Chapter 13
Teaching for Metacognition Project: Construction of Knowledge by Mathematics Teachers Working and Learning Collaboratively in Multitier Communities of Practice

Berinderjeet Kaur, Divya Bhardwaj and Lai Fong Wong

Abstract Teaching for metacognition project affirms a gradual shift in the centre of gravity away from the University-based, "supply side", "offline" forms of knowledge production conducted by university scholars for teachers towards an emergent school-based, demand-side, online, in situ forms of knowledge production conducted by teachers with support from fellow teachers, lead and senior teachers, and other experts such as university scholars and curriculum specialists. The project facilitates the participation of mathematics teachers in two-tier communities of practice. In this chapter, we describe the design of the project and the learning of two teams of teachers from two schools participating in the project. It is apparent from the findings that the teachers worked and learned collaboratively whilst participating in a first-tier and a second-tier community of practice. Their participation in the communities of practice enabled them to develop a deeper understanding of metacognition and also teaching for metacognition.

13.1 Introduction

Since 1998, in support of the Thinking Schools, Learning Nation vision (Goh 1997) of the Ministry of Education (MOE) in Singapore, all teachers are entitled to 100 h

B. Kaur (✉) · D. Bhardwaj
National Institute of Education, Nanyang Technological University, Singapore, Singapore
e-mail: berinderjeet.kaur@nie.edu.sg

D. Bhardwaj
e-mail: divya.bhardwaj@nie.edu.sg

L.F. Wong
Anderson Secondary School, Singapore, Singapore
e-mail: wong_lai_fong@moe.edu.sg

B. Kaur et al. (eds.), *Professional Development of Mathematics Teachers*, Mathematics Education – An Asian Perspective,
DOI 10.1007/978-981-10-2598-3_13

of training and core-upgrading courses each year. This entitlement is for teachers to keep abreast of current knowledge and skills. The professional development (PD) is funded by the MOE. There are many ways through which in-service mathematics teachers continue to develop themselves in Singapore. One of the ways is through participation in a research project facilitated by professors at the National Institute of Education, the sole institute for teacher education in Singapore. Two such past projects were Enhancing the Pedagogy of Mathematics Teachers (Teaching for Reasoning and Communication) (EPMT-RC) project (Kaur 2009, 2011) and the Think-Things-Through (T^3) project (Yeap and Ho 2009). In Singapore, these projects initiated the shift of PD activities from the "training model of PD" (Matos et al. 2009, p. 167) to the "hybrid model of PD" (Kaur 2011, p. 791).

In the training model of PD, teachers attend courses conducted by specialist officers from the mathematics Curriculum Planning and Development Division of the MOE or academics from the National Institute of Education. These courses are conducted for about 3 h per day spanning 4 to 10 consecutive days or days spread over some weeks. Almost always following the completion of such a course, there is no follow-up with the teachers about the use of the knowledge acquired and any impact that knowledge may have had on student achievement.

Research has shown that such courses are ineffective as teachers are likely to reject knowledge and skill requirements when (i) the requirements are imposed or encountered in the context of multiple, contradictory and overwhelming innovations; (ii) they are excluded from the development of the courses; (iii) PD is packaged in off-site courses or one-off workshops that are alien to the purposes and contexts of their work; or (iv) they experience them alone and are afraid of being criticized by colleagues or of being seen as elevating themselves on pedestals above them (Hargreaves 1995). Smylie (1989), in his survey of teachers' ratings of opportunities to learn in the US found that district-sponsored in-service workshops were at the bottom of the heap, ranked last out of 14 possibilities in terms of what teachers considered most valuable. Although such workshops are often accompanied by evaluations, seeking feedback on the duration, satisfaction, etc., efforts to measure what teachers learned have not been part of typical evaluation fare. In the same survey, Smylie found that teachers ranked direct classroom experience as their most important site for learning. Furthermore, for some teachers PD may not be an autonomous activity, i.e. chosen by a teacher in search of better ways of knowing and teaching mathematics (Castle and Aichele 1994).

The hybrid model of PD (Kaur 2011) integrates the "training model of PD" (Matos et al. 2009) with sustained support for teachers to integrate knowledge gained from the PD into their classroom practice. It exemplifies a critical development in the professional development of teachers in many parts of the world. This development reflects a gradual shift in the centre of gravity away from the University-based, "supply side", "offline" forms of knowledge production conducted by university researchers for teachers towards an emergent school-based, demand-side, online, in situ forms of knowledge production by teachers with support from university scholars.

In Singapore, the school mathematics curriculum is reviewed every 6–10 years. The last review was carried out in 2012. Since 1990, the framework for the school mathematics curriculum has been consistent. The primary goal of the school mathematics curriculum is mathematical problem-solving and the five interrelated components the framework places emphasis on; the learning of concepts, development of skills, attitudes, metacognition and mathematical processes such as thinking skills, heuristics, applications and modelling and reasoning, communication and connections (Ministry of Education 2012). In addition, the Ministry of Education (2010) identified competencies that have become increasingly important in the twenty-first century. These competencies include a confident person, a self-directed learner, an active contributor and a concerned citizen. In the US, the P21 Framework for twenty-first century learning has stressed that no twenty-first century skills implementation can be successful without developing core academic subject knowledge of mathematics and understanding among all students (Partnership for 21st Century Skills 2009).

Following the review of the curriculum in 2012, a group of teachers, university scholars and curriculum specialists examined the outcomes of three significant studies related to student achievement in mathematics. The studies are

(i) Programme for International Student Assessment (PISA) of 2009 (OECD 2010) and 2012 (OECD 2013);
(ii) Trends in International Mathematics and Science Study (TIMSS) of 2011 (Mullis et al. 2012; Kaur et al. 2013), and 2007 (Mullis et al. 2008; Kaur et al. 2012);
(iii) CORE 2 research at the National Institute of Education (NIE) by Hogan et al. (2013).

The findings of PISA and TIMSS showed that majority of Singapore students are very good in applying their knowledge in routine situations and this is definitely a consequence of what teachers do and use during their mathematics lessons. Hogan et al. (2013) found that there was a dominant use of performative tasks compared to knowledge-building tasks in grades 5 and 9 mathematics lessons that they studied. A performative task mainly entails the use of lower order thinking skills such as recall, comprehension and application of knowledge while a knowledge-building task calls for higher order thinking skills such as synthesis, evaluation and creation of knowledge. From the findings of the three studies, the group inferred that for students in Singapore to scale greater heights, teachers need to nurture metacognitive learners who are active and confident in constructing mathematical knowledge. Thus, a PD project—Teaching for Metacognition—was conceptualized as the greatest source of variance in the learning equation comes from teachers (Hattie 2009).

The project places emphasis on two key elements, knowledge-building tasks and teaching for metacognition, when used in tandem, create classroom discourse that facilitate students' active engagement in critical thinking, problem-solving, working collaboratively and articulating their thoughts and creating knowledge through their explorations. The goal of this chapter is to describe the Teaching for Metacognition project and examine the learning of two teams of teachers, from two schools participating in the project, in the two-tier communities of practice.

13.2 Teaching for Metacognition Project

Teaching for Metacognition project, is a hybrid model of PD (Kaur 2011). It is funded by the Academies Fund of the Ministry of Education and led by professors from the National Institute of Education, curriculum specialists from the Curriculum Planning and Development Division (Mathematics) of the Ministry of Education, a master teacher from the Academy of Singapore Teachers and a lead teacher from a secondary school. The aims of the project are threefold. The first is to provide teachers with knowledge and know-how of crafting knowledge-building tasks and how teachers may engage their students in metacognition during the learning of mathematics, i.e. metacognitive strategies. The second is to facilitate teaching for metacognition in the classrooms of teachers in the project. The third is to enthuse and support teachers to contribute towards the development of fellow mathematics teachers in Singapore and elsewhere.

13.2.1 Review of Literature

The conceptual framework of the project draws on research findings, specifically the characteristics of effective PD activities and teacher communities of practice. Relevant literature is reviewed in the following sections.

13.2.1.1 Successful PD Activities

High-quality and effective professional development programmes have been found to have a purpose as teachers are involved in shaping the foci of the programme so that it is related to their school work (Clarke 1994; Hawley and Valli 1999; Elmore 2002). These PD programmes are part of coherent programmes of teacher learning and development that support their instructional activities at school, such as adoption of new standards (Stiff 2002; Desimone 2009) and focus on how to teach and what to teach—the substance and the subject matter are keys (Stiff 2002; Desimone 2009). Lipowsky and Rzejak cited in Maaβ and Artigue (2013) noted that teachers viewed professional development initiatives as effective if they had clear relevance to their day-to-day teaching and the programmes had a clear focus on specific aspects of teaching or facilitation of student learning. Ball and Cohen (1999) have argued that "teachers' everyday work could become a source of constructive PD" (p. 6) through the development of a curriculum for professional learning that is grounded in the tasks, questions and problems of practice.

Such programmes include training, practice and feedback, and follow-up activities (Abdal-Haqq 1995). Ball (1996) claimed that the "most effective professional development model is thought to involve follow-up activities, usually in

the form of long-term support, coaching in teachers' classrooms, or ongoing interactions with colleagues" (pp. 501–502). Effective PD programmes are sustained (Clarke 1994; Abdal-Haqq 1995; Hawley and Valli 1999; Elmore 2002; Stiff 2002; Borasi and Fonzi 2002; Desimone 2009) and embedded in teacher work (Clarke 1994; Abdal-Haqq 1995; Hawley and Valli 1999; Carpenter et al. 1999; Elmore 2002). Lipowsky and Rzejak noted that effective programmes are also intensive, combining learning-off-job in courses with learning-on-job in school. Teachers learn best when observing, being observed, planning for classroom implementation, reviewing student work, and presenting, leading and writing (Stiff 2002). Therefore, opportunities for teachers to engage in active learning are certainly related to effectiveness of PD (Wilson and Berne 1999; Desimone 2009). They also value the exchange of experiences with colleagues (Lipowsky and Rezak). In addition, collective participation by teachers from the same school, grade or department allow for a powerful form of teacher learning through prolonged interaction and discourse (Wilson and Berne 1999; Desimone 2009; Stiff 2002). PD programmes that foster collaboration have been found to be effective (Clarke 1994; Abdal-Haqq 1995; Hawley and Valli 1999; Elmore 2002; Borasi and Fonzi 2002).

13.2.1.2 Teacher Communities of Practice

Matos et al. (2009) noted that in teacher PD, learning should not be defined as the acquisition of knowledge of a propositional nature, but rather be conceptualized as being situated in forms of co-participation in the practices of teachers. Teachers participating in such learning may be said to belong to a community of practice (CoP) (Lave and Wenger 1991). Such a community does not exist when a group of teachers from several schools are interacting in a given setting such as attending a series of seminars or workshops, nor with groups of teachers in the same school who are teaching the same subject or year level but do not have mutual relationship and shared goals.

According to Wenger (1998), a CoP is a group of persons sharing the same practice. It has three key features: the members of a community of practice have a mutual enterprise; a shared commitment; and a common repertoire. This repertoire can contain material objects and stories that are shared by members of the community. By virtue of the design of a project like the EPMT-RC (Kaur 2011), with at least a group of teachers per school voluntarily participating in it with a shared set of goals and commitment to one another, within each school, the conditions were favourable for the development of a community of practice. Furthermore, as participants of the project, the teachers also had a shared sense of accountability towards the work of the community, in this case, the learning facilitated by the project. Contemporary sociocultural theory of learning acknowledges that learning involves increasing participation in a CoP composed of experts and novices (Lave and Wenger 1991). In the EPMT-RC project, while the teachers were participating in the project they were the novices and the university scholars were the experts;

subsequently as communities of practice at the school level enlarged, the teachers who had participated in the EPMT-RC project took on the role of experts and the newcomers were the novices.

Gueudet et al. (2013) caution that teachers working together on resources may sometimes not be a CoP. Rather they could be mere collectives. Gueudet et al. emphasize that advantageous conditions in terms of material provisions or time allocations are not enough to engage a dynamic towards a CoP. In addition, engaging such a dynamic requires teachers to work on common resources and to share professional knowledge and beliefs about the teaching of mathematics. In their study, they claim that turning collectives into communities requires the "development of a synergy between teachers and resources" (p. 1014) and that this may be facilitated by material conditions such as common meeting times for teachers to work on shared tasks with the support of external agents such as university scholars or experts in the field.

13.2.2 Design of the Project

The project has five significant features. The features are:

(i) *Content focus*
The project is focused on what to teach and how to teach (Stiff 2002; Desimone 2009). It was specific to the pedagogy of mathematics. This focus is similar to that of most in-service courses conducted for mathematics teachers in Singapore as the main objective of such courses is to introduce teachers to new initiatives that arise from curriculum revisions. The secondary school teachers participating in the project are working with mathematical content that was appropriate for the grade levels of their students.

(ii) *Coherence*
The project is coherent with the needs of the teachers. It focusses on teaching for metacognition which is one of the five components of the school mathematics framework that nurtures mathematical problem solvers. In addition, it also addresses a gap in instruction identified by Hogan et al. (2013), i.e. the disproportionate use of knowledge-building tasks by teachers to engage learners in higher order thinking during mathematics lessons. The project supports the instructional activities of teachers at school, such as the adoption of initiatives (Stiff 2002; Desimone 2009). Ball and Cohen (1999) have argued that classroom activities can form the basis of constructive professional development, and many other researchers have also determined that effective PD is embedded in teacher work (Clarke 1994; Abdal-Haqq 1995; Hawley and Valli 1999; Carpenter et al. 1999; Elmore 2002).

(iii) *Duration*
The duration of the project is 2 years and comprises three phases. Teachers attended training workshops for a semester, followed by a semester of school-based work guided and monitored by the university scholars (PD providers), followed by another year (2 semesters) of self-directed school-based work. The duration of the project is significantly longer than most in-service courses that mathematics teachers usually attend.

(iv) *Active learning*
The project engages teachers in active learning (Wilson and Berne 1999; Desimone 2009). It includes training, practice and feedback, and follow-up activities (Abdal-Haqq 1995), consistent with Stiff (2002), who suggested that teachers learn best when observing, planning for classroom implementation, reviewing student work, and presenting, leading and writing. As stated earlier, Ball (1996) also claimed that the most effective professional development model includes follow-up activities in the form of long-term support, coaching in teachers' classrooms, and ongoing interactions with colleagues.

(v) *Collective participation*
In the project there is collective participation at two levels—school and project. At the school level, at least four teachers, with pairs of teachers teaching the same grade year and mathematics programme are participating. These teachers work together during the training workshops and also at school when implementing their learning in their classrooms. At the project level, teachers also work together building their knowledge by participating in sessions during which they critiqued their peers' work, and shared their experiences and difficulties encountered during the implementation of their newly gained knowledge. Teachers in the project are participating in two-tier communities of practice, the school community of practice and the project community of practice.

13.2.3 Participants of the Project

Forty in-service secondary mathematics teachers from seven secondary schools in Singapore are participating in the project. The project is facilitated by a professor from the NIE, a research associate and a lead teacher from a secondary school.

13.2.4 Implementation of the Project

The project has three phases spread over 2 school years. A school year comprises two semesters, each of 20-week duration. Details of the phases are as follows:

13.2.4.1 Phase I

Duration of this phase is the first semester of the first year of the project (i.e. from January till May). The phase started with the participants completing the Pre-Intervention Teacher Survey. The survey sought from teachers their understanding about performative tasks, knowledge-building tasks, metacognition and teaching for metacognition. The findings of data from the survey were used to plan the knowledge-building workshops for the participants. Figure 13.1 shows the survey item on mathematical tasks and Table 13.1 the corresponding responses of the teachers.

From Table 13.1, it is apparent that teachers were using significantly more performative tasks compared to knowledge-building tasks during their lessons. This may have been a consequence of several factors, such as (i) the lack of knowledge-building tasks commonly found in textbooks used by the teachers; (ii) inability to craft knowledge-building tasks using textbook tasks that focus on direct application of knowledge; and lastly (iii) the push to develop procedural fluency after the introduction of concepts so as to perform routine tasks with ease during tests.

<table>
<tr><td colspan="4">Mathematical Tasks
The following are examples of performative and knowledge building tasks.</td></tr>
<tr><td colspan="4">Topic: Scales and Maps</td></tr>
<tr><td colspan="2">Performative task</td><td colspan="2">Knowledge-building task</td></tr>
<tr><td colspan="2">The scale of map A is 1: 40 000.
A rectangular field is 3 cm by 2 cm on the map. Find the actual area of the field in km^2.

If the area of the field is now represented on map B with scale 1: 20 000, what is the area on the map?</td><td colspan="2">The scale of map A is 1: 40 000.
A rectangular field is 3 cm by 2 cm on the map. Find the actual area of the field in km^2.

If the field is now represented on map B with scale 1: 20 000, without computing any area, explain how will the size of the field be different on map B.</td></tr>
<tr><td colspan="4">Topic: Quadratic graphs and graphical solutions of simple quadratic equations</td></tr>
<tr><td colspan="2">Performative task</td><td colspan="2">Knowledge-building task</td></tr>
<tr><td colspan="2">Draw the graph of $y = x^2 - 2x - 3$ for $-2 \leq x \leq 4$.
Hence solve the equation $x^2 - 2x - 3 = -2$ graphically.</td><td colspan="2">Draw the graph of $y = x^2 - 2x - 3$ for $-2 \leq x \leq 4$.
Using your graph determine the number of solutions the equation $x^2 - 2x - 3 = a$ has.</td></tr>
<tr><td colspan="2">How often do you use performative tasks?
In ten consecutive lessons you would have used them</td><td colspan="2">How often do you use knowledge-building tasks? In ten consecutive lessons you would have used them</td></tr>
<tr><td>Please tick the appropriate response</td><td>√</td><td>Please tick the appropriate response</td><td>√</td></tr>
<tr><td>In all the lessons</td><td></td><td>In all the lessons</td><td></td></tr>
<tr><td>In 7 - 9 of the lessons</td><td></td><td>In 7 - 9 of the lessons</td><td></td></tr>
<tr><td>In 5 - 6 of the lessons</td><td></td><td>In 5 - 6 of the lessons</td><td></td></tr>
<tr><td>In 2 - 4 of the lessons</td><td></td><td>In 2 - 4 of the lessons</td><td></td></tr>
<tr><td>In 0 - 1 of the lessons</td><td></td><td>In 0 - 1 of the lessons</td><td></td></tr>
</table>

Fig. 13.1 Pre-intervention survey item on mathematical tasks used by teachers

Table 13.1 Responses of teachers to survey item on mathematical tasks

How often do you use **performative tasks**? In ten consecutive lessons, you would have used them		How often do you use **knowledge-building tasks**? In ten consecutive lessons, you would have used them	
N (%)		N (%)	
In all the lessons	20 (50.0)	In all the lessons	0 (0.0)
In 7–9 of the lessons	18 (45.0)	In 7–9 of the lessons	1 (2.5)
In 5–6 of the lessons	2 (5.0)	In 5–6 of the lessons	4 (10.0)
In 2–4 of the lessons	0 (0.0)	In 2–4 of the lessons	19 (47.5)
In 0–1 of the lessons	0 (0.0)	In 0–1 of the lessons	16 (40.0)

Our mathematics syllabus states that metacognition is "thinking about thinking".

a) What does metacognition mean to you? What is your understanding of metacognition?

b) Do you engage your students in metacognition during mathematics lessons? Yes / No.
If Yes, how do you engage your students in metacognition during mathematics lessons? Give an example of what you do.

Fig. 13.2 Pre-intervention survey item on teachers understanding of metacognition

Figure 13.2 shows the survey item that sought participants' perceptions about metacognition and teaching for metacognition.

Analysis of the qualitative data, of the survey item shown in Fig. 13.2, is reported in detail elsewhere (Kaur et al. 2016). From the analysis of the data it was apparent that teachers in the project had some knowledge about metacognition but their understanding was not comprehensive. Most of them generally tended to associate metacognition with higher order thinking and problem-solving. Only a few of them associated metacognition with awareness of thinking and reflection and critiquing of one's own thoughts. Thirty-two of the teachers claimed that they engaged their students in metacognition during mathematics lessons and all of them gave examples of how they did so. The examples showed that they did so by engaging their students in problem-solving and drawing on higher order thinking skills to solve mathematical tasks.

Seven 3-hour knowledge-building workshops were organized for the teachers. Table 13.2 shows the synopsis of the workshops.

13.2.4.2 Phase II

The second phase of the project was the second semester of the first year of the project. It was from July till November of the year. During this phase the school

Table 13.2 Synopsis of the workshops

Workshop	Synopsis of Workshop
1 and 2	*Performative and knowledge-building tasks* Teachers were introduced to performative and knowledge-building tasks by the professor and the lead teacher. They also demonstrated how a typical textbook task may be crafted into a knowledge-building one. Teachers were given four performative tasks and worked in school groups (according to their respective schools) to craft knowledge-building tasks. The first session drew to close with a show and tell activity during which all the groups shared their tasks with everyone in the project group and invited critique and suggestions During the second workshop, teachers in their school groups selected performative tasks from their textbooks and crafted knowledge-building tasks. Again they worked in their school groups, before sharing their knowledge-building tasks with all in the project group during the second half of the session. Following the two sessions, all the tasks created by the teachers in the project were made available to the project group through an e-portal. This was facilitated by the research associate of the project
3	*Teacher noticing* During this session, teachers watched a videorecorded lesson of the lead teacher in the project. In the lesson, the lead teacher was teaching for metacognition. During the first round of observing the lesson, the teachers were asked to note down on worksheet A (see Appendix A) what they observed After a short break the teachers were given worksheet B (see Appendix B) and asked to read it carefully before they viewed the videorecorded lesson once again. The prompts in worksheet B are adopted from McDuffie et al. (2014). During the second viewing of the videorecorded lesson, teachers noted down their observations on worksheet B. The session ended with a project group discussion on the four lenses that may be adopted for observing a lesson enacted by peers in the project
4–5	*Teaching for metacognition* In the first session, the paper Thinking about Thinking: Metacognition, Session 9 in The Learning Classroom: Theory into Practice—A Tele course for Teacher Education and Professional Development at the University of Stanford in the US (Darling-Hammond et al. 2001) was used as the resource for reading and discussion. Teachers worked in their school groups. Their deliberations were guided by the worksheet shown in Appendix C In the second session, teachers continued working in their school groups deliberating on aspects of metacognition. During the second half of the session, the school groups shared their examples of how teachers may engage students to reflect on what they know, direct their learning, create a culture of metacognition in the classroom and examples of strategies for developing metacognition. The session came to a close with teachers noting that there several strategies which may be used to engage students in metacognition [see list of strategies and examples in Kaur et al. (2016)]
6–7	*Planning a lesson to teach for metacognition* In the first session, teachers brainstormed as a project group and stated the mathematical norms and sociomathematical norms that would shape a knowledge and student-centred lesson. Next, they worked in their school groups and planned a lesson that they would teach in the next phase of the project. They were guided by the following instructions: • The lesson must use knowledge-building tasks and engage students in metacognitive strategies for learning • The lesson plan must state clearly (i) the lesson objectives (ii) the mathematical tasks used during the lesson (iii) the metacognitive strategies that would be developed (iv) the mathematical and sociocultural norms In the second session, the school groups engaged in a show and tell. They shared with the project group the specific instructional objective/s of their lesson, the mathematical tasks they planned to use, how they intended to engage the students in metacognition (stating specific strategies and key questions and prompts they planned to use), and what mathematical norms and sociomathematical norms guided their plan for the lesson. During the show and tell, the school groups invited critique and suggestions from fellow participants in the project group. Following the sharing, the school groups revised their lesson plans and prepared to teach the planned lesson during the next phase of the project

groups of the project worked collaboratively and implemented their planned lesson. They wrote a detailed lesson plan for the lesson they were carrying out. One teacher from the group taught the lesson to his/her students and the lesson was video-recorded. The school group met and viewed the lesson and prepared their presentation for the project group sharing meetings. Two project group meetings were held in October. During the project group meetings, the school group that presented solicited feedback from the project group. All participants in the project group except the teachers from the presenting school, participated in the feedback session. They used the four lens noticing feedback framework to give their feedback. The research team collected the feedback and the feedback was collated before it was sent back to the school group that presented.

Following the sharing sessions, the research team organized a meeting with every school group. Each meeting lasted between 2 and 3 h. A total of seven meetings were held. During the meetings the feedback from the project group was discussed and addressed. The feedback was very helpful as it provided the views of many more pairs of eyes reviewing the lesson. In addition, during the meetings the research team inducted the school group into a four-step approach to facilitate working and learning collaboratively when integrating their new knowledge into classroom practice. The four steps were as follows:

1. Plan and write a detailed plan of the lesson.
2. Enact and videorecord the lesson.
3. Watch the recorded lesson, compare it with the lesson plan and write the lesson narrative detailing the short comings and what the team would do differently the next time. A set of prompts were provided by the research team to guide the writing of the lesson narrative. The prompts were as follows:

    - Were the lesson objectives achieved? Was there any mismatch/deviation between the planned and enacted?
    - Were the mathematical tasks of knowledge-building type? How were the tasks enacted? Did they achieve the purpose they were intended for?
    - What were the metacognitive strategies that were developed? How were they developed? What challenges did the teacher encounter in developing them?
    - Did the teacher have any guiding mathematical norms that shaped the classroom discourse?
    - Did the teacher have any guiding sociomathematical norms that shaped the classroom interactions between the students, and also teacher–student?
    - What was the sequence of activities during the lessons? [e.g. teacher talk (demonstration), seat work, discussion/teacher talk (instructions), group work, student presentations, whole class discussion, etc.]
    - What was student engagement like during the lesson? [passive, active, problem-solving, explaining, problem posing, etc.]
    - Did the students say anything about the lesson? How similar or different it was from the teacher's normal lesson?
    - Would the teacher rate the lesson as one that taught for metacognition?

Table 13.3 Schedule of meetings to develop fellow teachers

School	Sharing session/conference
S1	Attended a conference in Korea in November [of the first year of the project]. Presented a paper showcasing their learning related to performative and knowledge-building tasks
S3	Organized a learning festival for staff in the school in November [of the first year of the project]. The teachers in the school group showcased lessons that taught for metacognition
S4	Teachers submitted a proposal to present their "Teaching for Metacognition Lessons" at the national Teachers Conference in June [of the second year of the project]

4. Write a reflection about the learning journey of the teacher's learning. Every member should do this individually, subsequently meet as a group and share with each other the reflections. The journal prompt was "*Describe in detail your learning journey during the planning, enacting and reviewing of your team's lesson that was carried out with the goal of teaching for metacognition*".

13.2.4.3 Phase III

Phase III of the project will be a year long and in the second year. Participants of the project will continue to work in their school groups and integrate their new knowledge into classroom practice. There will be periodic project meetings during which the school groups will showcase their lessons and invite critique and suggestions from the project group. In addition, the school groups will engage in activities through which they will contribute towards the development of fellow teachers both nationally and internationally. Table 13.3 shows a tentative schedule of activities that some schools have planned for developing fellow teachers who are not in the project, both nationally and internationally. At the time of writing this chapter the project had completed phases I and II only.

13.3 Learning of Teachers Working Collaboratively in Multitier Communities of Practice

In this section, we draw on the journals written by four teachers in the project to illustrate how the teachers in the project worked and learned collaboratively in the two-tier communities of practice. By virtue of the design of the project, with at least a group of teachers per school voluntarily participating in it with a shared set of goals and commitment to one another, within each school the conditions were favourable for the development of a community of practice. Furthermore, as

participants of the project, the teachers also had a shared sense of accountability towards the work of the community, in this case, the learning facilitated by the project. In addition, there were school groups that formed the first-tier communities of practice and the project group that formed the second-tier community of practice. In the project there were seven first-tier communities of practice and only one second-tier community of practice. All the seven school groups together formed the second-tier community of practice while each school formed its own first-tier community of practice. The four teachers belong to two school groups and each group had enacted a lesson that taught for metacognition during phase II of the project. The lessons they enacted are described elsewhere (Kaur et al. 2015). The two teachers from the first school are S1T1 and S1T2 and the two teachers from the second school are S2T1 and S2T2. The teachers S1T1 and S1T2 were involved in the first lesson their school group enacted in their school while S2T1 and S2T2 did the same in their school.

13.3.1 Working and Learning in First-Tier Communities of Practice

During the knowledge-building workshops, teachers worked collaboratively in their school groups to craft knowledge-building tasks, clarify their understanding about metacognition and teaching for metacognition, and planning a lesson that teach for metacognition. From the following excerpts, it is apparent that the school group continued to work collaboratively to plan the lesson, enact it, view it and reflect on it during phase II of the project in their respective schools.

S1T1 *When the team decided on conducting a lesson using problem posing as our pedagogy, there was some resistance from the team members as we were unsure where and how to start. We discussed our challenges and used the guidelines given by the NIE professor to kickstart the project. The most challenging part was creating the rubrics for the students to assess the quality of the questions created by them. We managed to produce the rubrics by listing down the key components of the rubrics through our discussions and from other sources. … After the lesson was conducted, we reviewed the video clips taken in the classroom. We compared the lesson flow with our lesson plan. We realized that certain aspects in the lesson plan were not carried out or could have been carried out better. Aspects like missing keywords that were supposed to be said out, skipping a portion of the lesson plan due to time constraints,* etc. *The video evidence also showed us that there are improvements needed to be made for our future projects. Improvements like time allocation, group discussions and students'*

presentations. The video review also allowed us to check if we have met our project objectives and reflect on it.

S1T2 *The sessions with the NIE professor and lead teacher prior to the planning of the actual lesson were instrumental in shaping what the group wanted to achieve in the lesson design. ... The team wanted to ensure that the lesson will be something innovative and one that the team has not designed before in the school. ... The team tried to also tie in the lesson design the department's pedagogy of Generative Activities, as the four design principles gel well with the concept of knowledge-building tasks and student-centred lesson. ... We had to do some reading up on problem-posing articles, and identified the types of activities that can be put into the lesson. At the same time, we were cognizant of the topics that are in the curriculum for the targeted class. ... The team prepared a lesson based on Sec 2 Algebra, and in the plan we highlighting good questions that were to be asked during the lesson.*

S2T1 *While planning, there were many considerations we had. ... these considerations posed many challenges. ... Bearing all the challenges in mind, we decided to carry out a knowledge-building task through linear graphs using ICT, focusing on the strategy "questioning by teacher" and "directed thinking".*

S2T2 *During the pre-planning stage, my team took into account the scheme of work when choosing the lesson idea for the project. We were also mindful about integrating other teaching methods, i.e. ICT, Learning Experience in order to enhance the collaboration and sharing amongst students. Therefore, we chose "Gradient of a Straight Line". When planning the lesson, we decided to focus on two metacognitive strategies, namely Awareness of knowledge and Monitoring activities during learning. We felt that these strategies could best support teaching and enhance metacognition in the students, and hence we developed our lesson around these strategies.*

13.3.2 Working and Learning in Second-Tier Community of Practice

During the knowledge-building workshops, teachers in the school groups presented their work to the project group and invited critique and suggestions. It was during these times that teachers worked and learned collaboratively in a second-tier community of practice. Furthermore, in phase II, when the school groups presented their lessons and invited critique and suggestions during the project group meetings,

once again teachers worked and learned collaboratively in a second-tier community of practice. From the following excerpts, it is apparent that teachers valued the feedback they gathered during their participation in the second-tier community of practice as it contributed towards their deeper understanding of how to enact lessons that teach for metacognition.

S1T2 *During the last session of the workshops, the team was asked questions when they presented their proposal of problem posing. Inputs from the NIE professor, lead teacher and project group teachers provided a better understanding to what we wanted to achieve in our lesson. … We learnt a lot when we reviewed the videorecorded lesson, and went through the other teachers' feedback [feedback from the project group that was collected during the school's group presentation in phase II]. For example, we realized that more opportunities can be given to students to play different roles, like presenting their feedback, besides being a critiquer. Also students could extend their learning by building their own 3D models.*

S2T1 *After reviewing, we realized that due to the nature of this lesson, "using discourse" seemed to be a better strategy to be used whereby students would need to first work and observe the lines drawn in different tasks before coming together in a pair to compare their answers. In the pair, each student is to explain how he/she obtains the observation and will have to justify and convince the other of the correct observation. This process helps students to concretize their thinking. … Hence, amendments need to be made to the students' worksheet to add in a "food for thought" segment to allow discourse of how the values of a and b affect* $y = ax + b$.

13.4 Conclusion

The Teaching for Metacognition project, described in this chapter, illustrates a form of PD for mathematics teachers that is gaining momentum in Singapore. This is so as the PD is nestled in the classrooms of the teachers and addresses their needs. The three phases of the PD, namely: ***Learn*** (Acquisition and co-construction of knowledge), ***Apply*** (integrate new knowledge into classroom practice) and ***Teach*** (develop fellow teachers nationally and/or internationally) appear to make the engagement of teachers in PD holistic. In addition the working and learning of the teachers in tiers of communities of practice facilitates critique, suggestions and openness that builds synergy.

Appendix A

Teacher Noticing – Round 1

You have viewed the video record of Teacher A.

What did you notice?

List at least five observations as completely as possible.

For example: teacher talk – the teacher spoke in a monologue for extended periods of time.

My
observations ……
……
……
……
……
……

Appendix B

Teacher noticing—Round 2 You may use the following prompts to guide you in viewing the videorecord for the four lenses. The prompts are adopted from McDuffie et al. (2014)	
Teaching lens • **How does the teacher elicit students' thinking and respond**? – What opportunities does the teacher create for diverse learners to communicate their mathematical thinking—show what they know? – How does the teacher implement the task in a way that maintains or changes the cognitive demand?	*Learning Lens* • **What specific math understandings and/or confusions are indicated in students' work, talk, and/or behaviour**? – How do students communicate what their understandings and sense making of others' thinking? – In what ways does student engagement reflect conceptual and/or procedural learning?

(continued)

(continued)

– What resources and knowledge does the teacher use/draw upon to support students' math understanding?	– What resources or knowledge do students draw upon to understand and solve the math task?
Task lens • **What is the nature of the task/s used in the lesson**? – What makes this a good and/or problematic task? How could it be improved? What is/are the central math idea/s in this task? – How does the task make thinking visible? – What resources or knowledge does this task activate and/or connect to?	*Power and participation lens* • **Who participates**? • **Does the classroom culture value and encourage most students to speak, only a few, or only the teacher**? • **Where does the majority of the math "work" take place in the classroom**? – Who holds authority for knowing mathematics? Do some students hold more status than others? – What evidence indicates that differences in approaches and perspectives are/are not respected and valued?

Appendix C

Thinking about thinking: metacognition	
Metacognitive knowledge– Reflecting on What we know • Awareness of knowledge • Awareness of thinking • Awareness of thinking strategies *Write down examples of each for mathematics lessons*	Metacognitive regulation– Directing our learning • Planning approaches to tasks • Monitoring activities during learning • Checking outcomes *Write down examples of each for mathematics lessons*
A Culture of Metacognition in the Classroom *What conditions support a metacognitive classroom environment?*	Strategies for learning • Predicting outcomes/evaluating work • Questioning by the teacher/self-assessing • Self-questioning/selecting strategies • Using directed or selected thinking • Using discourse/critiquing/revising *Write down examples of each for mathematics lessons*

References

Abdal-Haqq, I. (1995). *Making time for teacher professional development (Digest 95-4)*. Washington, DC: ERIC Clearinghouse on Teaching and Teacher Education.

Ball, D. L. (1996). Teacher learning and the mathematics reforms: What do we think we know and what do we need to learn? *Phi Delta Kappan, 77*, 500–508.

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Towards a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3–32). San Francisco: Jossey-Bass.

Borasi, R., & Fonzi, J. (2002). Professional development that supports school mathematics reform. Foundations series of monographs for professionals in science, mathematics and technology education. Arlington, VA: National Science Foundation.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Castle, K., & Aichele, D. B. (1994). Professional development and teacher autonomy. In D. B. Aichele & A. F. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 1–8). Reston, VA: National Council of Teachers of Mathematics.

Clarke, D. (1994). Ten key principles from research for the professional development of mathematics teachers. In D. B. Aichele & A. F. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 37–48). Reston, VA: National Council of Teachers of Mathematics.

Darling-Hammond, L., Austin, K., Cheung, M., & Martin, D. (2001). Thinking about thinking: Metacognition. In L. D. Darling-Hammond, K. Austin, S. Orcutt, & J. Rosso (Eds.), *The learning classroom: Theory into practice* (pp. 156–172). A telecourse for teacher education and professional development. Stanford University school of Education.

Desimone, L. M. (2009). Improving impact studies on teachers' professional development: Toward better conceptualisations and measures. *Educational Researcher, 38*(3), 181–199.

Elmore, R. F. (2002). *Bridging the gap between standards and achievement: The imperative for professional development in education*. Washington, DC: Albert Shanker Institute.

Goh, C. T. (1997). Shaping our future: "Thinking Schools" and a "Learning Nation". *Speeches, 21* (3), 12–20. (Singapore: Ministry of Information and the Arts).

Gueudet, G., Pepin, B., & Trouche, L. (2013). Collective work with resources: An essential dimension for teacher documentation. *ZDM—The International Journal of Mathematics Education, 45*, 1003–1016.

Hargreaves, A. (1995). Development and desire: A postmodern perspective. In T. R. Guskey & M. Huberman (Eds.), *Professional development in education: New paradigms and practices* (pp. 9–34). New York: Teachers College Press.

Hattie, J. (2009). Visible learning. Routledge.

Hawley, W. D., & Valli, L. (1999). The essentials of effective professional development: A new consensus. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 127–150). San Francisco: Jossey-Bass.

Hogan, D., Towndrow, P., Chan, M., Kwek, D., & Rahim, R. A. (2013). *CRPP Core 2 research program: Core 2 interim final report*. Singapore: National Institute of Education.

Kaur, B. (2009). Enhancing the pedagogy of mathematics teachers (EPMT): An innovative professional development project for engaged learning. *The Mathematics Educator, 12*(1), 33–48.

Kaur, B. (2011). Enhancing the pedagogy of mathematics teachers (EPMT) project: A hybrid model of professional development. *ZDM—The International Journal on Mathematics Education, 43*(7), 791–803.

Kaur, B., Areepattamannil, S., & Boey, K. L. (2013). *Singapore's perspective: Highlights of TIMSS 2011*. Singapore: Centre for International Comparative Studies, National Institute of Education.

Kaur, B., Bhardwaj, D., & Wong, L.F. (2015). Developing Metacognitive skills of mathematics learners. In The Korean Society of Mathematical Education. (Ed.) *The Korean Society of Mathematics Education Proceedings of the 2015 International Conference on Mathematics Education* (pp. 237–245). Seoul, Korea: The Korean Society of Mathematical Education.

Kaur, B., Boey, K. L., Areepattamannil, S., & Chen, Q. (2012). *Singapore's perspective: Highlights of TIMSS 2007*. Singapore: Centre for International Comparative Studies, National Institute of Education.

Kaur, B., Wong, L.F., & Bhardwaj, D. (2016). Mathematics subject mastery—A must for developing 21st century skills. In P.C. Toh & B. Kaur (Eds.), *Developing 21st century competencies in the mathematics classroom*.World Scientific.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Maaβ, K., & Artique, M. (2013). Implementation of inquiry-based learning in day-to-day teaching: a synthesis. ZDM—The International Journal of. *Mathematics Education, 45*, 779–795.

Matos, J. F., Powell, A., & Sztajn, P. (2009). Mathematics teachers' professional development: Processes of learning in and from practice. In R. Even & D. L. Ball (Eds.), *The professional education and development of teachers of mathematics* (pp. 167–183). New York: Springer.

McDuffie, A. R., Foote, M. Q., Bolson, C., Turner, E. E., Aguirre, J. M., Bartell, T. G., et al. (2014). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. *Journal of Mathematics Teacher Education, 17*, 245–270.

Ministry of Education, Singapore. (2010). MOE to enhance learning of 21st century competencies and strengthen art, music and physical education. Retrieved 5 Sept 2015 from www.moe.gov.sg

Ministry of Education, Singapore. (2012). *O-Level, N(A) Level, N(T) level mathematics teaching and learning syllabuses*. Singapore: Author.

Mullis, I. V. S., Martin, M. O., & Foy, P. (2008). *TIMSS 2007: International mathematics report*. Chestnut Hill, MA: TIMSS & PIRLS International Study Centre, Boston College.

Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011: International mathematics report*. Chestnut Hill, MA: TIMSS & PIRLS International Study Centre, Boston College.

OECD. (2010). *PISA 2009 results: What students know and can do: Student performance in reading, mathematics and science* (Vol. 1). OECD Publishing.

OECD. (2013). *PISA 2012 results: What students know and can do: Student performance in mathematics, reading and science* (Vol. 1). OECD Publishing.

Partnership for 21st Century Skills. (2009). *Assessment: A 21st century skills implementation guide*. Tucson, AZ: Author.

Smylie, M. A. (1989). Teachers' views of the effectiveness of sources of learning to teach. *Elementary School Journal, 89*, 543–558.

Stiff, L. V. (2002). March). Study shows high-quality professional development helps teachers most. *NCTM News Bulletin, 38*(7), 7.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. New York: Cambridge University Press.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. *Review of Research in Education, 24*, 173–209.

Yeap, B. H., & Ho, S. Y. (2009). Teacher change in an informal professional development programme: The 4-I model. In K. Y. Wong, P. Y. Lee, Kaur, B., P. Y. Foong, & S. F. Ng (Eds.) *Mathematics education: The Singapore journey* (pp. 130–149). Singapore: World Scientific.