Aircraft Takeoff Taxiing Model
Based on Lagrange Interpolation Algorithm

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Abstract. Research on aircraft takeoff taxiing model has a great significance upon evaluation of aircraft performance, design of airport runway and warfare simulation. Because the aircraft takeoff taxiing is a complex nonlinear process, existing numerical calculation models are ordinarily based on the energy method or assumption of moment equilibrium, which leads to complex modeling process and low precision. In this paper, we establish a takeoff taxiing model by numerical integration method which is easy to implement, and obtain the parameters of running distance calculation formula based on Lagrange interpolation algorithm [1]. At the end of this paper, we use the characteristic parameters of a specific aircraft as model inputs to validate the correctness and accuracy of this model by comparing the model output and actual flight data.

Keywords: Takeoff taxiing model · Warfare simulation · Lagrange interpolation algorithm · Numerical integration

1 Introduction

Takeoff taxiing is the first step of flight and a process of aircraft continuous acceleration on the runway until leave ground [2]. Aircraft takeoff taxiing model can imitate the taxiing course in different environmental conditions, according to the given engine thrust parameters and aerodynamic parameters [3]. Developing aircraft takeoff taxiing model is very meaningful: (1) it can calculate aircraft takeoff performance [4], including takeoff distance, taxiing time and takeoff speed, these are very helpful to the development of aircraft and the design of airport runway; (2) it can also be used to estimate the survival probability of aircraft during takeoff period, through combat simulation.

Takeoff taxiing is a complex acceleration process. Aircraft will be affected by gravity, runway support force, friction, air resistance, lift and engine thrust at each moment, and these forces will even change with the velocity. And the aerodynamic parameters of aircraft, such as instantaneous thrust, lift coefficient and drag coefficient, will also be affected by different of state of the aircraft. According to the characteristics above, we first use Lagrange interpolation algorithm to calculate the instantaneous thrust, lift and drag coefficient of taxiing distance calculation formula. Then we determine the takeoff speed through iterative method with high-precision termination

conditions. Finally, we use numerical integration method to establish the aircraft takeoff taxiing model based on the running distance calculation formula.

The remainder of this paper is organized as follows. In Sect. 2, the related work is shown. In Sect. 3, the key algorithm used in the model is introduced. In Sect. 4, the process of modeling aircraft takeoff taxiing is described. In Sect. 5, the correctness of this model is proved by an experiment. In the last section, a summary is made for this paper.

2 Related Work

As the takeoff taxiing is an important part of aircraft flight and the modeling of takeoff taxiing is important for aircraft development, runway design and warfare simulation, many researchers have proposed lots of schemes and technologies.

At present, there are many methods to establish the numerical model of the aircraft takeoff taxiing, such as look-up table method, analytic method, numerical integration method, energy method [5] and so on. Using table checking method to establish the takeoff taxiing model can accurately simulate aircraft takeoff taxiing process, but creating tables of different parameters for looking up needs a lot of flight tests, resulting in high modeling costs. Analytical method builds takeoff taxiing model based on the assumption of moment equilibrium [6] and needs to consider the force of the aircraft at every moment, and they will lead to complex modeling process and large calculation. In addition, the output data of aircraft takeoff taxiing model established by energy method [7] is rough and users cannot get the exact location of the aircraft during takeoff taxiing process. Numerical integral method combined with the advantages of energy method and analytical method, can be very convenient to build takeoff taxiing model.

Using numerical integral method to build takeoff taxiing model can have a good practical effect. But the selections of dynamic parameters, aerodynamic parameters and movement parameters of existing models are based on the fixed engine performance and aerodynamic characteristic curves or some fixed values, so the precision of models are not high. In literature [8], Johnson gets instantaneous thrust of aircraft by modeling aircraft engine using the similarity method, but the method is not universal and the modeling process is complex. In literature [7], the author uses empirical data to calculate the lift and drag coefficient of taxiing model, which results in low precision.

The establishment of takeoff taxiing model using numerical method needs to determine the takeoff speed of aircraft. The literature [9] points out that iterative method can be used to determine the takeoff speed, but the iterative process and the termination conditions are not specified. The literature [7] generates the formula of takeoff speed based on the assumption that the lift force is equal to the gravity at the moment of leaving ground, but the assumption does not take into account the rising acceleration of the aircraft.

In this paper, we first use Lagrange interpolation algorithm to calculate the instantaneous thrust and lift and drag coefficient. Then determine the takeoff speed of aircraft using iterative method. Finally, we use numerical integration method to establish the aircraft takeoff taxiing model based on the running distance calculation formula.

3   Key Algorithm

Establishing aircraft takeoff taxiing model according to running distance calculation formula needs to know the instantaneous thrust of engine under actual air pressure and temperature of airport. However, the general aircraft performance specification only gives a few thrust curves of engine in the international standard atmosphere, as shown in Fig. 1. Therefore, we use Lagrange 2D interpolation algorithm to determine the actual instantaneous thrust of engine. Its basic principles and procedures are as follows:

Fig. 1.   Thrust curve of engine F100-PW-200

Let $n + 1$ distinct interpolation points (nodes) $x_0, x_1, \cdots, x_n$, be given $y_0, y_1, \cdots, y_n$, together with corresponding numbers, which may or may not be samples of a function $y = f(x)$. Let Π_n denote the vector space of all polynomials of degree at most n. The classical problem addressed here is that of finding the polynomial $y = p_n(x) \in \Pi_n$ that interpolates $y = f(x)$ at the points $x_0, x_1, \cdots, x_n$, i.e.,

$$p_n(x_k) = y_k, \quad k = 0, 1, \cdots, n$$

The problem is well-posed; i.e., it has a unique solution that depends continuously on the data. Moreover, as explained in virtually every introductory numerical analysis text, the solution can be written in Lagrange form:

$$p_n(x) = \sum_{i=0}^{n} f(x_i) l_i(x) = f(x_0) l_0(x) + f(x_1) l_1(x) + \cdots + f(x_n) l_n(x) \tag{1}$$

And $l_i(x)$ is Lagrange basic polynomial:

$$l_i(x) = \prod_{j=0, j \neq i}^{n} \frac{(x - x_j)}{(x_i - x_j)}, i = 0, 1, \cdots, n \tag{2}$$

Let $n = 3$, the Lagrange interpolation polynomial is:

$$p_3(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2 \tag{3}$$

2D Lagrange interpolation is actually a surface interpolation of a two-variable function; it can be realized by the ideal of dimension reduction [10]. Let two-variable function $z = f(x, y)$, calculate estimated value z at point (x, y), the specific method can be carried out according to the following steps:

1. Find three nearest interpolation points x_i, x_{i+1}, x_{i+2} to x;
2. Let $x = x_i$, find three nearest interpolation points y_i, y_{i+1}, y_{i+2} to y, calculate estimated value $z_1 = p_3(y)$ at points (y_i, z_i), (y_{i+1}, z_{i+1}) and (y_{i+2}, z_{i+2}) with formula (3);
3. Let $x = x_{i+1}$ and x_{i+2}, get z_2 and z_3 by the same way;
4. Calculate estimated value $z = p_3(x)$ at points (x_i, z_1), (x_{i+1}, z_2) and (x_{i+2}, z_3) with formula (3).

4   Modeling Process

By consulting the open literature [9] we can know that in a small period of time, the distance increment Δs and the velocity increment ΔV have the following relationship:

$$\Delta s = \frac{(V - V_w) \Delta V}{\left[\frac{n P_s(H_p, M) \cos(\alpha + \varphi)}{m g} \right] - f - \theta - \frac{(C_x - f C_y) \rho_s S V^2}{2 m g}} \times \frac{1}{g} \tag{4}$$

In formula (4): m is aircraft mass; g is gravitational acceleration; V_w is the wind-speed along running direction; n is the number of engine; $P_s(H_p, M)$ is instantaneous thrust of one engine, which is related with airport pressure altitude H_p and instantaneous Mach M; α is angle of attack; φ is installation angle of engine; f is friction coefficient of airport runway; θ is average slope of airport runway; C_x is lift coefficient; C_y is drag coefficient; ρ_s is air density; S means wing area.

To establish aircraft takeoff taxiing model using numerical integral method, we must determine all parameters in the formula (4). Parameters: m, n, α, φ, S can be found in aircraft specification; V_W, f, θ can be assigned according to the actual situation of airport. According to the laws of thermodynamics, ρ_s can be determined by the following formula:

$$\rho_s = \rho_0 \times \frac{P_s}{P_0} \times \frac{T_0}{T_s} \tag{5}$$

In formula (5): ρ_0 means standard atmospheric density; P_0 is standard sea level atmospheric pressure; T_0 is Kelvin temperature 273.15 K; P_s actual atmospheric pressure at the airport; T_s airport actual Kelvin temperature.

4.1 Instantaneous Thrust

The instantaneous thrust of the engine P_s is related to the pressure altitude of the aircraft H_p and the instantaneous Mach M. Referring to the thrust characteristic curve in the international standard atmosphere given by aircraft specification, we can know the instantaneous thrust $P(H_i, M_j)(i = 1, 2, \cdots, s; j = 1, 2, \cdots, t)$ at some typical height and Mach points (H_i, M_j). As long as we know the pressure altitude of airport H_p and instantaneous Mach M, we can calculate the instantaneous thrust using 2D Lagrange interpolation introduced in Sect. 3 of this paper. The airport pressure altitude H_p is related to the actual atmosphere of the airport P_s, and the relationship is:

$$H_p = \frac{1 - (P_s/P_0)^{1/5.25588}}{225577} \times 10^5 \tag{6}$$

Instantaneous Mach M is the ratio of aircraft instantaneous velocity V and actual sound velocity v_s, and the calculation formula of v_s is as follows:

$$v_s = \sqrt{1.4 \times 287 \times T_s} \tag{7}$$

4.2 Lift and Drag Coefficient

The lift coefficient and drag coefficient are related to the angle of attack of aircraft, and the relationships are generally expressed by lift curve and polar curve. According to the characteristics of lift curve and polar curve, the lift coefficient and drag coefficient can be calculated by Lagrange three-point interpolation method. The approach is, finding the lift coefficient $C_{y(i)}(i = 0, 1, \cdots, p)$ corresponding to angle of attack α_i from the lift curve given by the aircraft specification, then calculating lift coefficient C_y about α using the following formula:

$$C_y = \sum_{i=r}^{r+2} \left\{ \left[\prod_{\substack{j=r \\ j \neq i}}^{r+2} \frac{\alpha - \alpha_j}{\alpha_i - \alpha_j} \right] \times C_{y(i)} \right\} \tag{8}$$

α_r, α_{r+1}, α_{r+2} are the nearest points of α. Similarly, we can find the drag coefficient $C_{x(i)}(i = 0, 1, \cdots, q)$ correspond to lift coefficient $C_{y(i)}$ from the polar curve, then calculate drag coefficient C_x about C_y using the following formula:

$$C_x = \sum_{i=l}^{l+2}\left\{\left[\prod_{\substack{j=l \\ j \neq i}}^{l+2}\frac{C_y - C_{y(j)}}{C_{y(i)} - C_{y(j)}}\right] \times C_{x(i)}\right\} \tag{9}$$

$C_{y(l)}$, $C_{y(l+1)}$, $C_{y(l+2)}$ are the nearest points of C_y.

4.3   Takeoff Speed

As the lift equals to gravity when the aircraft leaves ground, we can know that:

$$\frac{1}{2}C_{y2}\rho_s SV_t^2 + nP_s(H_p, M_t)\sin(\alpha_2 + \varphi) = mg \tag{10}$$

In this formula, C_{y2} is the lift coefficient at that moment; α_2 is the angle of attack; M_t is the instantaneous mach. From formula (10), we can get takeoff speed V_t:

$$V_t = \sqrt{\frac{2[mg - nP_s(H_p, M_t)\sin(\alpha_2 + \varphi)]}{\rho_s SC_{y2}}} \tag{11}$$

As $P_s(H_p, M_t)$ is the actual instantaneous thrust corresponding to the takeoff speed V_t when the aircraft leaves ground and V_t is unknown, we can use iterative method to solve it. The iterative equation is as follows:

$$V_{t(k+1)} = \sqrt{\frac{2[mg - nP_s(H_p, M_{t(k)})\sin(\alpha_2 + \varphi)]}{\rho_s SC_{y2}}} \tag{12}$$

And initial condition is $V_{t(0)} = \sqrt{\frac{2mg}{\rho_s SC_{y2}}}$; termination condition is $\left|V_{t(k+1)} - V_{t(k)}\right| < 10^{-6}$. The speed $V_{t(k+1)}$ at the end of iteration is what we want.

4.4   Takeoff Taxiing Model

Aircraft takeoff taxiing can be divided into two phases [11], in phase I aircraft will run with three wheels. With the increase of speed, aircraft will lift the front wheel, running with two wheels, this is the phase II. The speed of the aircraft to lift its front wheel is V_r, which is 0.8 times the takeoff speed V_t. Now we take phase I as an example to establish its numerical integration model; we assume that in this phase, the aircraft

angle of attack is α_1, and the corresponding lift and drag coefficient are C_{y1} and C_{x1}. Divide velocity interval $[V_w, V_r]$ into l identical cells with size ΔV_1, and in each cell we assume that velocity V and engine thrust $P_s(H_p, M)$ remain unchanged, and is the estimated value corresponding to the velocity at the left endpoint of the cells. Using numerical integration method we can get:

$$\Delta V_1 = \frac{V_r - V_w}{l} \tag{13}$$

$$\Delta S_i = \frac{(V_i - V_W)\Delta V_1}{\left[\frac{nP_s(H_p,M_i)cos(\alpha_1 + \varphi)}{mg}\right] - f - \theta - \frac{(C_{x1} - fC_{y1})\rho_s SV_i^2}{2mg}} \times \frac{1}{g} \tag{14}$$

$$V_{i+1} = V_i + \Delta V_1, V_0 = V_w \tag{15}$$

$$S_{i+1} = S_i + \Delta S_i, S_0 = 0 \tag{16}$$

$$t_{i+1} = t_i + \frac{\Delta S_i}{V_i}, t_0 = 0 \tag{17}$$

According to the above formula, we can get the speed V_i and the distance S_i at time t_i in phase I. The modeling method of phase II is the same as phase I, but the angles of attack are different and the initial state of phase II is the termination state of phase I. Aircraft takeoff taxiing model is the combination of the this two phase models.

5 Validation

Based on the modeling process above, we developed aircraft takeoff taxiing model using C++ programming language. This program can simulate the takeoff taxiing processes of various aircrafts under different environmental conditions. In order to prove the correctness of this model, we take the characteristic parameters of a certain aircraft as the model input, and compare the output data of the model and the actual test data. Figure 2
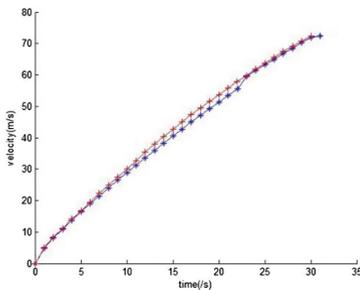

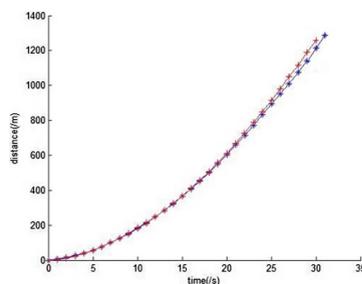**Fig. 2.** The velocity curves of the model output and flight test

Fig. 3. The aircraft taxiing distance curves of model output and flight test

shows the velocity curves of the model and aircraft. Through analysis, we find that the maximum absolute error of the output data and the actual flight data is approximately 2.5 m/s, and the maximum relative error is about 4.7 %. Figure 3 shows the aircraft taxiing distance curves. Through the comparison of the two curves we can find that the calculation results and actual measurement are very close. The calculation running distance is 1263 m, and the actual measurement is approximately 1257 m. Thus, we can say that the aircraft takeoff taxiing model is correct and high-precision.

The physical characteristics of this aircraft as shown in Table 1. Airport environmental data as shown in Table 2. Aerodynamic characteristics and thrust characteristic as shown in Tables 3 and 4.

Table 1. Physical characteristics of aircraft

Mass	60×10^3 kg
Number of Engine	2
Wing Area	$164.5m^2$
Installation Angle	$0°$
Stop AOA	$4°$
Takeoff AOA	$8°$

Table 2. Airport environmental data

Air Pressure	96525 Pa
Temperature	22.3°C
Wind Speed	2.0m/s
Gravity	$9.8m/s^2$
Average Slope	−0.0011
Friction	0.025

Table 3. Aerodynamic characteristics of aircraft

AOA/°	Lift coefficient	Drag coefficient
0	0.459	0.052
2	0.623	0.058
4	0.908	0.069
6	0.967	0.080
8	1.171	0.097
10	1.316	0.122
12	1.422	0.151

Table 4. Thrust characteristic of aircraft

Height/m	Mach	Thrust/N	Height/m	Mach	Thrust/N	Height/m	Mach	Thrust/N
0	0.0	93133	1000	0.0	85762	2000	0.0	77196
0	0.1	88303	1000	0.1	80556	2000	0.1	72793
0	0.2	84870	1000	0.2	77959	2000	0.2	71102
0	0.3	82980	1000	0.3	76244	2000	0.3	69514
0	0.4	82005	1000	0.4	75705	2000	0.4	69423

6   Conclusion

This paper shows the detail process of establishing aircraft takeoff taxiing model by numerical method. First of all, we summarize the limitations and the low-precision problems of the existing modeling methods. Then we introduce the principle of

Lagrange interpolation algorithm, as well as the steps of solving the 2D interpolation problems with Lagrange three-point interpolation algorithm. After that, we expound the method of determining the parameters in distance calculation formula and the process of establishing aircraft takeoff model. Finally, we prove the correctness and accuracy of the takeoff taxiing model, by comparing the model output data with the actual flight data of a specific aircraft.

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