Chapter 26
Modeling and Simulation of the Dynamic Response of a Generic Mechanical Linkage for Control Application Under the Consideration of Nonlinearities Imposed by Friction

Jitendra Yadav and Geeta Agnihotri

26.1 Introduction

In many industrial applications, involving a control mechanism using open kinematic chains along with spring or similar flexible elements, the positioning and dynamics of the target are prone to errors due to the presence of friction. Some of such applications are flight control mechanism, mechanical systems using single-acting hydraulic and pneumatic actuators, hydraulic copy turning, and robotics. In control applications, it is essential to model the dynamic response of the mechanical system to a sufficient accuracy, in order to obtain desired precision level. The mathematical model developed for obtaining the dynamic response would include the friction behavior of components. Further, the force of friction would depend upon the state variables such as velocity and displacement as has been shown in earlier research work. The objectives of the present work are to understand the system dynamics identifying the relevant parameters, to understand the inter-relationship among them, to determine their relative significance, and to estimate the extent of errors in system performance due to friction.

Researchers have attempted in past to model friction by including different effects and experimental observations, such as Coulomb friction, viscous friction, hysteresis, elastoplastic deformation at asperities, stiction, and Stribeck effect. The classical models fail to model behavior such as hysteresis and break-away.

J. Yadav (✉)
Mechanical Engineering Department, University of Petroleum and Energy Studies, Dehradun, Uttarakhand, India
e-mail: jyadav@ddn.upes.ac.in

G. Agnihotri
Mechanical Engineering Department, Maulana Azad National Institute of Technology, Bhopal, Madhya Pradesh, India
e-mail: dr.gagnihotri@gmail.com

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A method for using the classical Coulomb model is described in detail [1, 2]; this is due to the fact that the Coulomb model is discontinues at zero velocity. Due to this disadvantage, it is not widely used for simulation purposes. de Canudas et al. [3] proposed a dynamic model for friction that includes most of the friction phenomena, i.e., stiction, Stribeck effect, hysteresis, and varying break-away force. Nguyen et al. [4] simulated a dynamic system using LuGre model for friction. Researchers proposed many friction models [5–8]. Tripathi et al. [9] performed simulation of dynamic response of a mechanical system using the Coulomb and LuGre model for friction. Gafvert [10] compared friction models on rate dependency, model order, and damping.

26.2 Modeling of Friction

LuGre model

The complete LuGre model of dynamic friction is described as follows:

$$F_f(t) = \sigma_0 Z + \sigma_1 \dot{Z} + \sigma_2 V \tag{26.1}$$

Here,

$$\dot{Z} = V - \frac{|V|}{g(V)} Z \tag{26.2}$$

and

$$g(V) = \frac{1}{\sigma_0}\left[F_C + (F_S - F_C)e^{-\left(\frac{V}{V_S}\right)^2}\right] \tag{26.3}$$

Therefore,

$$F_f = \sigma_0 Z + \sigma_1 V - \frac{\sigma_0 |V| Z}{\left[F_C + (F_S - F_C)e^{-\left(\frac{V}{V_S}\right)^2}\right]} + \sigma_2 V \tag{26.4}$$

where,

- Z The lateral deflection of the bristle, expressed in microns
- σ_0 Average bristle stiffness
- σ_1 Micro-viscous friction coefficient
- σ_2 Constant for viscous friction
- F_C Static friction force, corresponding to Coulomb friction
- F_S Dynamic friction force
- V_S Stribeck velocity

Dahl Model

It is based on the stress–strain curve of classical mechanics which is governed by differential equation:

$$\frac{\mathrm{d}Z}{\mathrm{d}t} = v - |v|\frac{\sigma_0 Z}{F_\mathrm{C}} \tag{26.5}$$

$$F_\mathrm{f} = \sigma_0 Z \tag{26.6}$$

where Z is a state variable.

Dahl model is generalization of ordinary Coulomb model by introducing the system dynamics, i.e., presliding displacement, but it neither captures Stribeck effect nor stiction.

Coulomb Model

The classical Coulomb friction model is depicted in Fig. 26.1. The model shows discontinuity at zero velocity, which possess problem in the analytical solution of the dynamic system. For numerical computation using computer, this problem has been dealt with by obtaining piecewise solution for different sections and then integrating them. However, this is quite tedious and the method has to be devised separately for every individual system. The governing equation for the model is shown by Eq. 26.9.

$$F_\mathrm{f} = F_\mathrm{C}\mathrm{sgn}\,(v) \tag{26.7}$$

The problem of discontinuity in Coulomb model can be circumvented by the use of sigmoid function, as shown in Fig. 26.2. The sigmoid function is a monotonically increasing function with, asymptotically bound upper and lower limits of 0 and 1, respectively. The shape of the curve is controlled by the index n.

Sigmoid function is defined by Eq. 26.8 as a function of velocity.

$$F = \frac{1}{1 + \mathrm{e}^{-n.\mathrm{velocity}}} \tag{26.8}$$

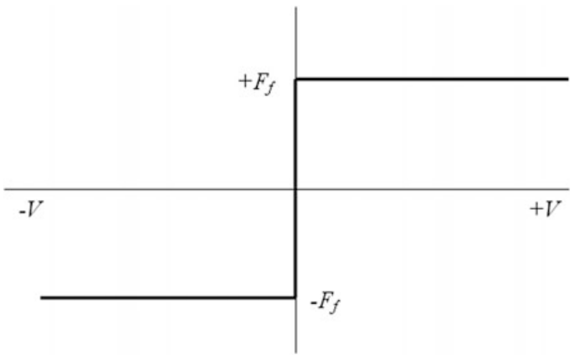

Fig. 26.1 Coulomb friction model

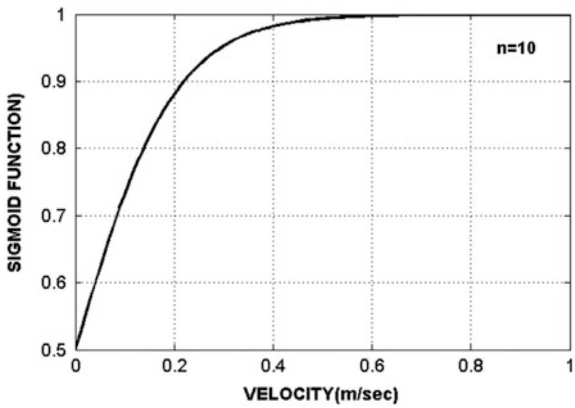

Fig. 26.2 Sigmoid function versus velocity plot

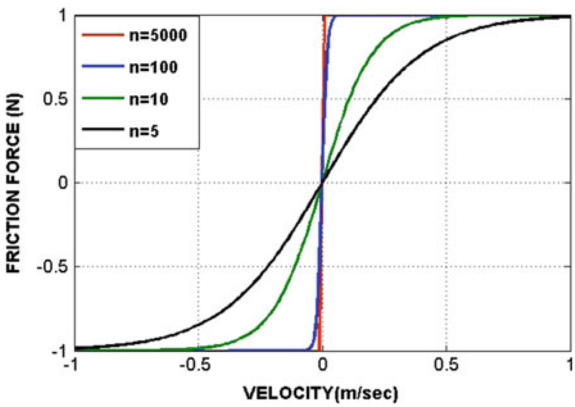

Fig. 26.3 Friction force versus velocity plot with the help of sigmoid function

The sigmoid function-based friction model can be expressed by Eq. 26.9.

$$F_{\mathrm{f}} = \mu mg\left[2\left(\frac{1}{1+\mathrm{e}^{-n.\mathrm{velocity}}} - 0.5\right)\right] \tag{26.9}$$

The friction force characteristic as a function of velocity by Eq. 26.11 for different values of index n is shown in Fig. 26.3.

The curves in Fig. 26.3 are confined to the velocity range −1.0 to +1 m/s. The velocity range is arbitrarily selected and can be extended to any value, as required by a particular dynamic system without any need for change in the computer program. It can be observed that for the value of index n = 5000 the curve can be compared quite well with the classical Coulomb model.

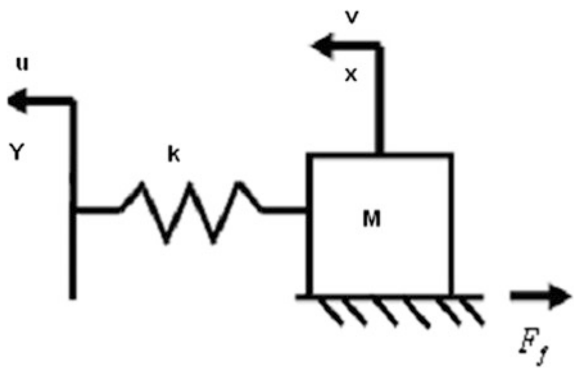

Fig. 26.4 Spring–mass system

Validation of Sigmoid Function-Based Friction Model

The proposed sigmoid function-based Coulomb model is validated by comparing the results with the results of Dahl model as both the models are the static models. For that a simple spring–mass system as shown in Fig. 26.4 is selected for the simulation.

For the system, the mass is driven with constant velocity on a rough surface in contact. The mathematical modeling of the identified problem can be a single degree of freedom, second-order spring–mass system dragged with a constant velocity (u), and the governing mathematical equation for the system is given by Eq. 26.10.

$$m\ddot{x} + k(y - x) - F_{\rm f} = 0 \tag{26.10}$$

In the equations, $\dot{y} = u$ is the driving velocity and $\dot{x}$ is the velocity of mass.
where

m is the mass of the moving body,
x is the position of the body,
y is the independent position of the wall,
k is the spring constant, and
$F_{\rm f}$ is the friction force.

The response of the system under the influence of sigmoid function-based model for the selected parameter as in Table 26.1, in terms of displacement, velocity, and friction force verses time plot, is depicted in Fig. 26.5.

The response of the system for Dahl model for the same parameters is shown in Fig. 26.6.

It is concluded from the comparison of responses shown in Figs. 26.5 and 26.6 that in case of both the models, there are friction-induced vibrations in the form of sinusoidal wave of constant frequency and amplitude and that are identical to both the models.

Though there is difference in friction force pattern as in the case of Coulomb model friction force in continuous and constant with time, it is intermittent with time in case of Dahl model that may be because of presliding displacement in case

Table 26.1 Values used in simulation for spring–mass system

Parameter	Value
Mass (m)	1 kg
Stiffness (K)	5 N/m
Micro-stiffness of bristle (σ_0)	100,000 N/m
Coulomb friction coefficient (μ)	0.1
Coulomb kinetic friction (F_C)	$\mu * \text{m g N}$
Driving velocity (u)	0.1 m/s
Sigmoid index (n)	5000

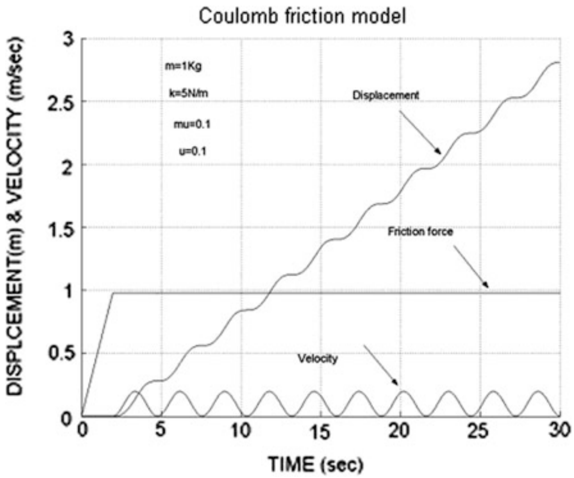

Fig. 26.5 Results of simulation using sigmoid function-based friction model

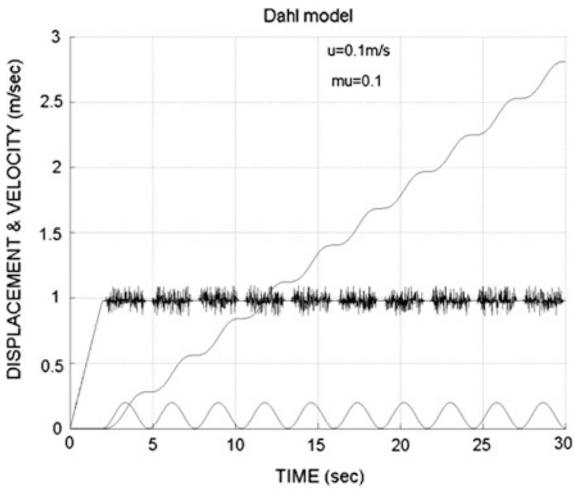

Fig. 26.6 Displacement, velocity, and friction force versus time plot for Dahl model

of Dahl model. So the sigmoid function-based model is validated by the result of simulation of the Dahl model.

26.3 Generalized Mechanical System for the Generic Linkage

The mechanical system used for carrying out simulation consists of a lever with moment of inertia as I, mass m, and a linear spring with stiffness k. The mass is subjected to force of friction. The spring is attached at one end to the mass, and at other with to a fixed support.

The lever is representative of a pedal, on which a time-varying force $F(t)$ is applied. The other end of the lever is connected to the cable. The schematic diagram of the system is shown in Fig. 26.7. The simulation is performed considering that the lever is initially pressed down completely. Under this situation, the spring is under tension, and the cable mass is displaced toward the left. Upon release of the pedal, the mass shall move toward right, under the spring force. The tension in the cable is $T(t)$. The governing equation is represented by a mathematical model depicted in Eq. 26.11.

$$\left[\frac{I}{\sqrt{l_2^2 - x^2}} + m\sqrt{l_2^2 - x^2}\right]\ddot{x} = \frac{-F(t)l_1\sqrt{l_2^2 - x^2}}{l_2} - F_f(t)\sqrt{l_2^2 - x^2} + kx\sqrt{l_2^2 - x^2} - \left[\frac{Ix(\dot{x})^2}{\left(l_2^2 - x^2\right)^{\frac{3}{2}}}\right] \tag{26.11}$$

Defining the Forcing Friction

The forcing function $F(t)$ has been defined as a variable force, which is maximum in the beginning, i.e., when the pedal is completely pressed down. Then this force is

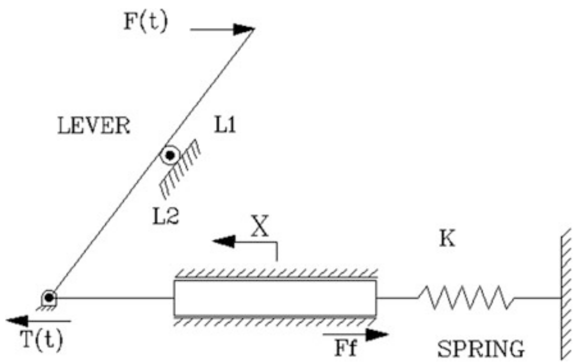

Fig. 26.7 Schematic diagram of generalized mechanical system

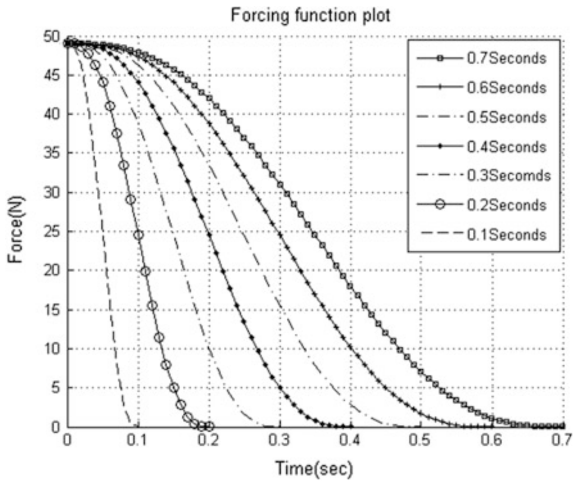

Fig. 26.8 Trajectory of applied force $F(t)$

gradually released, as is the case when the operator starts to release the pedal. Figure 26.8 shows the nature of forcing function, used in the simulation.

26.4 Results and Discussion

The dynamic model of the mechanical system shown in Fig. 26.7, the model of friction force, i.e., LuGre model and sigmoid function-based Coulomb friction model, and the model of forcing function described in the previous sections were

Table 26.2 Values used in simulation

Parameter	Value
Mass (m)	1 kg
Moment of inertia of lever	0.01234 kg m^2
Length l_1	0.25 m
Length l_2	0.06 m
Stiffness (K)	6600 N/m
σ_0	100,000 N/m
σ_1	$2\zeta\sqrt{(\sigma_0 \times m)}$ N s/m
σ_2	0.04 N s/m
F_C	Mass $*$ μ g N
F_s	1.5 $*$ F_C N
V_s	0.01 m/s
ζ	0.2–0.7
ω_n	5 rad/s
ω_d	4.33 rad/s
Coulomb friction coefficient	0.1
Sigmoid index (n)	5000

used for simulation of dynamic response of the mechanical system. The values for various parameters used for this simulation are given in Table 26.2.

Comparison of the Models

The simulation was carried out using two models of friction, the LuGre model and sigmoid function-based Coulomb friction model. Figure 26.9 shows the comparison between LuGre and Coulomb models on the basis of total settling time taken by the target mass to settle after the force is removed. It can be clearly seen from the figure that the Coulomb model takes longer settling time than the LuGre model. The time taken for lower values of release time is high for both the models as the

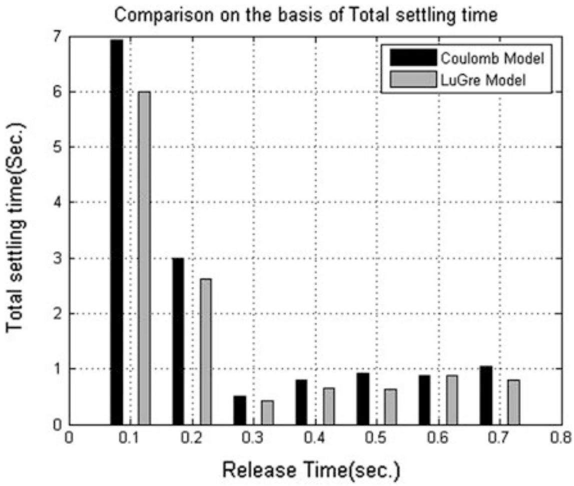

Fig. 26.9 Comparison on the basis of total settling time

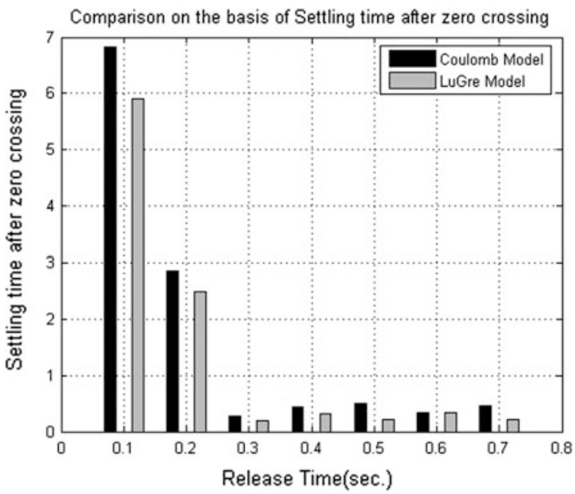

Fig. 26.10 Comparison on the basis of settling time after zero crossing

velocity of the system is high. The same phenomenon can be observed more clearly in the post-zero-crossing part of the dynamic response, as shown in Fig. 26.10. The time taken by the LuGre model to settle after zero crossing is less than Coulomb model for increasing values of release time. In the pre-zero-crossing part, the dynamics of the system is controlled mainly by the release force trajectory. Therefore, both the models show almost the same behavior, as evident from Fig. 26.11. The value for settling time before zero is increasing by a same value for both the models used.

In the present study, the computation time is the time required to obtain the dynamic response of the system numerically, for certain time duration. The time duration for all the results was taken as 8 s in this study. The computation time is

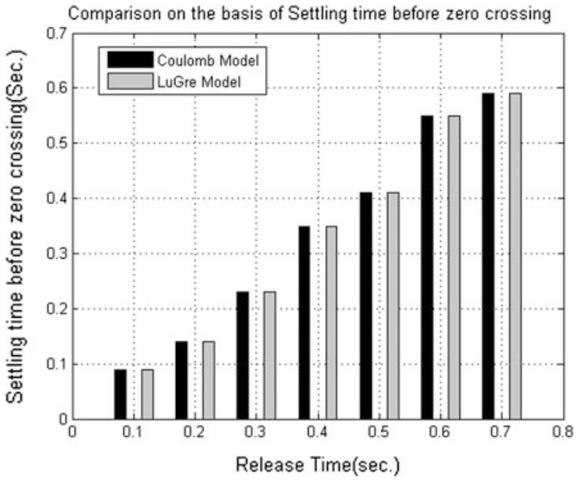

Fig. 26.11 Comparison on the basis of settling time before zero crossing

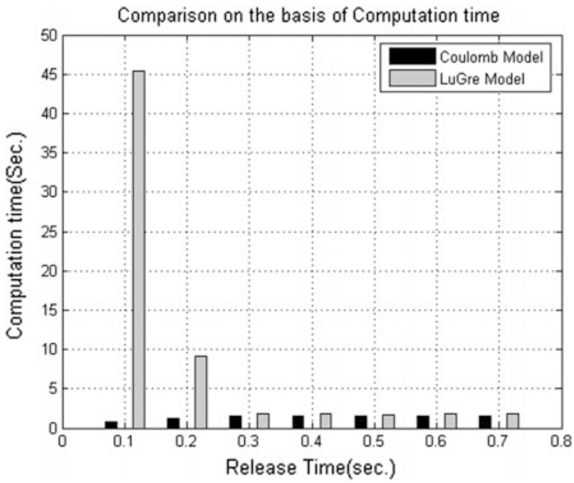

Fig. 26.12 Comparison on the basis of computation time

smaller in the case of Coulomb model for all the values of release time. For LuGre model, the value is very high for release times of 0.1 and 0.2 s. The values after 0.2 s are comparable to those of Coulomb model as shown in Fig. 26.12. This is due to the fact that the dynamics of bristles is much faster than that of the mechanical system. At higher velocities, the frequency of contact between the bristles increases, which causes the bristles to move faster. Numerical stiffness is caused due to difference between the speeds of friction dynamics and the dynamics of mass–spring system [10]. Due to this difference, smaller computational time steps are required, which results into longer computation times.

The time period for oscillations is higher for LuGre model for all values of release time as shown in Fig. 26.13 and the behavior is just opposite for frequency.

The value of maximum amplitude is almost same, as shown in Fig. 26.14.

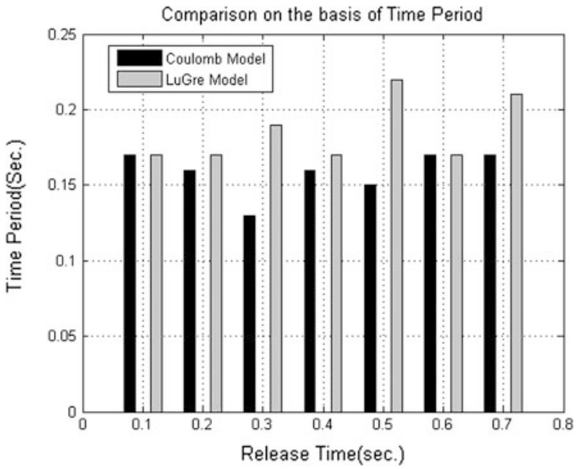

Fig. 26.13 Comparison on the basis of time period

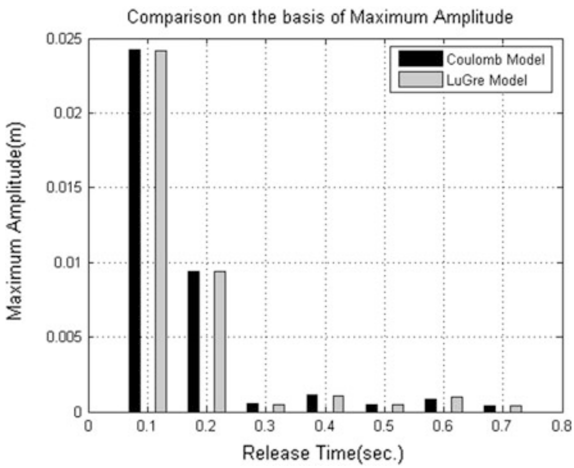

Fig. 26.14 Comparison on the basis of maximum amplitude

26.5 Conclusion

A typical mechanical actuation system for control applications was analyzed, with an objective to understand its dynamic behavior, in light of friction present in the system. The performance parameters were determined for different force trajectories, under which the system which moved from a starting condition. The friction force was modeled using LuGre model of dynamic friction. The dynamic response was compared with that obtained using the classical Coulomb model. Both the models predict dynamic response typical to an oscillating system. In a practical situation, where the motion of target mass is being controlled, it would be desirable that the target mass follows a smoother trajectory. The system parameters may be modified accordingly, and the model presented in this paper may be used for the design of such a system. The method of analysis and results presented in this paper may be useful in designing similar mechanical control systems in mechatronic systems such as hydraulic copy-turning machines, flight control mechanisms, single-acting hydraulic and pneumatic actuators, and robotics. This work can be extended for higher order mechanical systems.

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