Chapter 4
Lesson Study: The Fundamental Driver for Mathematics Teacher Development in Japan

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Abstract How do Japanese teachers develop knowledge and expertise for teaching mathematics effectively? Their journey begins while in university attending various teacher preparation programs. This undertaking does not end once they become teachers. They are expected to be lifelong learners to become effective educators. Lesson Study has been the fundamental driver of improvement in teaching and learning in Japan. This chapter describes how Lesson Study supports teachers in their continuous growth to become effective teachers of mathematics and provides empirical evidence based on current research projects conducted by the author as a part of Project IMPULS at Tokyo Gakugei University.

Keywords Lesson study · School-wide · Research steering committee · Knowledgeable other · Lifelong learning

4.1 Introduction

It is obvious that teachers cannot teach content beyond their knowledge (National Mathematics Advisory Panel 2008), but knowledge of content is not enough to teach effectively. Japanese mathematics educators and teachers identify three levels of expertise of mathematics teaching (Sugiyama 2008):

Level 1: The teacher can tell students the important basic ideas of mathematics such as facts, concepts, and procedures

Level 2: The teacher can explain the meanings and reasons of the important basic ideas of mathematics in order for students to understand them

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Level 3: The teacher can provide students with opportunities to understand these basic ideas, and support their learning so that the students become independent learners

(Trans. Takahashi 2011)

Level 1 teaching does not require any special knowledge. In fact, having received decent grades in mathematics in grade school may be all that is necessary. But for Level 2 and Level 3 teaching, special knowledge and expertise are required. Sugiyama (2008) writes that during the early twentieth century, which is early in the evolution of the Japanese public education system, most elementary school teaching was at Level 1. Instructors simply told their students the facts and expected them to memorize those facts through practice, and contemporary textbooks were designed to support this form of instruction. Certainly it is important for teachers to be able to tell students basic facts, but today in Japan a teacher must provide instruction at Level 2 or 3 to be considered a professional.

To teach at Level 2, one must possess knowledge of mathematics beyond what is needed in everyday life or what is required to solve problems in school textbooks. For example, knowing the "invert and multiply" rule for division of fractions is enough to be a Level 1 teacher, but for Level 2 a teacher must be able to explain how multiplying by the reciprocal of a fraction produces the quotient. This type of knowledge is important for helping students understand mathematics (e.g., Ball et al. 2008). But while Level 2 is considered professional teaching, Japanese mathematics educators believe that all mathematics teaching should be at Level 3, because they have seen that Level 2 teaching does not enable students to develop mathematical proficiency with understanding.

A majority of current government-authorized mathematics textbooks in Japan are designed to support Level 3 teaching. These textbooks are designed for teachers to present students with a problem that the students have not yet learned how to solve. The texts provide structure and allow the teachers to guide the conversation in such a way that students can arrive at a new understanding as a result of their own efforts in solving the new problem. The philosophy behind Level 3 teaching is that students should be given a reasonable amount of independent work, such as problem-solving, in order to develop the knowledge, the understanding, and the skills of mathematics (National Research Council 1989; Polya 1945).

Japanese mathematics educators can safely assume that most university students have knowledge of mathematics for Level 1 teaching. Their concern, therefore, is to move those students toward Level 2. But there is not enough time in the preservice program to equip the future teachers with Level 2 knowledge of all the contents they might be required to teach. So, Japanese universities focus on training students to acquire Level 2 knowledge through a careful study of teaching materials (Sugiyama 2008). They offer courses for elementary mathematics teacher preparation that focus mainly on examining the contents of mathematics for elementary grades and developing a deeper understanding of these contents. This process is

called *kyouzai kenkyuu*—in other words, "studying teaching materials for establishing deeper understanding for better teaching" (Watanabe et al. 2008).

For example, there are several formulas for finding the area of basic geometric figures. Most students who come to a teacher preparation program already know those formulas and can use them to find the area of basic figures. Using contents from published textbooks for elementary grades, university courses help the prospective teacher see how the formulas are developed, how they are related to each other, how they are related to other areas in mathematics, and potential difficulties students might have with learning the formulas. Investigating a topic in this way is typical of *kyouzai kenkyuu* and is an essential part of teachers' preparation for everyday teaching; hence these courses also introduce the prospective teachers to *kyouzai kenkyuu* as a critical step in preparing lessons.

Preparing student teachers for Level 3 teaching is even further beyond the scope of what can be accomplished during the teacher preparation programs at the university. Japanese educators believe that teachers cannot master Level 3 teaching simply by listening to lectures, reading textbooks, and watching videos. Learning to teach at Level 3 is demanding and time-consuming, and a career-long process. But the universities do help prospective teachers understand what Level 3 teaching is, and teaches them the pathway to it.

4.2 Helping Practicing Teachers Increase Their Knowledge and Expertise

When designing professional development programs for practicing teachers, it is useful to recognize that professional development falls into two categories; phase 1 and phase 2. Phase 1 professional development (phase 1 PD) focuses on increasing a teacher's knowledge for teaching mathematics, while phase 2 professional development (phase 2 PD) focuses on developing expertise for teaching mathematics—that is, the ability to use new knowledge in the classroom (Takahashi 2011).

Moving from Level 1 to Level 2 can be achieved through phase 1 PD, and most university courses in teacher preparation programs, which may include reading books, listening to lectures, and observing well-designed mathematics classes, fall into the category of phase 1. Practicing teachers may need phase 1 PD from time to time to update their knowledge for teaching. On the other hand, Level 3 teaching requires very different classroom practices and skills than Level 2 teaching, and learning these practices and skills requires phase 2 PD. To develop this expertise requires considerable teaching experience with a reflection component. Japanese teachers and researchers use Lesson Study to develop the deeper knowledge and the expertise necessary to make Level 3 teaching available for their students.

4.3 Lesson Study as a Fundamental Driver for Mathematics Teacher Development

Lesson Study has been the primary mechanism of professional development for both prospective teachers and practicing teachers since the Japanese public education system started (Lewis 2000; Lewis and Tsuchida 1998a; Makinae 2010; Murata and Takahashi 2002; Takahashi 2000; Takahashi and Yoshida 2004; Yoshida 1999a). In Lesson Study, teachers conduct intensive *kyouzai kenkyuu*—study the standards, read relevant research articles, examine available curricula, and other materials—and work together to design a lesson focused on a problematic topic while also addressing a broader research theme related to teaching and learning. The lesson they design, known as a "research lesson" (*kenkyuu jugyou*), is taught by one teacher from the planning team while the other team members observe. The planning team and observers then conduct a post-lesson discussion focusing on how students responded to the lesson in order to gain insights into the teaching–learning process. Figure 4.1 illustrates the typical process of Lesson Study.

4.3.1 Introduction to Lesson Study During Teacher Preparation

Japanese teachers acquire first-hand experience of Lesson Study during their student teaching. Each student teacher has the opportunity to carefully observe lessons taught by the cooperating teacher and by other student teachers. Based on the observation of the lessons, student teachers write a detailed lesson plan and teach the lesson based on that lesson plan. After each lesson is taught, the cooperating teacher, the student teacher who taught the lesson, and other student teachers who observed the lesson have a mini-version of a post-lesson discussion. This is based

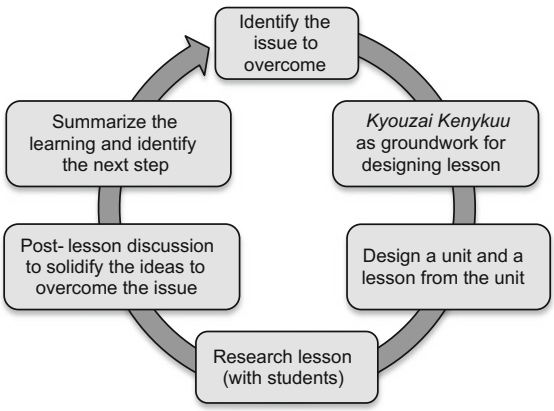

Fig. 4.1 Lesson study cycle to impact student learning

on the careful observation of the students during the lesson. At the end of the student teaching, the school conducts a formal research lesson for the student teachers by having other teachers at the school observe them teach; this research lesson is an initiation of the student teachers into the teaching profession. Through this experience of practicing Lesson Study during student teaching, each prospective teacher learns the basics of Lesson Study. For example, how to observe students during lessons, how to prepare a lesson plan for a research lesson, how to be part of the post-lesson discussion, and how to write a summary report of a Lesson Study cycle.

4.4 An Example of School-Wide Lesson Study to Support Teachers Becoming Life-Long Learners

During Lesson Study, teachers have the opportunity to look closely at teaching practices and to judge, based on student learning, whether the lesson properly supports the students in learning mathematics. Researchers credit Japanese Lesson Study with enabling the implementation of new teaching approaches (Lewis 2002; Lewis and Tsuchida 1998b; Stigler and Hiebert 1999; Yoshida 1999b).

Although Lesson Study is commonly used by teachers and schools to improve teaching and learning in general, Lesson Study is also used to seek practical ideas for the effective implementation of the Course of Study (COS), the national curriculum (Murata and Takahashi 2002). This is a very common focus of school Lesson Study work during the transition period from one COS to a new COS.

During this transition stage, Japanese schools, especially public schools, typically conduct school-wide Lesson Study events for all the teachers at the school to work collaboratively to address the new curriculum implementation. The following case study shows how this process worked at one public elementary school (Takahashi 2014b).

4.4.1 The School Research Organization and the Research Steering Committee

The school the author examined in this section (Takahashi 2014b) is a public elementary school in Tokyo with about 760 students in grades 1 through 6, and 64 teachers and staff. Immediately after the Japanese Ministry of Education released a revision of COS, the teachers at the school decided to focus their Lesson Study work over the next two years on developing students' ability to express their ideas and learn from each other, which was a new point of emphasis in the revised COS.

During the two years of the school-wide Lesson Study, all full-time teachers at the school worked within a structure based on existing grade-level groups (see Fig. 4.2).

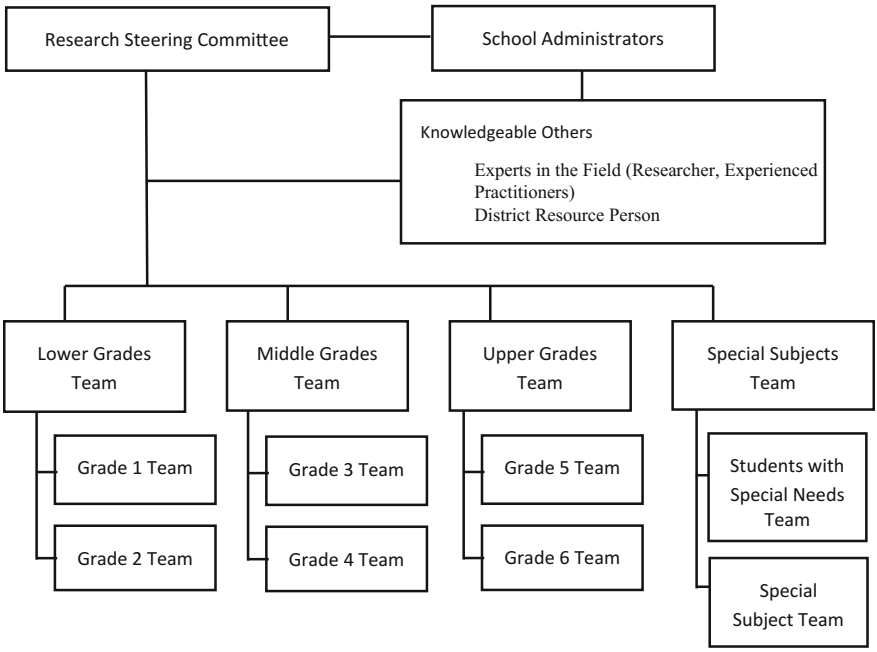

Fig. 4.2 Structure of the school research organization (reprinted from Takahashi 2014b with permission of Springer)

Grade-level groups typically exist in Japanese elementary schools to facilitate the sharing of responsibilities for running school events and for academic activities. Most public schools have time for grade-level meetings in their weekly schedule, typically about one hour. Teachers have desks in a common work area so that they can collaborate on a regular basis. In order to conduct the school-wide lesson effectively, each grade-level group was made responsible for crafting a plan for a research lesson, conducting their research lesson in front of the rest of the faculty, serving as panelists during the post-lesson discussion, and supporting the other teams' research lessons. The school also had grade-band teams, which consisted of all the teachers from adjacent grades, such as grade 1 and grade 2. Although the responsibility for lesson planning belonged to each grade group, most of the lesson planning was done in grade-band meetings in order to maintain consistencies across the grades and to help the teachers develop a shared view of the scope and sequence of the curriculum in adjacent grades. Finally, the grade-band meetings provided more opportunities for each teacher to participate in research lesson planning, a valuable experience especially for novice teachers not only to learn how to design lessons but also to deepen their understanding of the topics they teach.

Following common practice, the school organized a research steering committee, which consisted of representatives of each grade level and the lead teacher for

mathematics,[1] who was appointed chairperson of the committee by the principal on the basis of his leadership ability and knowledge of mathematics teaching and learning. The committee led the school's efforts and maintained the cohesiveness of ideas across the grades. Among other things, the research steering committee was responsible for the following:

- Develop a master plan for the school research.
- Schedule and lead monthly meetings to find strategies to address the school's research theme based on the ideas of the teachers.
- Publish a monthly internal newsletter to record the findings from each research lesson.
- Plan, edit, and publish the school research reports, including those for the research open house.
- Arrange for knowledgeable others to present lectures, teach demonstration lessons, and give final comments at research lessons.

The first task of the research steering committee was to propose a focus for the school's research. That proposal was discussed by the entire faculty at the first faculty meeting of the school year. The following was the approved research theme and focus of study:

Research theme: The development of individual thinking and the expression of these thoughts.

Focus of study: Seeking effective ways to support students' individual problem solving skills and better facilitation of whole-class discussion in teaching through problem solving.

The research theme articulated a goal for students while the focus of study expressed the faculty's idea about a path toward accomplishing the goal.

Each grade-level team developed a lesson plan for a research lesson and conducted the research lesson and post-lesson discussion to address the theme. Most of the research lessons were scheduled on one of the half-day professional development days in order for all full-time teachers to be able to observe the lessons and participate in the discussions. As a result, each full-time teacher had the opportunity to be a part of eight research lessons during one school year. The school also invited two distinguished mathematics educators to give lectures, one professional development day in the first month of the school year (April) and another during the summer break, about the issues and trends in mathematics education and ideas for implementing the new COS.

Throughout the two years of the project, the research steering committee met between the research lessons to summarize the ideas that had been proposed by each lesson planning team and addressed during the post-lesson discussion. They published their summaries as a school research newsletter each month. These

[1]The lead teacher has his or her own self-contained class but also has responsibility for providing support for the upper grade teachers and for preparing curriculum materials for the school.

newsletters documented the process of this long-term collaborative effort, and, more important, they allowed the teachers to share what was discussed and helped other teams build off the results of previous research lessons.

4.4.2 Lesson Plans and Their Development

In each stage of lesson plan development, members of the research steering committee reviewed the lesson plan and provided feedback to the team. Through this process, they tried to ensure that all the lesson plans developed by the school were of sufficient quality to contribute to the school's effective implementation of the new COS. In order to do so, the committee distributed to each teacher the following list of questions to guide them toward higher quality lesson plans:

- Does the lesson plan provide sufficient information for the teacher to understand the task and the flow of the lesson?
- Does the lesson plan provide sufficient information about how the planning team decided to teach the lesson as described by the plan?
- Do the objectives of the lesson plan clearly address the Course of Study?
- Are the tasks appropriate for the students given the date of the lesson?
- Are the key questions clear? Will they encourage students to think mathematically and help them complete the task independently?
- Does the lesson plan include reasonable anticipated student responses and indicate how the teacher will help students overcome any misunderstandings?
- Does the lesson plan include a plan for formative assessment and a plan to accommodate individual student differences during the lesson?

4.4.3 Disseminating the Results of the School Research

Toward the end of Year 2, the school faculty and staff hosted a half-day public open house to share their findings. All content specialists of the district and principals of other area schools were invited to the open house, and many other schools sent their teachers. In all, a total of 612 participants, including teachers, administrators, educators, and parents attended this event.

The public open house consisted of three major parts: public research lessons, research presentations by the school's research steering committee, and a panel discussion by experts in the field of mathematics education who had been involved with the school's research project. There were 28 mathematics lessons conducted simultaneously based on 25 different lesson plans available for the participants to observe at the beginning of the open house. All 25 lesson plans were in a booklet given to each participant on arrival at the school. The participants were able to witness strategies for the effective implementation of the COS in live lessons and

were able to bring these ideas back to their own school as a set of lesson plans. The presentation given by the members of the steering committee informed the participants about the philosophy and the rationale behind the strategies being used at the school. The presentation also provided educators from other schools an opportunity to learn how the school conducted its research using Lesson Study and what the faculty at the school had learned by going through this process.

Two sets of research reports were also made available for teachers and administrators of other schools as summaries of the school research effort of Year 1 and of Year 2. Since the school used a district grant to produce them, all the research reports were made available for free. In the second year, the school compiled a report covering the entire two-year study. The report was produced as four booklets: three of them were distributed at the public open house and the last was sent to all the schools in the district at the end of the school year. An English translation of one of these booklets is available at http://www.impuls-tgu.org/en/resource/readings/page-26.html.

4.4.4 The Results from the School-Wide Lesson Study

The Japanese national standards released in 2008 contained a new emphasis on having students learn to express their ideas and learn from each other, as a way to help students develop their own thinking. The teachers at this school chose to spend two years working through Lesson Study to research changes in practice that would address this new emphasis. Some of what they learned—and what they put into practice—is evident in the booklet they published for the open house. Here are a few points from the booklet

- Students were able to express their ideas using not only words but also mathematical expressions and diagrams. Because of the cohesive use of diagrams, such as tape diagrams, area diagrams, number line diagrams, and of expressions and equations throughout the grades, whole-class discussions became deeper and productive. Moreover, students were able to express their ideas in similar ways regardless of who was teaching the lessons.
- By crystallizing what was expected of students in each stage of problem solving (e.g., understanding the problem, solving the problem, reflecting upon the solution) and at the major points of teacher instruction, students were able to learn independently.
- By preparing effective key questions for each stage of problem solving, students were able to express their ideas in various ways and to talk to each other clearly by focusing on what should be discussed.
- By planning blackboard writing, the flow of the lessons became more coherent. Students became able to look back at what they learned by looking at the board. Then they could use it to put the various ideas together in integrated and expanded ways, and to evaluate their learning during the lessons by themselves.

Each teacher was deeply involved in planning only one research lesson per year, which may not seem like enough to support such profound growth. But the school's work over the two years was carefully organized to support teacher learning in various ways. Each teacher at the school had at least two opportunities to critique lesson plans from another team during the planning process through the grade-band meetings. Teachers observed and discussed the lessons of all the other grades at the school. And the newsletters published by the research steering committee helped each successive team build on what was learned before.

4.4.5 Supporting the School-Wide Lesson Study

When implementing new ideas of teaching and learning, teachers must figure out what the necessary changes will look like in their own classrooms and with their own students. To do so, teachers need to conduct their own research, and Lesson Study provides an organized way to do so. Since Lesson Study is tied directly to teachers' practice, teachers can minimize the gap between research and practice. Outside of Japan, many Lesson Study projects have been conducted by a few volunteers within a school or across school districts. Individual teachers can certainly improve their own teaching by participating in such volunteer groups. But in Japan, as this case study illustrates, improving teaching is a responsibility of all teachers at a school, to be worked on together.

Although teachers work hard to improve teaching and learning by collaborating with their colleagues through Lesson Study, they can be limited by what they do not know. In order to maximize the effect of the collaboration, Japanese school administrators usually provide additional supports for expanding teachers' knowledge. These include a structure to support collaboration (grade-level teams and grade-band teams) and distributing leadership in the form of a research steering committee that comprises teachers from multiple grades, and access to new knowledge and expertise via an outside expert.

Researchers have noted the importance of outside expertise provided by the so-called "knowledgeable other" in making Lesson Study effective (Lewis et al. 2006; Lewis and Tsuchida 1998b; Takahashi and Yoshida 2004; Yoshida 1999b). The following section describes the roles of the knowledgeable other in school-wide Lesson Study.

4.4.6 The Role of the Knowledgeable Other

It is common practice among Japanese schools to bring in an outside expert who is knowledgeable about the school research theme. This person is referred to as a "knowledgeable other." Based on a study conducted by the Takahashi (2014a), the knowledgeable other is responsible for

(1) bringing new knowledge from research and the standards,
(2) showing the connection between the theory and the practice, and
(3) helping others learn how to reflect on teaching and learning.

Each of the responsibilities is elaborated below.

4.5 Bringing New Knowledge from Research and the Standards

As in many other countries, most Japanese elementary school classroom teachers have to teach all subjects. In order to update their knowledge regarding mathematics teaching and learning, teachers need support from people who have access to the latest research and the standards. One of the important responsibilities of a knowledgeable other is to help classroom teachers deepen their understanding of the content, the curriculum, ideas behind the textbooks, and pedagogical ideas.

When teachers engage in lesson study, they are expected to deepen their knowledge of mathematics teaching and learning by reading teacher resources such as teaching guides, recent journal articles, and curriculum materials, as well as carefully studying the textbooks that the school uses. But Japanese educators often emphasize that simply reading about what research and the standards say is not enough.

Like students, teachers best learn new ideas with concrete examples. A research lesson serves as a rich source of concrete examples from which teachers can learn, if given proper guidance. Thus, Japanese schools customarily invite a knowledgeable other to their research lesson and ask that person to provide "final comments," lasting 10, 30 min, or more at the end of the post-lesson discussion.

In the final comments, knowledgeable others typically begin their comments by providing new knowledge pertaining to the teaching and learning of the topic of the lesson, drawing from the COS and from the textbooks. They then examine key ideas in the lesson plan. Finally, they reflect on the actual events of the lesson, bringing up specific evidence of what students had learned from the lesson, and make suggestions for future consideration.

In order to do so, the knowledgeable others may prepare in advance some handouts for the teachers based on the draft lesson plan they receive a week before. These handouts mainly elaborate on that part of the COS related the topic of the lesson so that the teachers other than the lesson planning team can understand the fundamental ideas involved. Although a similar curriculum investigation is often done by the lesson planning team as a part of their lesson planning research, the handout aims to go beyond the investigation by the team. This is the first place where a depth of understanding of the contents and the curriculum is required of the knowledgeable other.

Another important responsibility of the knowledgeable others is to elaborate on the ideas behind the textbook pages. Japanese textbooks are thin but contain rich

content with a focused and coherent organization (Watanabe et al. 2010). The underlying rationale for textbook content is sometimes subtle, and it can be difficult for teachers to see the connections between the problems on different pages, in different units, and from different grades. The knowledgeable other can help teachers see those connections.

All of the above aims to deepen the teachers' knowledge and understanding of the contents of the research lesson.

4.5.1 Showing the Connection Between Theory and Practice

Japanese teachers understand that they are responsible for implementing the COS, using theory and research findings to improve students' attainment of the curriculum. Although there are plenty of materials available for teachers to help them implement the curriculum, reading these resources or listening to experts' lectures are usually not enough to develop the expertise to use them in Level 3 teaching. In Lesson Study, teachers have the opportunity to plan lessons based on knowledge acquired by reading or listening, to teach the lesson based on a carefully designed lesson plan, and to reflect on the teaching and learning using evidence from the lesson. Through this process, teachers can try out new ideas or practices and evaluate the effectiveness of their lesson plan in the post-lesson discussion.

School-wide lesson study almost always focuses on a research theme selected by the faculty. Therefore, to help the school translate theory into practice, another important role of the knowledgeable other is to connect what they observed during the research lesson to the school's research theme. The knowledgeable other should try to highlight concrete evidence from the lesson that is relevant for assessing the progress of the school toward its research theme.

Knowledgeable others may also suggest possible directions the school should take in order to pursue the research theme, and may also offer professional viewpoints and opinions about the school research and the research lessons.

4.5.1.1 Helping Others Learn How to Reflect on Teaching and Learning

Another role of the knowledgeable other is to help the school conduct effective post-lesson discussions. Thus, he or she should give the teachers the opportunity to reflect upon important lessons learned from the discussion and on what else they could learn if the discussion were improved. In order to do so, the knowledgeable other should be able to not only summarize the discussion, but also to effectively contribute to the discussion by raising important issues that were not addressed during the post-lesson discussion.

Sharing what the knowledgeable other observed during the lesson helps the teachers see what can they learn if they have good "eyes for observing students."

In fact, knowledgeable others often say that observing lessons with experienced Lesson Study practitioners was the best way to develop good eyes for observing students.

4.6 Recommendations

Stigler and Hiebert (1999) argue that Japanese mathematics lessons better exemplify recent reform ideas than do US lessons. One of the reasons Japanese teachers are able to use reform ideas effectively in their classroom is their participation in Lesson Study. Lesson Study provides the opportunity for classroom teachers to work collaboratively to seek effective implementation of new ideas, rather than struggle in isolation to understand how the ideas look in his/her own classroom.

Since the early research on Lesson Study published late 1980s (e.g., Lewis and Tsuchida 1998a; Stigler and Hiebert 1999; Yoshida 1999b), researchers, educators, and teachers around the world have attempted to replicate its success at transforming Level 1 and 2 teaching to Level 3 teaching focused on problem solving (e.g., Hart et al. 2011). Although many schools and teachers have tried to use ideas from Lesson Study in various ways, only a few cases have been documented in which there was strong evidence of impact on teaching and learning (e.g., Gersten et al. 2014; Lewis et al. 2006).

One of the reasons behind is that the "Lesson Study" outside Japan based on early research documents may have left some important aspects of Lesson Study in Japan. For example Fujii (2014) argues that many activities described as "Lesson Study" are often very different from Lesson Study in Japan.

In order to overcome such dilemma and seek vital impacts on student and teacher learning, several new projects with careful examination of critical aspects of Lesson Study in Japan to design a comprehensive program to support teachers and schools have been conducted in the US, UK, and some other counties. Although full reports of these projects may not be published, some important aspects of Lesson Study may be revealed (e.g., Takahashi and McDougal 2016).

For educators who try to improve mathematics teaching and learning, it is important to understand why Lesson Study has been less consistently impactful outside of Japan and design a program carefully so that the teachers can receive appropriate support in order to experience an authentic Lesson Study process.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389–407. doi:10.1177/0022487108324554

Fujii, T. (2014). Implementing japanese lesson study in foreign countries: Misconceptions revealed. *Mathematics Teacher Education and Development, 16*(1), 65–83.

Gersten, R., Taylor, M. J., Keys, T. D., Rolfhus, E., & Newman-Gonchar, R. (2014). *Summary of research on the effectiveness of math professional development approaches. (REL 2014–010).* Retrieved from Washington, DC: http://ies.ed.gov/ncee/edlabs

Hart, L. C., Alston, A., & Murata, A. (Eds.). (2011). *Lesson study research and practice in mathematics education*. Now York: Springer.

Lewis, C. (2000, April 2000). *Lesson study: The core of Japanese professional development.* Paper presented at the AERA annual meeting.

Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Philadelphia: Research for Better Schools, Inc.

Lewis, C., & Tsuchida, I. (1998a). A lesson is like a swiftly flowing river: How research lessons improve Japanese education. *American Educator, 22*(4), 12–17, 50–52.

Lewis, C., & Tsuchida, I. (1998b). A lesson like a swiftly flowing river: Research lessons and the improvement of Japanese education. *American Educator, 22*(4).

Lewis, C., Perry, R., Hurd, J., & O'Connell, M. P. (2006). Lesson study comes of age in North America. *Phi Delta Kappan, 88*(04), 273–281.

Makinae, N. (2010). *The origin of lesson study in Japan.* Paper presented at the The 5th East Asia Regional Conference on Mathematics Education: In Search of Excellence in Mathematics Education, Tokyo.

Murata, A., & Takahashi, A. (2002). Vehicle to connect theory, research, and practice: How teacher thinking changes in district-level lesson study in Japan. In *Proceedings of the twenty-fourth annual meeting of North American chapter of the international group of the Psychology of Mathematics Education* (pp. 1879–1888).

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Retrieved from Washington, DC.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, D. C.: National Academy Press.

Polya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton, New Jersey: Princeton University Press.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Sugiyama, Y. (2008). *Introduction to elementary mathematics education*. Tokyo: Toyokan publishing Co.

Takahashi, A. (2000). Current trends and issues in lesson study in Japan and the United States. *Journal of Japan Society of Mathematical Education, 82*(12), 15–21.

Takahashi, A. (2011). The Japanese approach to developing expertise in using the textbook to teach mathematics rather than teaching the textbook. In Y. Li & G. Kaiser (Eds.), *Expertise in mathematics instruction: An international perspective* (pp. 197–219). New York: Springer.

Takahashi, A. (2014a). The role of the knowledgeable other in lesson study: Examining the final comments of experienced lesson study practitioners. *Mathematics Teacher Education and Development, 16*(1), 4–21.

Takahashi, A. (2014b). Supporting the effective implementation of a new mathematics curriculum: A case study of school-based lesson study at a Japanese public elementary school. In I. Y. Li & G. Lappan (Eds.), *Mathematics curriculum in school education* (pp. 417–441). New York: Springer.

Takahashi, A., & McDougal, T. (2016). Collaborative lesson research: maximizing the impact of lesson study. *ZDM*, 1–14. doi:10.1007/s11858-015-0752-x

Takahashi, A., & Yoshida, M. (2004). How can we start lesson study? Ideas for establishing lesson study communities. *Teaching Children Mathematics, 10*(9), 436–443.

Watanabe, T., Takahashi, A., & Yoshida, M. (2008). Kyozaikenkyu: A critical step for conducting effective lesson study and beyond. In F. Arbaugh & P. M. Taylor (Eds.), *Inquiry into Mathematics Teacher Education, Association of Mathematics Teacher Educators (AMTE) Monograph Series* (Vol. 5, pp. 131–142).

Watanabe, T., Takahashi, A., & Yoshida, M. (2010). Supporting focused and cohesive curricula through visual representations: An example from Japanese textbooks. In B. Reys, R. Reys, & R. Rubenstein (Eds.), *2010 Yearbook: Contemporary issues in mathematics curriculum* (pp. 131–143). Reston, VA: National Council of Teachers of Mathematics.

Yoshida, M. (1999a, April). *Lesson Study [jugyokenkyu] in elementary school mathematics in Japan: A case study.* Paper presented at the American Educational Research Association Annual Meeting, Montreal, Canada.

Yoshida, M. (1999b). *Lesson study: A case study of a Japanese approach to improving instruction through school-based teacher development.* (Dissertation), University of Chicago, Chicago.