Chapter 10
Multi-objective Nurse Rerostering Problem

Shih-Min Wu, Tenda Okimoto, Katsutoshi Hirayama and Katsumi Inoue

Abstract How to schedule a limited number of nurses in hospital wards staffed 24 h a day is important issue for the satisfactory patient care and potentially improve nurse retention. Nurse Scheduling Problem (NSP) is a combinatorial optimization problem, in which a set of nurses must be assigned into a limited set of working slots, subject to a given set of hard and soft constraints. Various sophisticated algorithms have been developed for solving a NSP. It is natural to consider the scheduled nurse's unexpected absences, e.g., illness, accident and injury. Nurse Rerostering Problem (NRP) is a dynamic NSP where the aim is to reschedule the current roster so that the number of changes of assignments between current and modified schedules is minimized. In this paper, the focus is laid on NRP with multiple criteria and the "egalitarianism" among nurses in a modified schedule. A formal framework of Multi-Objective Nurse Rerostering Problem (MO-NRP) is defined where the aim is to find trade-off solutions among "optimality" and "stability". Also, a novel solution criterion called an egalitarian solution for a MO-NRP is introduced.

Keywords Nurse rerostering problem · Multi-objective weighted constraint satisfaction problem · Pareto optimality · Stability · Egalitarianism

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10.1 Introduction

Nurse Scheduling Problem (NSP) [1, 6, 8, 12] is one of the widely investigated application problems in operations research (OR) and artificial intelligence (AI). It is well known that a NSP can be represented as an weighted constraint satisfaction problem (WCSP) [1, 14] where the aim is to find an assignment that satisfies all hard constraints and minimizes the sum of all violated costs of soft constraints. In order to provide the satisfactory patient care and potentially improve nurse retention, creating a good schedule for nurses is an important issue. However, since there are many constraints which must be satisfied, making an ideal schedule for both nurses and the hospital is intractable, and that is why the scheduler (e.g. head nurse in many cases) spends a lot of time to find a feasible schedule. Various sophisticated complete and incomplete algorithms have been introduced for solving a NSP in order to generate better nurse schedules and solve large-scale problems [2, 7, 9, 15].

Nurse Rerostering Problem (NRP) [16, 19, 21] is a dynamic NSP where the aim is to reschedule the current roster/schedule so that the number of changes of assignments between current and modified rosters/schedules is minimized. It is natural to consider the scheduled nurse's unexpected absences, e.g., illness, accident and injury of a nurse, after the scheduler created a roster with difficulty. When an absence is announced, the scheduler must find a nurse who can fill the vacancy of the absentee and the current schedule must be rebuilt as soon as possible. Most previous works on NRP have been investigated the stability of a modified schedule, i.e., a modified schedule should be similar to the previous one as much as possible.

The *egalitarianism* among nurses is an expected property of a NRP. Assume that the number of changes of all assignments in a modified schedule is small and it is also optimal (i.e. all hard constraints are satisfied and the sum of the violation costs of soft constraints is minimized). However, what happen if one nurse needs to change her assignments a lot in a modified schedule, while other nurses not. Clearly, the nurse who should change a lot complains about the modified schedule.

In this paper, the focus is laid on NRP with multiple criteria and the egalitarianism among nurses in a modified schedule. A formal framework for *Multi-Objective Nurse Rerostering Problem* (MO-NRP) is defined which is the extension of a mono-objective NRP. In this framework, the both stability and optimality are considered simultaneously. More specifically, MO-NRP is modeled by using the framework of a multi-objective WCSP [20] where the aim is to find an assignment that satisfies all hard constraints and minimizes the sum of violated costs of all objective functions.

Furthermore, a novel solution criterion called egalitarian solution for a MO-NRP is defined. In an egalitarian solution, the nurses share the changes of their shift works, i.e., minimize the maximal number of changes of assignments among nurses.

In a MO-NRP, since trade-offs exist among objectives, there does not generally exist an ideal assignment, which minimizes all objectives simultaneously. Thus, the optimal solutions of a MO-NRP is characterized by using the concept of *Pareto optimality*. An assignment is Pareto optimal if there does not exist another assignment that weakly improves all of the objectives. Solving a MO-NRP is to find Pareto front which is a set of cost vectors obtained by all Pareto optimal solutions (i.e. trade-off solutions among stability and optimality). MO-NRP can be represented using a graph called a constraint graph [22] in which nodes correspond to variables and each edge represents a constraint. In a MO-NRP, even if a constraint graph has the simplest tree structure, the size of Pareto front becomes exponential in the number of variables, i.e., all assignments are Pareto optimal solutions in the worst case.

The rest of the paper is organized as follows. In the next section, the formalizations of NRP and MO-WCSP are provided. Afterwards, the framework for MO-NRP is presented and the formal definition of an egalitarian solution for a MO-NRP is defined. Finally, we conclude this paper and give some future works.

10.2 Preliminaries

In this section, the models of nurse rerostering problem (NRP) and multi-objective weighted constraint satisfaction problem (MO-WCSP) are briefly described.

10.2.1 Nurse Rerostering Problem

Nurse Rerostering Problem (NRP) [16, 19, 21] is a dynamic nurse scheduling problem where the aim is to reschedule the current roster so that the number of changes of assignments between current and modified schedules is minimized, i.e., solving a NRP is to find a stable solution. In general, the constraints are dependent on the requirements of both nurses and hospitals. The following is the hard and soft constraints, which are frequently used in previous works. Note that some hard constraints are used as soft constraints and vice versa. It depends on the hospitals.

Hard Constraints

- H1: Prohibited working patterns must be avoided (e.g. one should not assign a nurse for 7 consecutive works and 3 consecutive night shifts).
- H2: In order to provide the satisfactory patient care, there exists the required number of nurses for each shift in a day (e.g., at least 3 nurses must be assigned to the morning and 2 nurses for evening and 1 nurse for night shifts).
- H3: For each nurse, the number of day-offs in a current schedule should not be less than that in a modified schedule.
- H4: Each newcomer should be assigned together with a skillful nurse, i.e., she has to work with a head nurse or a highly experienced nurse.
- H5: Nurses must rest at least 16 h between two consecutive shift works, e.g., in case a nurse is assigned to the night shift (0:00–8:00), morning (8:00–16:00) and evening shifts (16:00–24:00) should not be assigned.

Soft Constraints

S1: For each shift work (morning/evening/night), the required skill level of assigned nurses should be satisfied (e.g. for each shift work, at least one head nurse or one highly experienced nurse must be assigned).
S2: Day-offs of nurses in a current schedule should not be changed, i.e., the scheduled day-offs after modification must be same as much as possible.
S3: Requests of nurses (e.g. the preferred working patterns and specially the day-off requests) should be satisfied as much as possible.

Objective: Minimize the number of changes of shift works between current and modified schedules, i.e., the aim is to find a stable solution.

10.2.2 Multi-objective WCSP

Multi-Objective Weighted Constraint Satisfaction Problem (MO-WCSP) [20] is the extension of a mono-objective WCSP [1, 14] where the aim is to find an assignment that satisfies all hard constraints and minimizes the sum of all violated costs of soft constraints. Let k be the number of objectives. MO-WCSP is defined by a tuple MO-WCSP = $\langle X, D, C, S, \Phi\rangle$, where $X = \{x_1, x_2, ..., x_n\}$ is a set of variables, $D = \{d_1, d_2, ..., d_m\}$ is a set of domains, $C = \{C^1, C^2, ..., C^k\}$ is a set of hard and soft constraints, $S = \{S^1, S^2, ..., S^k\}$ is a set of valuation structures, and $\Phi = \{\phi^1, \phi^2, ..., \phi^k\}$ is a set of multi-objective functions. For each objective i $(1 \leq i \leq k)$, $C^i = C^i_h \cup C^i_s$ is the union of hard and soft constraints, where C^i_h is a set of hard constraints and C^i_s shows a set of soft constraints, $S^i = (E^i, \sum, <)$ is the valuation structure, where $E^i = \mathbb{N} \cup \{\infty\}$, $\sum$ is the standard sum over $\mathbb{N}$ and all elements of E are ordered by the operator $<$, and $\phi^i : C^i \rightarrow E^i$ is a cost function. Let A be an assignment to all variables. For an objective i, the valuation of A for constraint $c \in C^i$ is defined as:

$$\phi^i(A, c) = \begin{cases} 0 & c \in C^i_h \ \textit{is satisfied by } A, \\ \infty & c \in C^i_h \ \textit{is violated by } A, \\ \phi^i(A, c) & c \in C^i_s, \end{cases}$$

and the overall valuation of A is given by

$$\phi^i(A) = \sum_{c \in C^i} \phi^i(A, c).$$

Then, the sum of the violation costs of all cost functions for k objectives is defined by a cost vector, denoted

$$\Phi(A) = (\phi^1(A), \phi^2(A), ..., \phi^k(A)).$$

Finding an assignment that minimizes all objective functions simultaneously is ideal. However, in general, since trade-offs exist among objectives, there does not exist such an ideal assignment. Therefore, the "optimal" solution of a MO-WCSP is characterized by using the concept of Pareto optimality. This problem can be represented using a graph (called a constraint graph [22]), in which each node corresponds to a variable and each edge represents a constraint. In a MO-WCSP, even if a constraint graph has the simplest tree structure, the number of Pareto optimal solutions is often exponential in the number of variables in the worst case.

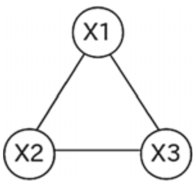
Definition 1 (*Dominance*) For a MO-WCSP, two cost vectors $\Phi(A) = (\phi^1(A), \phi^2(A),..., \phi^k(A))$ and $\Phi(A') = (\phi^1(A'), \phi^2(A'),..., \phi^k(A'))$, we call that $\Phi(A)$ dominates $\Phi(A')$, denoted by $\Phi(A) \prec \Phi(A')$, iff $\Phi(A)$ is partially less than $\Phi(A')$, i.e., it holds

- $\phi^i(A) \leq \phi^i(A')$ for all objectives i, and
- there exists at least one objective i', such that $\phi^{i'}(A) < \phi^{i'}(A')$.

Definition 2 (*Pareto optimal solution*) For a MO-WCSP, an assignment A is said to be Pareto optimal solution, iff there does not exist another assignment A', such that $\Phi(A') \prec \Phi(A)$.

Definition 3 (*Pareto Front*) For a MO-WCSP, a set of cost vectors obtained by Pareto optimal solutions is said to be Pareto front. Solving a MO-WCSP is to find Pareto front.

Example 1 (*MO-WCSP*) Consider the complete graph (i.e. each node has constraints with all other nodes) of a bi-objective WCSP with three variables x_1, x_2 and x_3 (see Fig. 10.1). Each node represents a variable and each edge corresponds to a constraint between two variables. Each variable takes its value from finite, discrete domain {0, 1}. The table shows the cost vectors for each constraint. For example, for the constraint between x_1 and x_3 (middle in the table), in case x_1 takes the value 0 and x_3 takes 1, the obtained cost vector is (0, 3), i.e., the violation cost is 0 for objective 1 and 3 for objective 2. The cost ∞ in the table means that it violates a hard constraint. Pareto optimal solutions of this problem are $\{\{(x_1, 0), (x_2, 1), (x_3, 0)\}, \{(x_1, 0), (x_2, 1), (x_3, 1)\}\}$ and the obtained Pareto front is $\{(3, 6), (4, 5)\}$.



x_1	x_2	*cost*	x_1	x_3	*cost*	x_2	x_3	*cost*
0	0	(∞,0)	0	0	(3,1)	0	0	(1,3)
0	1	(0,1)	0	1	(0,3)	0	1	(2,∞)
1	0	(∞,1)	1	0	(3,2)	1	0	(0,4)
1	1	(∞,0)	1	1	(2,1)	1	1	(4,1)

Fig. 10.1 Example of a bi-objective WCSP with three variables x_1, x_2 and x_3. Each node represents a variable and each edge corresponds to a constraint between two variables. Each variable takes its value from discrete domain {0, 1}. Table shows the cost vectors for each constraint. Pareto optimal solutions are $\{\{(x_1, 0), (x_2, 1), (x_3, 0)\}, \{(x_1, 0), (x_2, 1), (x_3, 1)\}\}$ and Pareto front is $\{(3, 6), (4, 5)\}$

10.3 Multi-objective Nurse Rerostering Problem

In order to consider minimizing the number of constraint violations (optimality) and the number of changes of assignments (stability) simultaneously in a NRP, i.e., NRP with multiple criteria, a Multi-Objective Nurse Rerostering Problem (MO-NRP) is formalized. Moreover, a novel solution criterion called an egalitarian solution for a MO-NRP is defined. First, let us describe the following basic terms for a MO-NRP.

- $N = \{1, ..., n\}$ is a set of ID-numbers for nurses.
- $M = \{1, ..., m\}$ is a set of days in a scheduling period.
- $X = \{x_{11}, ..., x_{nm}\}$ is a set of variables.
- $W = \{o, m, e, n\}$ is a set of shift works, where o = {day-off}, m = {morning} (8:00–16:00), e = {evening} (16:00–24:00) and n = {night} (0:00–8:00).
- $L = \{l_1, ..., l_5\}$ is a set of skill levels of nurses where l_1 = {head nurse}, l_2 = {highly experienced}, l_3 = {experienced (i.e. more than 3 years)}, l_4 = {few years experience (i.e. 1-2 years)} and l_5 = {newcomer}.
- $\alpha_l : N \rightarrow L$ is a mapping which provides the skill level of a nurse, e.g., for a head nurse $i \in N$, her skill level can be obtained by $\alpha_l(i) = l_1$.

A $(n \times m)$-table is said to be a master schedule and is denoted as $MS_{current}$ for a current schedule and MS_{mod} for a modified schedule after unexpected absences of a nurse. One can see that $MS_{current}$ is a solution of NSP and MS_{mod} is that of NRP.

Definition 4 (*Stability*) For two master schedules $MS_{current}$ and MS_{mod}, each $w_{ij} \in W$ in $MS_{current}$ and each $w'_{ij} \in W'$ in MS_{mod}, and a non-negative integer r, MS_{mod} is said to be *r-stable*, iff the sum of the changes of assignments is bounded by r, i.e.,

$$\sum_{i,j} g(w_{ij}, w'_{ij}) \leq r, \ where\ g(w_{ij}, w'_{ij}) = \begin{cases} 0 & w_{ij} = w'_{ij}, \\ 1 & otherwise. \end{cases}$$

Example 2 Consider a master schedule for a week of 7 nurses. Table 10.1 (left) represents the current master schedule $MS_{current}$ which satisfies all hard constraints provided in Sect. 10.2, i.e. hard constraints from H1 to H5. Assume that nurse n_5 has an unexpected absence on Monday and cannot work her morning shift work m. Table 10.1 (right) shows a modified master schedule MS_{mod}. The morning shift of n_5 on Monday has been changed from m to absence in MS_{mod} (denoted by ╱). From the hard constraint H2, i.e., at least 3 nurses must be assigned to the morning shift and 2 for evening and 1 for night shifts, nurse n_1 works the morning shift work m instead of n_5 in MS_{mod}. In order to satisfy all hard constraints, nurse n_1 changes her shift works (i.e. evening shifts e) on Friday, Saturday and Sunday in $MS_{current}$ to night shift n on Friday, day-off o on Saturday and morning shift m on Sunday in MS_{mod}. Also, nurse n_5 changes her night shift n on Friday, day-off o on Saturday and morning shift m on Sunday in $MS_{current}$ to evening shifts e on these three days in MS_{mod}. Since the number of changes of shift works between $MS_{current}$ and MS_{mod} is 8 (including the absence of nurse n_5 on Monday), MS_{mod} is $r = 8$-stable.

Table 10.1 Example of $MS_{current}$ for a week of 7 nurses (left) and a modified schedule MS_{mod} (right). Nurse n_5 had an unexpected absence on Monday (denoted by /). Red fonts show the modified shift works. Nurse n_1 changes her shift works (i.e. evening shifts e) on Friday, Saturday and Sunday in $MS_{current}$ to night shift n on Friday, day-off o on Saturday and morning shift m on Sunday in MS_{mod}. Nurse n_5 changes her night shift n on Friday, day-off o on Saturday and morning shift m on Sunday in $MS_{current}$ to evening shifts e on these three days in MS_{mod}. The MS_{mod} is $r = 8$-stable

$MS_{current}$							
Nurse (Level)	M	T	W	T	F	S	S
n_1 (l_1)	o	m	m	m	e	e	e
n_2 (l_2)	e	e	n	o	m	m	m
n_3 (l_3)	m	m	m	e	e	n	o
n_4 (l_3)	m	e	e	n	o	m	n
n_5 (l_4)	m	m	e	e	n	o	m
n_6 (l_4)	n	n	o	m	m	e	e
n_7 (l_5)	e	o	m	m	m	m	m

MS_{mod}							
Nurse (Level)	M	T	W	T	F	S	S
n_1 (l_1)	**m**	m	m	m	**n**	**o**	**m**
n_2 (l_2)	e	e	n	o	m	m	m
n_3 (l_3)	m	m	m	e	e	n	o
n_4 (l_3)	m	e	e	n	o	m	n
n_5 (l_4)	/	m	e	e	**e**	**e**	**e**
n_6 (l_4)	n	n	o	m	m	e	e
n_7 (l_5)	e	o	m	m	m	m	m

The framework for MO-NRP is defined as follows.

Definition 5 (MO-NRP) A multi-objective nurse rerostering problem is a tuple

$$\text{MO-NRP} = \langle X, W, L, C, S, MS_{current}, \Phi \rangle,$$

where X is a set of variables, W is a set of domains, L is a set of skill levels, C and S are same as a MO-WCSP, $MS_{current}$ is the current schedule, $\Phi = \{\phi^{opt}, \phi^{stable}\}$ is a set of cost functions where ϕ^{opt} is a cost function for optimality and ϕ^{stable} is that for stability. For a value assignment A to all variables, the sum of the violation costs and the number of the changes of assignments are given by a vector $\Phi(A) = (\phi^{opt}(A), \phi^{stable}(A))$. Solving a MO-NRP is to find Pareto optimal solutions so that

1. all hard constraints are satisfied,
2. the sum of the violation costs of soft constraints is minimized (i.e. optimality),
3. the number of the changes of assignments is minimized (i.e. stability).

In previous works on NRP, the aim is to find an assignment so that the number of the changes of assignments between current and modified schedules is minimized, i.e., solving a NRP is to find a stable solution. On the other hand, in a MO-NRP, bi-objectives are considered simultaneously, namely optimality and stability. In this framework, one can easily define several objective functions (i.e. $\phi^{opt_1}, \phi^{opt_2}, \ldots, \phi^{opt_p}$) instead of only one objective function ϕ^{opt} by considering each soft constraint as an objective function. For the simplicity, this paper defines ϕ^{opt} for optimality like classic NSP. Such simplification can be done by aggregating all objective functions which is called an AOF technique [17] (or in other words, linear sum and scalarization methods). Note that this technique can be utilized among objective functions for optimality and not for objective functions for optimality and stability, i.e., it makes no sense to aggregate the costs and the number of changes.

Definition 6 (s*-vector*) Let MS_{mod} be a modified master schedule. For a nurse i ($1 \leq i \leq n$), let s_i be the number of changes of assignments from a current master schedule to MS_{mod}. The number of changes of assignments for all nurses is said to be a s-vector w.r.t. MS_{mod} and denoted by $v_s = (s_1, ..., s_n)$.

Definition 7 (*Equivalence*) For two s-vectors $v_s = (s_1, ..., s_n)$ and $v_{s'} = (s'_1, ..., s'_n)$ w.r.t. MS_{mod}, v_s and $v_{s'}$ are said to be equivalent, iff it holds

$$\sum_{i=1}^{n} s_i = \sum_{i=1}^{n} s'_i$$

Let V_s be a set of equivalent s-vectors w.r.t. MS_{mod} and $\preceq_{lex}$ be the total preorder over V_s defined $\forall v_s, v_{s'} \in V_s$ as $v_s \preceq_{lex} v_{s'}$ if and only if lexically reordered v_s precedes lexically reordered $v_{s'}$. For example, let $v_s = (4, 1, 3, 2, 2)$ and $v_{s'} = (4, 0, 3, 2, 3)$ be two equivalent s-vectors (i.e. $\sum_{i=1}^{5} s_i = 4 + 1 + 3 + 2 + 2 = 12 = 4 + 0 + 3 + 2 + 3 = \sum_{i=1}^{5} s'_i$). The corresponding reordered vectors are $\overline{v_s} = (4, 3, 2, 2, 1)$ and $\overline{v_{s'}} = (4, 3, 3, 2, 0)$. Compare the 1st components of $\overline{v_s}$ and $\overline{v_{s'}}$. In case they are same, the 2nd components are compared. Continue to compare until one of two components is smaller than the another one. In this example, for the 3rd components, since 2 of $\overline{v_s}$ is smaller than 3 of $\overline{v'_s}$, the vector v_s is lexically smaller than v'_s (i.e. $v_s \preceq_{lex} v_{s'}$).

Definition 8 (*Egalitarianism*) For a modified master schedule MS_{mod} and a s-vector v_s w.r.t. MS_{mod}, v_s is said to be an egalitarian solution of MS_{mod}, iff there does not exist another equivalent s-vector $v_{s'}$ w.r.t. MS_{mod}, such that

$$v_{s'} \preceq_{lex} v_s,$$

i.e., minimizing the maximal number of changes among nurses.

Example 3 Consider the master schedules in Table 10.2. The MS_{mod} is the master schedule presented in Table 10.1 and the MS'_{mod} shows an alternative master schedule. The s-vectors v_s w.r.t. MS_{mod} and $v_{s'}$ w.r.t. MS'_{mod} are

$$v_s = (4, 0, 0, 0, 4, 0, 0), \quad v_{s'} = (2, 0, 1, 1, 4, 0, 0).$$

Since the number of changes of assignments is 8, the MS'_{mod} is also $r = 8$-stable, i.e., v_s and $v_{s'}$ are equivalent. The lexically reordered vectors of v_s and $v_{s'}$ are

$$\overline{v_s} = (4, 4, 0, 0, 0, 0, 0), \quad \overline{v_{s'}} = (4, 2, 1, 1, 0, 0, 0).$$

Table 10.2 MS_{mod} (left) is the modified schedule used in Example 2. MS'_{mod} (right) is an alternative modified schedule which is also 8-stable like MS_{mod}. The MS'_{mod} is more egalitarian than MS_{mod}

MS_{mod}							
Nurse (Level)	M	T	W	T	F	S	S
n_1 (l_1)	**m**	*m*	*m*	*m*	**n**	**o**	**m**
n_2 (l_2)	*e*	*e*	*n*	*o*	*m*	*m*	*m*
n_3 (l_3)	*m*	*m*	*m*	*e*	*e*	*n*	*o*
n_4 (l_3)	*m*	*e*	*e*	*n*	*o*	*m*	*n*
n_5 (l_4)	/	*m*	*e*	*e*	**e**	**e**	**e**
n_6 (l_4)	*n*	*n*	*o*	*m*	*m*	*e*	*e*
n_7 (l_5)	*e*	*o*	*m*	*m*	*m*	*m*	*m*

MS'_{mod}							
Nurse (Level)	M	T	W	T	F	S	S
n_1 (l_1)	**m**	*m*	*m*	*m*	*e*	**o**	*e*
n_2 (l_2)	*e*	*e*	*n*	*o*	*m*	*m*	*m*
n_3 (l_3)	*m*	*m*	*m*	*e*	**n**	*n*	*o*
n_4 (l_3)	*m*	*e*	*e*	*n*	*o*	*m*	**m**
n_5 (l_4)	/	*m*	*e*	*e*	**e**	**e**	**n**
n_6 (l_4)	*n*	*n*	*o*	*m*	*m*	*e*	*e*
n_7 (l_5)	*e*	*o*	*m*	*m*	*m*	*m*	*m*

The s-vector $v_{s'}$ w.r.t. MS'_{mod} is more egalitarian than v_s, i.e., $v_{s'} \preceq v_s$. Compared to MS_{mod}, four nurses (i.e. n_1, n_3, n_4 and n_5) share the changes of their shift works in MS'_{mod}, while only two nurses (i.e. n_1 and n_5) changes their assignments in MS_{mod}.

10.4 Experiments

In this section, an egalitarian solution for a MO-NRP is computed by using the Lp solver (Lp solve IDE 5.5.2.0). In the experiments, a master schedule is created. Then, the modified schedules are generated by absenting any nurse in the master schedule. The experimental setting is as follows.

- Period: one week (from Monday M to Sunday S).
- The number of nurses: 7 (n_1, n_2,..., n_7).
- Hard constraints
  - One should not assign a nurse for 7 consecutive works and 3 consecutive night shifts (H1).
  - At least 2 nurses must be assigned to the morning and 2 nurses for evening and 1 nurse for night shifts (H2).
  - For each nurse, the number of day-offs in a current schedule should be same in a modified schedule (H3).
  - Nurses must rest at least 16 hours between two consecutive shift works (H5).
- Soft constraint: In each day, at least one head nurse or one highly experienced nurse must be assigned (S1).
- Objective 1: Minimize the number of violations of the soft constraint.
- Objective 2: Minimize the number of changes between the master and modified schedules.

The following shows the Lp program we used in the experiments in order to compute the number of changes of assignments (i.e. objective 2). Let $N = \{i \mid 1, ..., n\}$ be a set of nurses, $M = \{j \mid 1 \leq j \leq m\}$ be a set of days in a schedule period, and $W = \{k \mid 1 \leq k \leq 4\}$ be a set of shift works, where 1=day-off, 2=morning, 3=evening and 4=night.

$$Minimize\ 1 - \sum x_{ijk} \tag{10.1}$$

subject to

$$\sum_{k \in W} x_{ijk} = 1 \tag{10.2}$$

$$x_{ijk} + x_{i(j+1)k} + x_{i(j+2)k} + x_{i(j+3)k} + x_{i(j+4)k} + x_{i(j+5)k} + x_{i(j+6)k} \leq 6 \tag{10.3}$$

$$x_{ij4} + x_{i(j+1)4} + x_{i(j+2)4} \leq 2 \tag{10.4}$$

$$\sum_{i \in N} x_{ij2} \geq 2, \quad \sum_{i \in N} x_{ij3} \geq 2, \quad \sum_{i \in N} x_{ij4} \geq 1 \tag{10.5}$$

$$x_{i'j'1} = 1 \tag{10.6}$$

(1) represents the objective function 2, i.e., minimize the number of changes between the master and modified schedules. (2) is the constraint that no one can work several shifts in a day, e.g., morning and evening in the same day. (3) and (4) represent the forbidden shift patters for H1, i.e., 7 consecutive works and 3 consecutive night shifts. (5) is the constraint for H2 and (6) shows that a nurse i' has unexpected absent on a day j'.

In the experiments, we aggregate the objective function 1 and 2 and find the optimal solution which minimizes the sum of the costs. Table 10.3 shows a master schedule which satisfies all hard constraints, and the number of violations of the soft constraint is zero. Table 10.4 represents two modified schedules we computed. The both schedules satisfy all hard constraints and the number of violations of the soft constraint is zero.

The s-vectors v_s w.r.t. MS_{mod} and $v_{s'}$ w.r.t. MS'_{mod} are

$$v_s = (0, 0, 0, 2, 0, 2, 2), \quad v_{s'} = (1, 2, 0, 1, 0, 2, 0).$$

Table 10.3 Master schedule for one week with 7 nurses. This schedule satisfies all hard constraints and the number of violations of the soft constraint is zero

$MS_{current}$: Master schedule							
Nurse Level	M	T	W	T	F	S	S
n_1 (l_1)	*m*	*o*	*n*	*e*	*o*	*m*	*m*
n_2 (l_2)	*o*	*m*	*m*	*o*	*m*	*m*	*m*
n_3 (l_3)	*e*	*e*	*o*	*m*	*m*	*o*	*e*
n_4 (l_3)	*n*	*o*	*m*	*m*	*e*	*o*	*e*
n_5 (l_4)	*m*	*m*	*e*	*o*	*e*	*e*	*o*
n_6 (l_4)	*e*	*e*	*o*	*n*	*n*	*e*	*o*
n_7 (l_5)	*o*	*n*	*e*	*e*	*o*	*n*	*n*

Table 10.4 Modified schedules where the nurse n_6 has unexpected absent

MS_{mod}							
Nurse (Level)	M	T	W	T	F	S	S
n_1 (l_1)	*m*	*o*	*n*	*e*	*o*	*m*	*m*
n_2 (l_2)	*o*	*m*	*m*	*o*	*m*	*m*	*m*
n_3 (l_3)	*e*	*e*	*o*	*m*	*m*	*o*	*e*
n_4 (l_3)	**e**	*o*	*m*	*m*	*e*	*o*	**n**
n_5 (l_4)	*m*	*m*	*e*	*o*	*e*	*e*	*o*
n_6 (l_4)	/	*e*	*o*	*n*	*n*	*e*	**e**
n_7 (l_5)	**n**	*n*	*e*	*e*	*o*	*n*	**o**

MS'_{mod}							
Nurse (Level)	M	T	W	T	F	S	S
n_1 (l_1)	**e**	*o*	*n*	*e*	*o*	*m*	*m*
n_2 (l_2)	**m**	*m*	*m*	*o*	*m*	*m*	**o**
n_3 (l_3)	*e*	*e*	*o*	*m*	*m*	*o*	*e*
n_4 (l_3)	*n*	*o*	*m*	*m*	*e*	*o*	**m**
n_5 (l_4)	*m*	*m*	*e*	*o*	*e*	*e*	*o*
n_6 (l_4)	/	*e*	*o*	*n*	*n*	*e*	**e**
n_7 (l_5)	*o*	*n*	*e*	*e*	*o*	*n*	*n*

Since the number of changes of assignments is 6 in both schedules (i.e. they are $r = 6$-stable), v_s and $v_{s'}$ are equivalent. Their lexically reordered vectors are

$$\overline{v_s} = (2, 2, 2, 0, 0, 0, 0), \quad \overline{v_{s'}} = (2, 2, 1, 1, 0, 0, 0).$$

The s-vector $v_{s'}$ w.r.t. MS'_{mod} is an egalitarian solution for this problem instance.

10.5 Related Work

Compared to NSP, there exists few works on NRP. Moz et al. [19] proposed two integer multicommodity flow models for a NRP. The first one is a directed multilevel acyclic network based model where the aim is to optimize an integer multicommodity flow in a multi-level network by adding some constraints. The other one is the extension of the first one which is an aggregation based model (i.e. aggregate the nodes of this network). They empirically showed that the second model outperforms the first, both the solution quality and runtime. Hattori et al. [11] formalized a dynamic NSP by using the framework of dynamic weighted MaxCSP which can effectively deal with dynamic changes to a problem. They introduced provisional constraints which allow variables to keep the same values so that one can obtain stable solutions that are close to previous ones. Pato et al. [21] worked on a utopic Pareto genetic heuristic which considers the trade-offs between two objectives, i.e., (i) minimize the gap between the number of scheduled duties and the number of duties each nurse should perform during the period, and (ii) minimize dissimilarity regarding the previously announced roster for the same period. Maenhout et al. [16] developed an evolutionary meta-heuristic which revises and re-optimizes a schedule for a set of heterogeneous nurses. Compared to these existing works, this paper focuses on NRP with multiple criteria and also the egalitarianism among nurses.

There exists very limited work on NSP with multiple criteria [5, 8]. The goal programming is the most widely used method where the aim is to find a solution which is as close as possible to each of the objectives in the order of the given priorities [3]. Others are the well-known tabu search based approach [5], Pareto simulated annealing approach based on the scalarization [13], modified harmony search [2] and adaptive neighborhood search [15]. Compare to these existing works, this paper focuses on a dynamic multi-objective NSP (i.e. MO-NRP).

NRP can be an application problem of Minimal Perturbation Problem (MPP) [10, 24] which is a dynamic CSP where the aim is to find a solution that minimizes a given distance function. The distance function measures the number of changing variables. Minimizing perturbations results in minimizing the number of changes in the assignment. Solving a MPP is finding a stable solution like NRP. Compared to MPP, this paper focuses on MO-NRP and also the egalitarianism among nurses.

10.6 Conclusion

In order to provide satisfactory patient care and potentially improve nurse retention, creating a good schedule for nurses and hospitals is important issue. NSP is a combinatorial optimization problem, in which a set of nurses must be assigned into a limited set of working slots, subject to a given set of hard and soft constraints. It is natural to consider the scheduled nurse's unexpected absence. NRP is a dynamic NSP where the aim is to reschedule the current roster after the unexpected absence of a nurse so that (i) all hard constraints are satisfied and (ii) the number of the changes of assignments between current and modified schedules is minimized. Most previous works on NRP focused on the stability, i.e., the new schedule should be similar to the current one as much as possible. The contribution of this paper is twofold:

- A formal framework of Multi-Objective Nurse Rerostering Problem (MO-NRP) is first defined by using the framework of a multi-objective weighted constraint satisfaction problem. The aim of a MO-NRP is to find trade-off solutions among "optimality" and "stability" of a modified schedule.

- A novel solution criterion in a MO-NRP is introduced, namely an egalitarian solution. By considering the egalitarianism among nurses, the following situation can be avoided; some nurses change their assignments a lot, while others not, i.e., in an egalitarian solution, the changes of assignments are shared among nurses.

As a perspective for further research, we intend to apply our approach to some real problems and analyze the trade-off solutions for a MO-NRP. More specifically, for existing NSP benchmarks in INRC-II (the second international nurse rostering competition), we model them as MO-NRP by assuming all soft constraints as objective functions and find an egalitarian solution. In order to solve the problems, we will use existing SAT/ASP solvers [4, 23]. We are also interested in multi-objective setting and egalitarian solutions in sport and transport timetables [18].

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