Chapter 14
Wrong Limits and Wrong Assumptions: Kenny Norwich and Willy Wong Fail to Derive Equal-Loudness Contours

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14.1 Introduction

What is a loudness contour? It is a plotted curve, which is obtained for a given person by having that research subject adjust the intensity of a "comparison" tone of a given waveform frequency until it seems as loud as a "reference" tone of a constant intensity and constant frequency. This method has been used in psychology laboratories for decades. For each different reference-tone intensity, a different loudness contour ensues.

Kenneth Howard (Kenny) Norwich and Willy Wong claim to derive equal-loudness contours mathematically, from theoretical first-principles [1, 2]. This is the only occurrence of such a feat, to the present author's knowledge. Further, full comprehension of what Norwich and Wong did requires reading *two* papers [1, 2], of which the more recent one [1] gives a backward derivation of what appears in the older one [2]. Indeed, further reading proves to be necessary [3–6]. All of the needed papers are examined here, to provide a synopsis that is not found in the literature. The synopsis reveals wrong limits and wrong assumptions underlying the Norwich and Wong derivations.

14.2 Norwich and Wong: A Loudness Equation

Norwich and Wong [1] introduce an "Entropy Equation", in which loudness is denoted L. Acoustical stimulus intensity, in units of power, is denoted I. Norwich and Wong ([1], Eq. (11)) write

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$$L = \frac{k}{2} \ \ln \ \left(1 + \gamma \left(\frac{I}{I_{th}}\right)^n\right) . \tag{14.1}$$

Investigation reveals that this is also Eq. (9) of [2], but that has k instead of k/2 (i.e., the ½ was presumably absorbed into “k”). “k” appears in earlier Norwich papers, as “a proportionality constant” [3], its value “determined by the arbitrary scale units of the experimenter” ([4], p. 269). Norwich and Wong assume that k is *independent of intensity and frequency*. The term I_{th} in Eq. (14.1) is the “threshold intensity”. Norwich and Wong ([1], p. 931) now name L_{th}, “the loudness threshold” (actually the *threshold loudness*); from Eq. (14.1),

$$\begin{aligned} L(I_{th}) = L_{th} &= \frac{k}{2} \ \ln\left(1 + \gamma \left(\frac{I_{th}}{I_{th}}\right)^n\right) \\ &= \frac{k}{2} \ \ln \ (1 + \gamma) . \end{aligned} \tag{14.2}$$

They then state that ([1], Eq. 12)

$$L = \begin{bmatrix} L - L_{th}, & L > L_{th} \\ 0, & otherwise \end{bmatrix} . \tag{14.3}$$

(This equation will be explored soon.) Altogether, from Eqs. (14.1) and (14.2) and L = L–L_{th} in Eq. (14.3),

$$\begin{aligned} L &= \frac{k}{2} \ln\left(1 + \gamma \left(\frac{I}{I_{th}}\right)^n\right) - \frac{k}{2} \ \ln \ (1 + \gamma) \\ &= \frac{k}{2} \ \ln \left(\frac{1 + \gamma \left(\frac{I}{I_{th}}\right)^n}{1 + \gamma}\right) \quad for \ I > I_{th}. \end{aligned} \tag{14.4}$$

This appears in [1] as Eq. (13), with the γ misprinted as “y”. Equation (14.4) also appears in [2], as Eq. (10) but with k instead of k/2.

14.3 Norwich and Wong: The “Weber Fraction”

Norwich and Wong [1] now differentiate Eq. (14.4). They get ([1], Eq. 14)

$$\frac{dL}{dI} = \frac{k}{2} \ \frac{n\gamma I^{n-1} I_{th}^{-n}}{1 + \gamma \left(\frac{I}{I_{th}}\right)^n} \tag{14.5}$$

which does not depend on which of Eq. (14.1) or Eq. (14.4) is used for loudness. Norwich and Wong [1] then reorder Eq. (14.5) to obtain dI/I. They then replace dL by ΔL, the difference limen (i.e., the subjectively just-noticeable difference) in loudness. Likewise, they replace dI by the intensity change corresponding to ΔL, namely, ΔI ([1], p. 932). Altogether,

$$\frac{\Delta I}{I} = \frac{2\Delta L}{nk}\left(1 + \frac{1}{\gamma}\left(\frac{I_{th}}{I}\right)^{n}\right), \tag{14.6}$$

which they name the Weber fraction ([1], Eq. (15)).

The *empirical* values of the Weber fraction had already been studied by many psychologists. Early-on, there was Riesz [7]. He mentioned an equation that "can be made to represent ΔE/E [his notation for ΔI/I] as a function of intensity at any frequency", by adjusting three equation parameters ([7], p. 873), each parameter itself being an empirical equation of frequency. Norwich and Wong [1] transformed Eq. (14.6) into the empirical equation used by Riesz [7], first by assuming (after Fechner [8]) that ΔL is constant with intensity, and then by defining two new positive quantities, S_∞ and S_0–S_∞:

$$S_\infty = \frac{2\Delta L}{nk}, \tag{14.7a}$$

$$S_0 - S_\infty = \frac{2\Delta L}{nk\gamma}. \tag{14.7b}$$

Hence ([1], Eq. (22))

$$\gamma = \frac{S_\infty}{S_0 - S_\infty}. \tag{14.7c}$$

Altogether, then,

$$\frac{\Delta I}{I} = S_\infty + (S_0 - S_\infty)\left(\frac{I_{th}}{I}\right)^{n} \tag{14.8}$$

where $\Delta I/I > 0$. Equation (14.8) has the same general form as Eq. (2) of Riesz [7]. Equation (14.8) is also Eq. (16) of [1] and Eq. (1) of [2].

14.4 Norwich and Wong: Derivation of Equal-Loudness Contours

Norwich and Wong [1] then proceed to derive equal-loudness contours. First is the theoretical intensity of the comparison tone, as a function of its frequency, found by equating comparison-tone loudness to that of a constant-intensity 1 kHz reference tone. With the parameters of the reference tone denoted by ^ (caret),

$$L(I,f) = L\left(\hat{I},\ 1\ kHz\right) \tag{14.9}$$

(in [1] as Eq. (37)). Substituting appropriate terms from Eq. (14.4) gives

$$\frac{k}{2}\ \ln\left(\frac{1+\gamma\left(\frac{I}{I_{th}}\right)^{n}}{1+\gamma}\right) = \frac{\hat{k}}{2}\ \ln\left(\frac{1+\hat{\gamma}\left(\frac{\hat{I}}{\hat{I}_{th}}\right)^{\hat{n}}}{1+\hat{\gamma}}\right) \tag{14.10}$$

which is Eq. (12) of [2], but with k there instead of k/2, and with "prime" in place of "caret", and with a script I in place of caret-I. Equation (14.10) can be re-arranged to solve for I/I_{th}; ten times its logarithm to base 10 gives the intensity of the matching comparison tone in decibels *sensation level* (dB SL):

$$10\ \log_{10}\left(\frac{I}{I_{th}}\right) = 10\ \log_{10}\left(\left(\frac{1}{\gamma}\right)^{\frac{1}{n}}\left[(1+\gamma)\ \left(\frac{1+\hat{\gamma}\left(\frac{\hat{I}}{\hat{I}_{th}}\right)^{\hat{n}}}{1+\hat{\gamma}}\right)^{\frac{\hat{k}}{k}} - 1\right]^{\frac{1}{n}}\right). \tag{14.11}$$

It transpires that Eq. (13) of [2] is the term within the largest brackets here, *but* with k instead of k/2, and with "prime" instead of "caret", and with a script I in place of caret-I.

In Norwich and Wong [1], the frequency for the ^ parameters is taken to be 1 kHz. $\hat{I}_{th}$ is taken from Wegel [9]. S_0 and S_∞ are evaluated from Riesz [7], allowing values for γ (Eq. (14.7c)). k is assumed to be independent of frequency and intensity ([1], p. 931). k and $\hat{k}$ are hence assumed equal; $\hat{k}/k$ therefore disappears from Eq. (14.11).

The reference tone for making loudness contours is here 1 kHz. *Empirically*, each intensity of the 1 kHz tone would have an associated plot of the points {tone frequency, tone sensation level} that subjectively match the loudness of the 1 kHz reference tone. Equation (14.11) gives the respective *theoretical* equal-loudness contours; they appear in Fig. 5 of Norwich and Wong [1] and in Fig. 4 of Wong and Norwich [2]. They are bowl-shaped, being lowest in-between the lowest and highest waveform frequencies.

Norwich and Wong note elsewhere [5] that Fletcher and Munson [10] had produced equal-loudness contours, likewise using an SL scale. Unlike the theoretical curves of Norwich and Wong [1] and of Wong and Norwich [2], however, the Fletcher and Munson [10] contours *increase* in-between the lowest and highest waveform frequencies, forming hill-shaped plots. Norwich and Wong [1] hence replaced Riesz's "n" by an equation for the Stevens exponent as a function of frequency. They employed ([1], p. 935) "a function similar to the one suggested by Marks" [11]. Their theoretical equal-loudness contours were now hill-shaped like those of Fletcher and Munson [10].

14.5 Examining Norwich and Wong (1): The Loudness Equation at the Loudness Threshold

Let us carefully scrutinize what Norwich and Wong [1] did. According to them, Eq. (14.4) above derives from Eq. (14.1) above. That is, loudness allegedly obeys *both* Eqs. (14.1) *and* (14.4). *But this is not possible*. First, note that the upper line of Eq. (14.3) should read $L \neq L–L_{th}$, not $L = L–L_{th}$. Now consider loudness limits. As $I \rightarrow I_{th}$ from $I > I_{th}$, $L \rightarrow L_{th}$ (following Eq. (14.1)); but according to Eq. (14.4), $L \rightarrow 0$. Thus, we have $L = 0$ *and* $L = L_{th}$ at the same intensity, I_{th}. Note also that as $I \rightarrow 0$ from $I > 0$, then after Eq. (14.1), $L \rightarrow 0$; but after Eq. (14.4), $L \rightarrow (–k/2)\ln(1 + \gamma)$, a *negative* loudness ($k, \gamma > 0$). Of course, a negative loudness is an impossibility.

14.6 Examining Norwich and Wong (2): The Constancy of K

Crucially, Norwich and Wong [1] assume that the parameter k is independent of intensity and frequency. *But Norwich and his co-authors never test that assumption, even up to the present day.* Therefore, it is tested here, as follows. k can only be obtained by fitting Eq. (14.1) or Eq. (14.4) to empirical plots showing loudness growth with intensity. To do so, first the term γ/I_{th}^{n} in Eq. (14.1) was replaced by a symbol γ'. Equation (14.1) was then fitted to thirty-seven loudness-intensity plots from the peer-reviewed experimental literature. Following that literature, Eq. (14.1) was put into logarithmic form in {*ln* I, *ln* L} coordinates, before fitting to logarithms of loudnesses.

Figure 14.1 shows the fitted values of k, versus the respective tone frequency, or for white noise. The empirical loudness-growths were taken from: [12], white noise (geometric means of magnitude estimates: series 1–3); [13], 1 kHz tone (Figs. 2, 3,

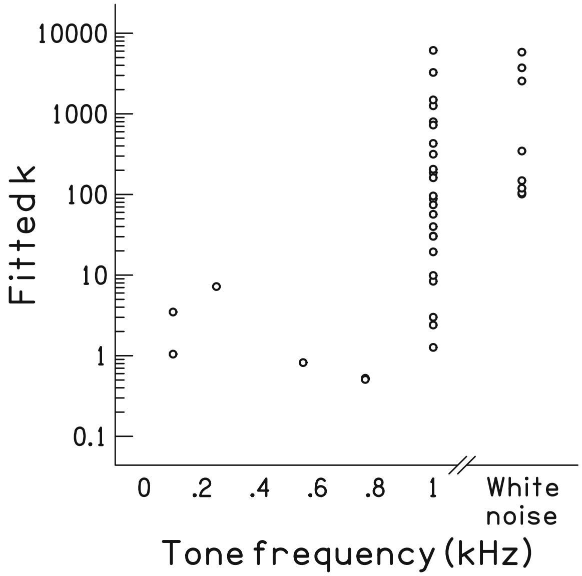

Fig. 14.1 The fitted value of the free parameter k of Eq. (14.1) as a function of tone frequency or for white noise (see text). The 37 dots represent more loudness-growth plots than in all of Norwich et al.'s relevant publications, from the very first in 1975 up to the present day

6, 7, 8, and 10); [14], 0.1 kHz tone (Fig. 2, crosses; Fig. 2, circles), 0.250 kHz tone (Fig. 3, geometric means of circles); [15], 1 kHz tone (subjects # 8, 9, 10, 11, 12, 13); [16], 1 kHz tone (curves 1–7); [17], 0.550 kHz tone (subject AWS), 0.765 kHz tone (subjects EWB, RSM); [18], white noise (Fig. 2: binaural, magnitude production; binaural, magnitude estimation; monaural, magnitude

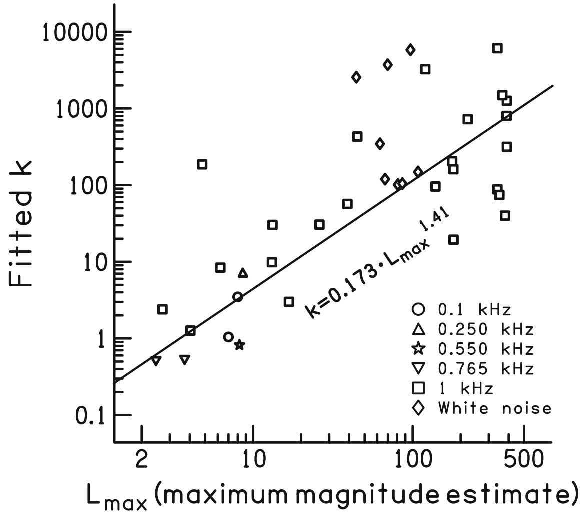

Fig. 14.2 The fitted value of the free parameter k of Eq. (14.1) (values of Fig. 14.1) versus the corresponding maximum magnitude estimate from each empirical loudness-growth plot. The line $k = 0.173 \cdot L_{max}^{1.41}$ is fitted to the data points (see text)

production; monaural, magnitude estimation); [19], 1 kHz tone (Fig. 1, circles; Fig. 1, squares), white noise (Fig. 7, circles and crosses); [20], binaural 1 kHz tone (cross-modality-matching, high range day 2; low range day 2).

Regarding Fig. 14.1: k is *not* constant. For 1 kHz tones and for white noise, multiple k values are evident. Figure 14.2 shows k versus the maximum loudness available from each empirical loudness-growth plot (i.e., the loudness at the highest respective applied stimulus intensity). A power function, converted to logarithmic form (as per the literature), was fitted using sum-of-squares-of-residuals. The latter favors higher loudnesses, hence the data points were weighted by the square root of the absolute value of each loudness. From this, $k = 0.173 \cdot L_{max}^{1.41}$. Norwich and co-authors never mention such a relation. Regardless, k is not constant with L_{max}. Hence, k cannot generally be constant, although Norwich and Wong [1] think otherwise.

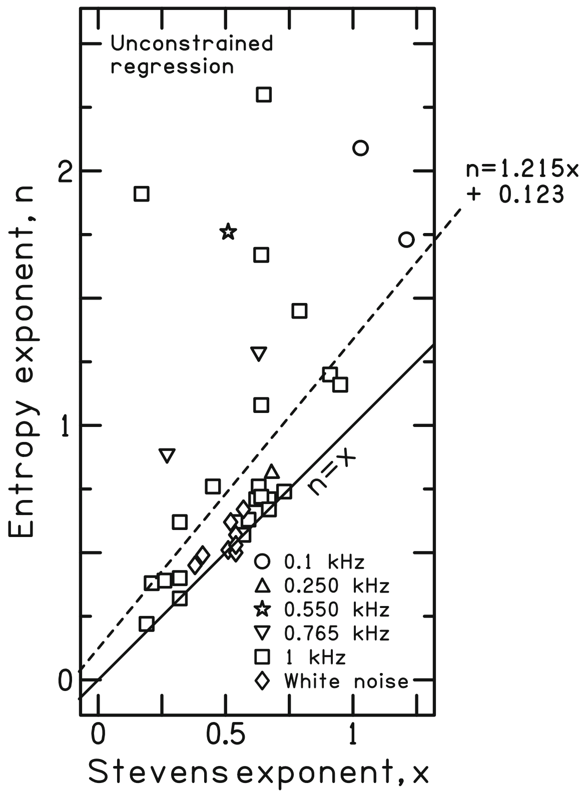

Fig. 14.3 The fitted value of the free parameter n (the Entropy exponent) of Eq. (14.1), versus the fitted value of the Stevens exponent, x, for the loudnesses used in Fig. 14.1. The line n = x indicates putative equality of n and x. The line n = 0.123 + 1.215x is fitted to the data points (see text)

14.7 Examining Norwich and Wong (3): Entropy Exponent, Stevens Exponent

Recall that Norwich and Wong [1] replace n, the exponent of Eq. (14.1) and of Eq. (14.4), with x, the exponent of Stevens' power law: they assume that n = x. But, just as for k, *Norwich and his co-authors never test that assumption, even up to the present day.* Values of n arise from the same curvefitting described just above. Therefore, there is just one parameter remaining to be quantified, namely x. It is obtained through the same procedures described above, but with $L = aI^x$ as the fitted equation. Having thereby obtained values of n and of x, we may examine the notion that n = x. Figure 14.3 shows n versus x.

A straight line fitted to the data points gives n = 0.123 + 1.215x. The reasons for the numbers 0.123 and 1.215 are not known. Nonetheless, n does *not* equal x, contrary to Norwich and Wong [1].

To thoroughly establish any actual relation between n and x, further analysis was done, as follows. Norwich's "Entropy Theory" (e.g., [6]) specifies that the maximum transmitted information during the perception of a stimulus, called $I_{t,\ max}$ units

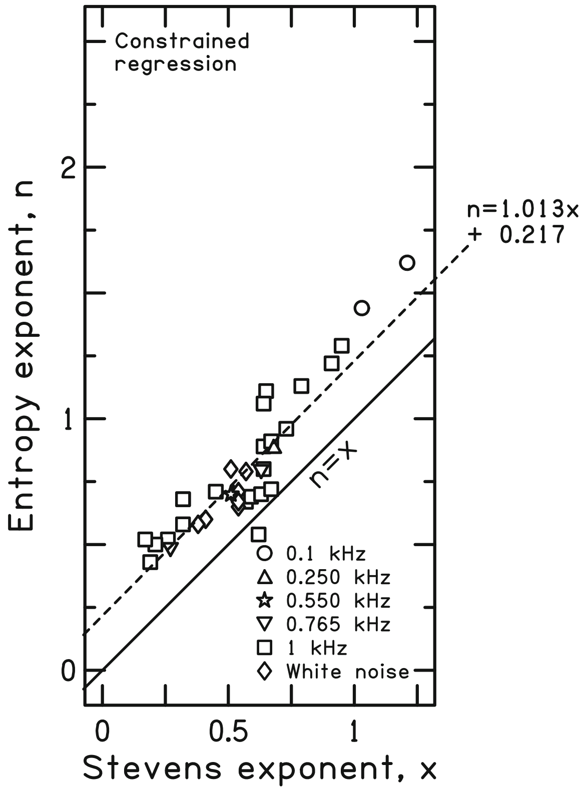

Fig. 14.4 n versus x, for the magnitude estimates used in Fig. 14.1, when the fit of Eq. (14.1) is constrained to produce $I_{t,\ max} = 2.5$ bits/stimulus (see text). The Entropy exponents n may therefore differ from those in Fig. 14.3. The line n = 0.217 + 1.013x is fitted to the data points, as described in the text

of information, is related to maximum loudness L_{max} and minimum loudness L_{min} as

$$I_{t,\ max} = \frac{L_{max} - L_{min}}{k}. \tag{14.12}$$

k in the denominator is the same k as investigated above. Empirically $L_{max} \gg L_{min}$, allowing L_{min} to be ignored. The fitting of the Entropy equation to loudness growth can be stopped for a value of k yielding $I_{t,\ max}$ = 2.5 bits/stimulus, which is the average value found in the literature [6]. Figure 14.4 shows the relation of n to x in such circumstances. A line fitted to the data points in the same manner described above is n = 0.217 + 1.013x. The reasons for the values 0.217 and 1.013 are unknown. Again, however, n does *not* equal x, contrary to Norwich and Wong [1]. The difference may seem trivial, but it is well-known that empirical values of x can be as low as 0.3 (e.g., [11]), in which case the difference between n and x is not, in fact, trivial.

14.8 Examining Norwich and Wong (4): Frequency-Dependence of the Exponent

Recall from above that Norwich and Wong ([1], p. 935) employed "a function similar to the one suggested by Marks" [11] to make their theoretical equal-loudness contours hill-shaped like those of Fletcher and Munson [10]. The

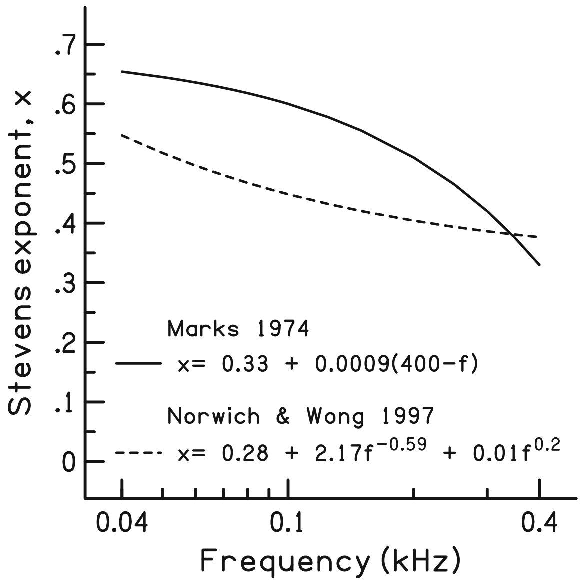

Fig. 14.5 x as a function of frequency, from the equation of Marks and the equation of Norwich and Wong. (The curves are only shown for frequencies below 400 Hz, Marks' limit of validity for his equation.)

respective equation of Marks [11] describes the Stevens exponent x as a function of frequency f (in Hertz), and is

$$x = 0.33 + 0.0009 \cdot (400 - f) \quad where \quad f < 400Hz, \tag{14.13}$$

which applies "over low frequencies (f) and not too high sound pressure levels" ([11], p. 74). Compare Eq. (14.13) to the Norwich and Wong equation "similar to the one suggested by Marks" [1], Eq. (40), but with no restrictions of sound pressure level or frequency:

$$x = 0.28 + 2.17f^{-0.59} + 0.01f^{0.2}. \tag{14.14}$$

Equations (14.13) and (14.14) are not the same. Indeed, we do not know the units of the constants in Eqs. (14.13) and (14.14), hence we do not even know whether the x's represented by the two equations have the same units! Figure 14.5 shows the curves generated by Eqs. (14.13) and (14.14). The curves clearly differ, and in fact, they intersect only once.

14.9 Conclusions

Norwich and Wong [1] present a backwards derivation of arguments made in Wong and Norwich [2]. Wrong arguments in one paper prove to be wrong in the other. In particular, Norwich and Wong [1] make several assumptions which prove to be unjustified. All of this is somewhat surprising, given that Norwich and Wong [1] identify several prominent tenured professors (Lawrence Marks, Lester Krueger, Lawrence Ward) as reviewers of their paper. It is even more alarming that Wong and Norwich [2] was reviewed by a *theorist*, William Hellman.

The present conclusions reveal that embarrassing and needless errors occur consequent to the embarrassing and needless errors of inappropriate limits, unjustified assumptions, and quantities having incompatible units. Indeed, Norwich's entire Entropy Theory is riddled with mistakes (see [21–30]). Professor Norwich evidently tolerates a culture of errors in his laboratory; papers authored independently by his co-authors may prove similarly faulty.

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