Exchange at the Estuary Mouth

For application in the analysis of water exchange through the estuary mouth, this method uses steady-state mean salinities in transversal sections to calculate a non-dimensional parameter named *tidal exchange ratio*, defined by Fischer et al. (1979). This method was implemented with the introduction of a second parameter (*volumetric exchange ratio*), enabling its use for others coastal transition environments, such as bays (MacDonald 2006).

A portion of the water volume that enters an estuary forced by tidal flood currents is composed of water that left the estuary the previous ebb but retained in the estuarine plume. The remainder is water that we may think of as “new” ocean water volume (V_O); this water volume is what contributes to the dilution of pollutants inside the estuary, and knowledge of this new ocean water volume is important in the one-dimensional analysis of dilution of the concentration of substances or pollutants introduced into estuaries (Fischer et al. op. cit). The *tidal exchanged ratio* (TER) has been defined as the ratio of new ocean water (V_O) to the total volume of water that enters the estuary during a flood tide (V_f), which has already been defined as the tidal prism ($V_f = T_{PR}$, Chap. 2),

$$\text{TER} = \frac{V_O}{T_{PR}}. \quad (6.88)$$

This ratio varies between the limits $\text{TER} = 0$ (when $V_O = 0$, in the absence of new water) and $\text{TER} = 1$ when $V_O = T_{PR}$ (i.e., the new water volume is equal to the total volume of water entering the estuary during the flood); the condition $\text{TER} = 1$ or $V_O = T_{PR}$ is the most favorable to dilution of pathogenic substances discharged in natural coastal environments.

The new water volume (V_O), is usually is not possible to be predicted theoretically; it is dependent on coastline circulation, which transports the estuarine plume, controls the ebb flow along the coast and delivers the supply of new ocean water for the flood. Without a favorable water mass renewal, the water mass exchange with the continental shelf offshore of the estuary mouth will be ineffective ($V_O \ll V_{PR}$), and eventually estuarine polluted water may return to the estuary. As previously discussed, using ADCP equipment, the tidal prism T_{PR} may be determined with the u-velocity component normal to the cross section of the current velocity at the estuary mouth, $u = u(y, z, t)$. With vertical velocity profiles measured at time intervals during the food ($0 \leq t \leq T/2$), this volume may be calculate by numerical integration (Chap. 5, Eq. 5.52).

The tidal exchange ratio, defined in Eq. (6.88), may be determined with known mean volumes, salinities and densities at the transverse cross section at the estuary mouth, and tidal exchange ratio is solved taking into account the volume and mass conservation during the ebb and flood through the estuary mouth. To achieve this, let us follow the Fischer et al. (1979) empirical determination, using the same symbols and definitions:

T_{PR}	Total volume of seawater entering the estuary on the flood tide (tidal prism);
V_O	Volume of the new ocean entering the estuary during the flood tide;
V_e	Volume of the estuarine water leaving the system on the ebb tide;
$V_Q = T_P Q_f$	Volume of river discharge (fresh water) entering the estuary during the tidal cycle;
V_{fe}	Part of the volume (V_e) which flowed out of the estuary on the previous ebb;
$S_f(\rho_f)$	average salinity (density) of water entering the estuary on the flood tide;
$S_e(\rho_e)$	Average salinity (density) of water leaving the estuary on the ebb tide;
$S_0(\rho_0)$	Salinity (density) of ocean water

In steady-state conditions, the salinity and the water volume in the estuary remain constant. Then, the salt mass balance of water entering and leaving the estuary will be:

$$\rho_e S_e V_e = \rho_f S_f T_{PR}. \quad (6.89)$$

The dimension of each side of the equation is units of mass [M], because the product (ρS) numerically represents the salt concentration per volume unit. The salt balance of the water mass that enters the estuary during the flood, which has an increased salinity from the new ocean water (V_O), and includes the volume of water that is re-entering the estuary having previously flowed out of the estuary on the ebb tide, is given by:

$$\rho_e S_e V_e = \rho_f S_f T_{PR} + \rho_0 S_0 V_O. \quad (6.90)$$

The second member of this equation is the mass of salt that flows into the estuary mixed with the new water mass but without the estuarine plume influence. In Eqs. (6.89) and (6.90), the quantities ρ_e , ρ_v and ρ_0 are densities of the water masses which were introduced to maintain the equation with its dimension correctly [M]. However, the following simplification will be made $\rho_e \approx \rho_f \approx \rho_0$.

The water volume entering the estuary on the flood (T_{PR}), added to the volume of fresh water discharged into the estuary during the tidal cycle (V_Q) must be equal to the volume of water leaving the estuary during the ebb tide, then

$$T_{PR} + V_Q = V_e. \quad (6.91)$$

As the flood water (V_{PR}) is composed with some water volume that flowed out of the estuary on the previous ebb (V_{fe}) plus the new water volume (V_O) entering into the estuary, another relationship may be written for V_f ,

$$V_{PR} = V_{fe} + V_O. \quad (6.92)$$

Combining Eqs. (6.89), (6.90) and (6.92), the new water volume is given by:

$$V_O = S_e \frac{(V_e - V_f)}{(S_0 - S_e)}, \quad (6.93)$$

and inserting this result in the definition of tidal exchange ratio (6.88),

$$TER = \frac{V_O}{T_{PR}} = \frac{S_e}{(S_0 - S_e)} \left(\frac{V_e}{T_{PR}} - 1 \right), \quad (6.94)$$

and using the Eq. (6.89) transformed as $\left(\frac{V_e}{T_{PR}} = \frac{S_f}{S_e} \right)$, the result for the tidal exchange ratio is given by:

$$TER = \frac{(S_f - S_e)}{(S_0 - S_e)}. \quad (6.95)$$

This result indicates that the TER is directly dependent on the differences in the mean salinity values at the estuary mouth in the flood (S_f) and ebb (S_e) tides, and is inversely proportional to the difference between the non-diluted salinity at the adjacent coastal ocean (S_0) and the average salinity leaving the estuary mouth during the ebb (S_e). $S_f = S_0$ or $S_f = S_e$ implies that $TER = 1$ and $TER = 0$, which corresponds to the best and the worst tidal exchange, respectively.

Solving Eq. (6.89) for the flood salinity, S_f , and combining with the TER of Eq. (6.95) and the equality $V_Q = V_e - T_{PR}$ (Eq. 6.91), it follows that a useful expression of the tidal exchange ratio when in function of $V_Q = TQ_f = R$ (Fischer et al. 1979) is

$$TER = \frac{S_e}{(S_0 - S_e)} \frac{V_Q}{V_f}, \quad (6.96)$$

where V_Q is the volume of the river discharge entering the estuary during the tidal cycle.

This result indicates that besides the river discharge (Q_f), to determine TER and simulate nearly steady-state conditions during the flood and ebb tides, accurate observational data are required (hydrographic and current velocity), measured at the cross section at the estuary mouth during one of or more tidal cycles. Exemplifying, from salinity values measured at a cross section with an area, A , the averaged value (S_e) leaving the estuary during the ebb tide interval ($0 \leq t \leq T/2$) is calculated as follows: firstly, its cross-section mean value $S_e = S_e(t)$ is calculated by:

$$S_e(t) = \frac{1}{A(t)} \iint_A S_e(y,z,t) dydz, \quad (6.97a)$$

Then, it follows that,

$$S_e = \frac{2}{T} \int_0^{T/2} S(t) dt = \frac{2}{T} \int_0^{T/2} \left[\frac{1}{A(t)} \iint_A S(y,z,t) dA \right] dt, \quad (6.97b)$$

where dA indicates the area element at the mouth cross-section. Similar procedure may be used to calculate mean velocities and the corresponding values of the tidal prism ($V_f = V_{PR}$).

Pioneering studies Nelson and Lerseth (1972), quoted Fischer et al. (1979) describe measurements of the tidal exchange ratio at the entrance of San Francisco Bay (California, USA). Salinity and current velocity were measured throughout the tidal cycle at a number of positions along transect at the Golden Gate Bridge. Measurements were made on two occasions with different tide conditions, and TER values were calculated by Eq. (6.95). In this article, the authors found that

increasing the flood tide in the range from 0.30 to 2.3 m increased the tidal exchange rate (TER) from 0.1 to 0.5, respectively, and thus reducing the effectiveness of pollutant discharges.

To illustrate TER results estimates in the Curimataú river estuary (Rio Grande do Norte, Brazil) were analysed by Miranda et al. (2005, 2006), using observational data measured during two neap and spring semi-diurnal tidal cycles in the vicinity of the estuary mouth. Due the strong river discharge, during the neap tide, the estuary was classified as partially mixed and highly stratified (type 2b), evolving in the spring tide to a partially mixed and low stratification (type 2a). In the first attempt to classify the estuary, $S_f < S_e$ and the $TER < 0$ (Eq. 6.95) has no physical meaning due to the abnormal river discharge. In the spring tide, the estimated mean salinity values were $S_f = 34.97\text{‰}$, $S_e = 34.03\text{‰}$ and $S_0 = 36.8\text{‰}$, and the calculated $TER = 0.3$. This result indicates a small new oceanic water volume intrusion (V_O) into the estuary and therefore less effective conditions for the dilution of pollutants input into the estuary.

6.6 Mixing Diagrams

Mixing diagrams are very useful for investigating the presence of sources and sinks of natural components and/or pollutant concentrations in the estuarine water, tidal river or in the adjacent coastal sea. This diagram is a Cartesian orthogonal coordinate system used to correlate a given concentration versus salinity, which has the coastal sea as its main source and has a well known longitudinal variation.

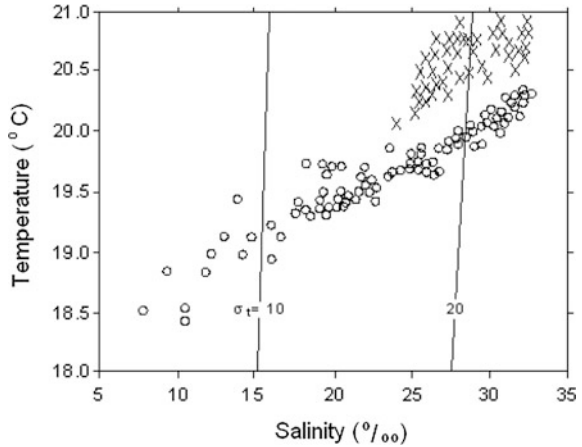
The classical T-S diagram was introduced in the oceanographic literature by Björn Helland-Hansen in papers published in 1916 and 1918, as a pioneering study on classification, distribution and mixing of oceanic water mass. It is a diagram with temperature and salinity (heat and salt concentrations) in the ordinate and abscissa axis, respectively. It has also been used as a basic mixing diagram for estuarine water mass classification.

If the water body is homogeneous in salinity and temperature the image of these properties on the T-S diagram is a single point, representing the final stage of the irreversible mixing generate by the advection and diffusion processes. If there is no homogeneity in the water mass, due to the variations of these properties, the S and T pairs of points will appear as a set of aligned points on the diagram; the point distributions indicate the occurrence of changes in the heat and salt concentrations during mixing.

As the density anomaly at atmospheric pressure (Sigma-t or σ_t) is dependent only on the salinity and temperature, it is possible to draw in the T-S diagram a set of parametric curves, which represents the state equation of seawater at atmospheric pressure; this diagram is named *state diagram of seawater*.

With a few exceptions, salinity and temperature in estuarine water respond more quickly to mixing processes (advection and turbulent diffusion) than to air-sea interaction processes. Although these properties have small temporal variability, a

Fig. 6.15 Scatter T-S diagram with salinity (S) and temperature (T) values sampled during neap (o) and spring (x) tidal conditions in the estuarine channel of Bertioga, in July, 1991, showing estuarine water mass with high and low stratification, respectively. Continuous lines indicate the density in Sigma-t units

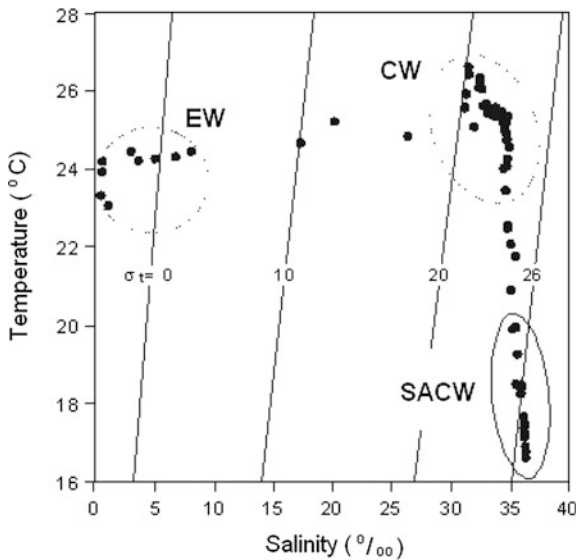


well defined correlation may exist in the T-S diagram, if the set of (S, T) pairs are sampled during complete tidal cycles. The set of sampled data is named *scatter* T-S diagram, which may be used to identify whether or not the tidal river zone (TRZ), the mixing zone (MZ) or the adjacent coastal sea have heat and salt sources or sinks.

The scatter T-S diagram of the Bertioga estuarine channel (Fig. 5, Chap. 1) with hydrographic data sampled during two complete tidal cycles, in neap and spring tidal conditions, 5 km landward from its mouth is shown in Fig. 6.15.

In the neap tide experiment, the temperature interval change was 3 °C (from 18.5 to 20.8 °C); however salinities varied over a large interval from ≈8 to 33‰. In

Fig. 6.16 Scatter T-S diagram in the mixing (MZ) and near-shore turbidity (NTZ) zones of the Itajaí-açu estuarine system, in a longitudinal section showing thermohaline characteristics of the Estuarine Water Mass (EW), the Coastal Water Mass (CW) and the South Atlantic Central Water Mass (SAW) (adapted from Schettini et al. 1998)



the spring tide, the temperature and salinities varied from 20 to 21.0 °C, and ≈ 24 to 33‰, respectively (Fig. 6.15). In the spring, tide the tidal currents were more intense than in the neap tide, and more energy was available to raise the potential energy of the water column and non-isopicnal vertical turbulent diffusion.

This diagram is also an indicator of the vertical salinity stratification, and we may observe that the estuarine water mass changed from highly to moderately stratified, from the neap and spring tides, respectively, with the (S,T) points presenting a relatively large and small scatter, respectively. The positive correlation between temperature and salinity indicates that the main source and sink of the heat and salt concentrations in the estuary were the adjacent coastal waters and the river water, respectively.

Thermohaline characteristics of the mixing zone (MZ) and the near-shore turbidity zone (NTZ) of the Itajaí-açu river (Santa Catarina State, Brazil) estuarine system were almost synoptically sampled, and the analysis using the scatter T-S diagram (Fig. 6.16) was presented in Schettini et al. (1998).

In this diagram (Fig. 6.16), the following water masses were identified: the Estuarine Water (EW), Coastal Water (CW) and the South Atlantic Central Water (SACW). It can also be observed that the less dense water formation of the estuarine plume is due to the non-isopicnal mixing of the EW and CW water masses, and the upper part of the oceanic water (SACW) with $20\text{ °C} < T < 16\text{ °C}$ and $35\text{‰} < S < 36\text{‰}$ is in agreement with the mean values during the summer, which

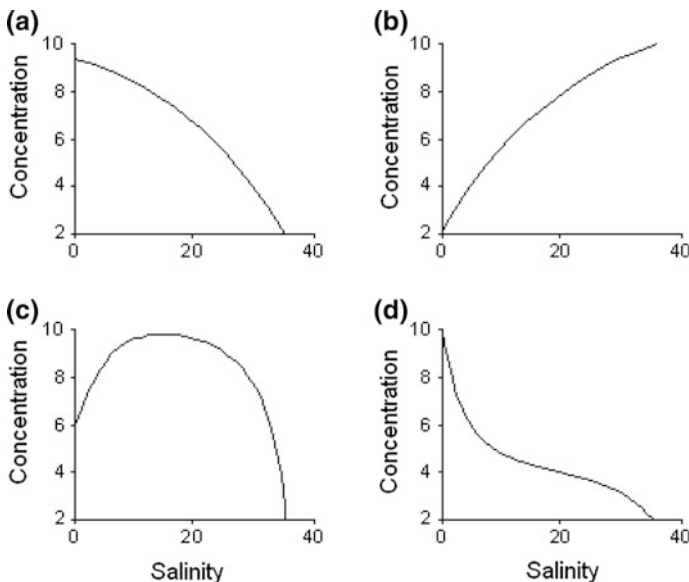


Fig. 6.17 Mixing diagrams schematically showing the sources and sinks of conservative or non-conservative properties' concentrations along an estuary: **a** source at the TRZ and sink in the MZ; **b** source in the coastal ocean and sink at the estuary head; **c** source at MZ and sinks at the head and in the coastal ocean; **d** and source at the TRZ and sink at the MZ

were calculated by Castro and Miranda (1998) for the continental shelf waters offshore of Itajaí (Santa Catarina, Brazil). This water with a higher density is able to be advected into the estuary by barotropic and baroclinic gradient pressure forces, generating high vertical stratification in the MZ during seasons with high fresh water discharge.

The T-S Diagram can be altered to become a *mixing diagram*, by substituting the temperature (heat concentration), in the ordinate axis, by an alternative property's concentration. According to the immediate purpose of the investigation, the correlation of a given property with salinity may be performed in several ways: with instantaneous concentration values during high and low tide, or with mean concentration values during tidal cycles. Among the numerous correlations which can be expected, some possibilities to identify concentrations of sources or sinks of the property are shown schematically in Fig. 6.17a–d.

If the property concentration is not at steady-state and the salinity at the coastal ocean has short temporal variations, these correlations may be more complex than those depicted in the figure.

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